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TRANSITIONS INDUCED BY SUPERPOSED ELECTRIC AND MAGNETIC FIELDS IN CHOLESTERIC AND FERROCHOLESTERIC LIQUID CRYSTALS

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Se examinează tranziţiile (fero)colesteric-(fero)nematic induse de câmpuri electrice şi magnetice suprapuse în celule cu cristal lichid având diferite rapoarte de confinare. Plecând de la estimarea densităţii de energie liberă a (fero)colestericului cu anizotropie dielectrică şi magnetică pozitivă şi apoi utilizând ecuaţiile Euler-Lagrange s-a obţinut o relaţie generală între câmpurile electrice, magnetice si raportul de confinare. Aceasta reprezintă o diagramă de fază care dă valorile câmpurilor critice (electric şi magnetic) care corespund diferitelor rapoarte de confinare.

The (ferro)cholesteric-(ferro)nematic transitions induced by superposed electric and magnetic fields in liquid crystal cells with different confinement ratios are examined. Starting with the estimation of the free energy density of a (ferro)cholesteric with positive dielectric and magnetic anisotropies, and using Euler-Lagrange equations, a general relationship between electric and magnetic fields and confinement ratios was obtained. This represents a 3D phase diagram giving critical electric and magnetic field values corresponding to different confinement ratios.

Keywords: nematic liquid crystals, cholesteric liquid crystals, ferronematics, ferrocholesterics, confinement ratio

1. Introduction

Electro-optical properties of cholesteric liquid crystals are important for many technical applications such as liquid crystal displays (LCD), diffraction gratings, memory and bistability devices, etc. In all cases the cholesteric liquid crystal is confined between two glass plates covered with a thin conductive layer which has been previously processed for planar or homeotropic alignment.

As known, cholesteric liquid crystals display a rotational director configuration. When planar aligned cholesterics with positive electric/magnetic anisotropies are subjected to electric/magnetic fields the colesteric pitch increases

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and becomes infinite when the external field reaches a critical value. As the final liquid crystal texture is identical to that of a homeotropic oriented nematic, this phenomenon is called cholesteric-nematic transition. Theoretical estimations and experimental results [1-6] confirmed that the texture of a cholesteric liquid crystal strongly depends on the confinement ratio $r = d / p$ (*d* is the cell thickness and *p* is the cholesteric pitch).

When ferroparticles are embedded into nematic or cholesteric liquid crystals ferronematics and ferrocholesterics are obtained. Brochard and de Gennes [7, 8] were the first who studied the nematic mixtures containing ferroparticles. They assumed that the magnetic moments of the ferroparticles are aligned parallel to the director and the critical field for Freedericksz transition is dramatically decreased. This result was confirmed only for lyotropic liquid crystals. To overcome this difficulty Burylow and Raikher [9] developed another approach to explain the behavior of thermotropic ferronematics under magnetic fields.

The aim of this paper is to investigate (ferro)cholesteric-(ferro)nematic transitions under the action of both electric and magnetic fields when using liquid crystal cells with different confinement ratios.

2. Theoretical considerations

The cholesteric-nematic transition

The free energy density of cholesteric liquid crystal due to the action of both magnetic and electric fields may be expressed as

$$f = f_{elastic} + f_{magnetic} + f_{electric} \tag{1}$$

where

$$f_{elastic} = \left(\frac{\partial\theta}{\partial z}\right)^2 \left(\frac{K_1}{2}\cos^2\theta + \frac{K_3}{2}\sin^2\theta\right) + \left(\frac{\partial\varphi}{\partial z}\right)^2 \left(\frac{K_3}{2}\sin^2\theta + \frac{K_2}{2}\cos^2\theta\right)\cos^2\theta + \frac{K_2}{2}\left(-2\cos^2\theta \cdot q_o \frac{\partial\varphi}{\partial t} + q_0^2\right) \tag{2}$$

is the elastic free energy density of a planar aligned cholesteric liquid crystal,

$$f_{magnetic} = -\frac{1}{2}\mu_0^{-1}\chi_a B^2 \sin^2\theta \tag{3}$$

is the free energy density due to the magnetic field and

$$f_{electric} = -\frac{\varepsilon_0\varepsilon_a}{2} E^2 \sin^2\theta \tag{4}$$

is the free energy density due to the electric field .

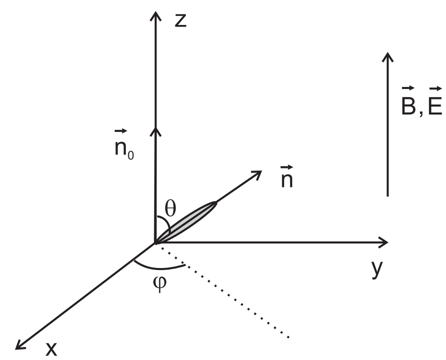

Fig.1 The molecular director under the electric and magnetic field

The Euler Lagrange equations are

$$\frac{d}{dz}\left(\frac{\partial f}{\partial \theta_z}\right)-\frac{\partial f}{\partial \theta}=0 \quad (5.1)$$

$$\frac{d}{dz}\left(\frac{\partial f}{\partial \varphi_z}\right)-\frac{\partial f}{\partial \varphi}=0 \quad (5.2)$$

where $\theta_z=\frac{\partial \theta}{\partial z}$ and $\varphi_z=\frac{\partial \varphi}{\partial z}$.

By solving Eq. (5.2) one obtains:

$$\varphi_z=\frac{K_2 q_0}{K_3\sin^2\theta+K_2\cos^2\theta} \quad (6)$$

After solving Eq. (5.1) and introducing Eq. (6) the free energy density may be written as:

$$f(\theta,\theta_z)=\frac{1}{2}\theta_z^2\left(K_1\cos^2\theta+K_3\sin^2\theta\right)-\frac{K_2^2q_0^2}{2\left(K_3\sin^2\theta+K_2\cos^2\theta\right)}\cos^2\theta$$
$$-\frac{1}{2}\mu_0^{-1}\chi_a B^2\sin^2\theta-\frac{1}{2}\varepsilon_0\varepsilon_a E^2\sin^2\theta+\frac{K_2q_0^2}{2} \quad (7)$$

In the vicinity of the transition point we may assume that $\theta=\frac{\pi}{2}-\xi$ where ξ is a very small angle. If higher order terms in ξ are neglected, Eq. (7) becomes:

$$f\left(\xi,\xi_z\right)=\frac{K_3}{2}\xi_z^2-\frac{\xi^2}{2}\left(\varepsilon_a E^2+\mu_0^{-1}B^2\chi_a-\frac{K_2q_o^2}{K_3}\right)+\frac{K_2^2q_0^2}{2}-\frac{\varepsilon_0\varepsilon_a E^2}{2}-\frac{\mu_0^{-1}B^2\chi_a}{2} \quad (8)$$

The corresponding Euler-Lagrange equation gives

$$K_3\frac{d^2\xi}{dz^2}-\left(\frac{K_2^2q_0^2}{K_3}-\mu_0^{-1}B^2\chi_a-\varepsilon_0\varepsilon_a E^2\right)\xi=0 \quad (9)$$

A solution of Eq. 9 satisfying the boundary conditions $\xi(\pm\frac{d}{2}) = 0$ is

$$\xi = \xi_0 \cos\frac{\pi z}{d} \tag{10}$$

After substituting it into Eq.(9) we get the equation

$$-\frac{K_3\pi^2}{d^2} = -\frac{K_2^2 q_0^2}{K_3} + \mu_0^{-1}B^2\chi_a + \varepsilon_0\varepsilon_a E^2 \tag{11}$$

Introducing the critical fields for magnetic and electric cholesteric-nematic transitions

$$B_c^2 = \frac{K_3\pi^2}{\mu_0^{-1}\chi_a d^2}, E_c^2 = \frac{K_3\pi^2}{\varepsilon_0\varepsilon_a d^2} \tag{12}$$

and the confinement ratio

$$r = \frac{d}{p} = \frac{dq_0}{2\pi}$$

one obtains

$$\frac{r^2}{\frac{K_3^2}{4K_2^2}} - \frac{B^2}{B_c^2} - \frac{E^2}{E_c^2} = 1 \tag{13}$$

The equation represents a hyperboloid separating two domains in a 3D space. It may also be considered a phase diagram separating the homeotropic nematic phase and the homogeneous cholesteric one.

Using the following material parameters: $\chi_a = 7\times10^{-7}$, $K_1 = 17,2\times10^{12}$ N, $K_2 = 7,5\times10^{-12}$ N, $K_3 = 17,9\times10^{-12}$ N and $d = 400$ µm the hyperboloid given by Eq.(13) was plotted. (Fig.2)

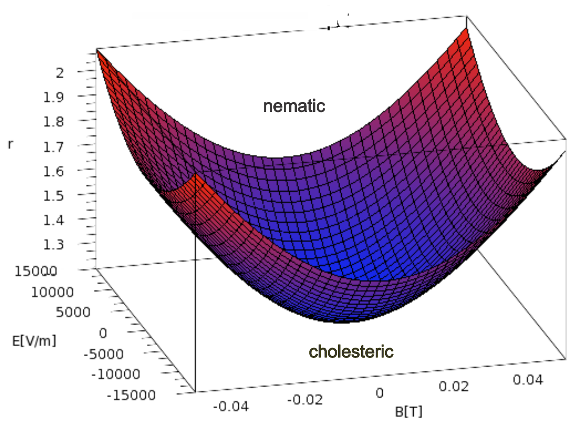

Fig.2 The 3D phase diagram giving critical fields for cholesteric-nematic transition and the corresponding confinement ratio.

Using this plot the critical fields E_c and B_c corresponding to a known confinement ratio may be determined. When $E < E_c$, $B < B_c$, we get the homogeneous (planar) cholesteric phase and for $E > E_c$, $B > B_c$ the homeotropic nematic phase.

The ferrocholesteric-ferronematic transition

In the case of ferrocholesteric liquid crystals, we have to consider the contribution of the ferroparticles to the free energy density.

According to Brochard and de Gennes, the magnetic contribution of the ferroparticle to the free energy density is:

$$f_m = -M_s f \vec{m}\vec{B} \qquad (14)$$

where M_s is the saturation magnetization of the particles, f is the volume fraction of magnetic particles and $\vec{m}$ is the magnetic moment of ferroparticles.

Assuming a negligible interaction between magnetic particles embedded into the liquid crystal, the entropic contribution to the free energy has to be considered.

$$f_s = \frac{f k_B T}{V} \ln f \qquad (15)$$

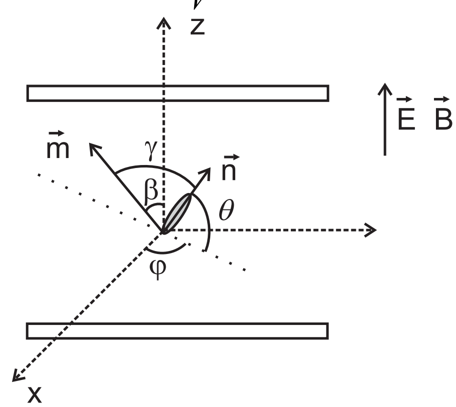

Fig.3. Orientations of nematic director and magnetic moment of magnetic particle in a ferrocholesteric

In Eq.(15) V is the volume of the mixture.

In order to fit the experimental data Burylov and Raikher [9] considered that the interaction energy W between ferroparticles and nematic mixture has a finite value; consequently, a new contribution to the free energy density was considered:

$$f_a = \frac{fW}{a}\left(\vec{n}\vec{m}\right)^2 \qquad (16)$$

where a is the ferroparticle mean diameter.

Considering Eqs.(14-16) the free energy density of the ferrocholesteric due to the magnetic particles is:

$$f = -M_s\left(\vec{m}\vec{B}\right) + f\frac{W}{a}\left(\vec{n}\vec{m}\right)^2 + \left(\frac{fk_BT}{V}\right)\ln f \qquad (17)$$

Therefore the free energy desity of the ferrocholesteric is

$$f_{ferro} = f_{elastic} + f_{electric} + f_{magnetic} + f_{ferroparticle} \qquad (18)$$

The procedure used to determine the critical electric and magnetic fields as a function of the confinement ratios, is similar to the one presented before for the cholesteric-nematic transition. Therefore, using Eqs. 2, 3, 4 and 17 the free energy density of the planar aligned ferrocholesteric is

$$f_v(\theta) = (\theta_z)^2\left(\frac{K_1}{2}\cos^2\theta + \frac{K_3}{2}\sin^2\theta\right) + (\varphi_z)^2\left(\frac{K_3}{2}\sin^2\theta + \frac{K_2}{2}\cos^2\theta\right)\cos^2\theta +$$
$$+\frac{K_2}{2}\left(-2\cos\theta q_0\frac{\partial\varphi}{\partial z} + q_0^2\right) - \frac{1}{2}\mu_0^{-1}\chi_a B\sin\theta - \frac{1}{2}\varepsilon_0\varepsilon_a E^2\sin^2\theta - M_s fB\cos\beta + \qquad (19)$$
$$+f\frac{W}{a}\left[\cos\theta\sin\beta\left(-\cos\varphi\cos\gamma - \sin\varphi\sin\gamma\right) + \cos\beta\sin\theta\right]^2 + \left(\frac{fk_BT}{V}\right)\ln f$$

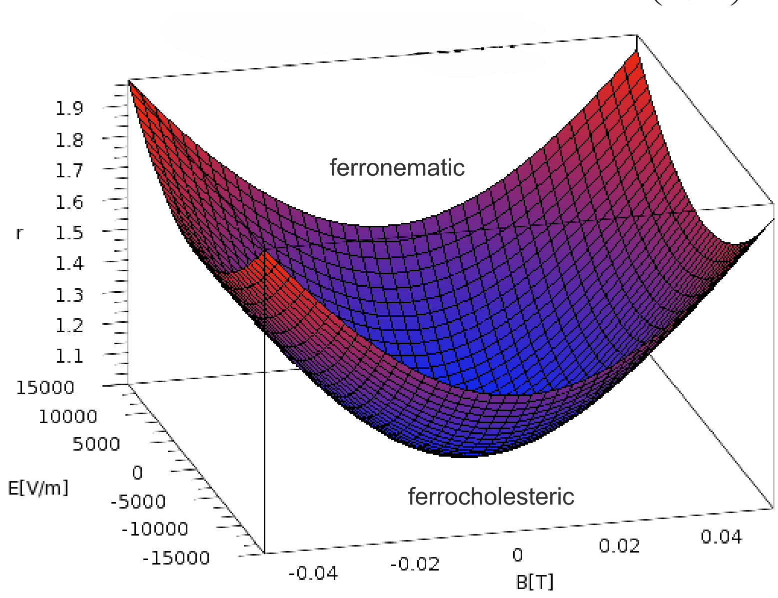

Fig.4 A phase diagram separating ferronematic and ferrocholesteric domains

The significance of β, θ, γ and φ is given in Fig. 3.

Using the Euler Lagrange equations for β, γ and φ and assuming the magnetic moment of the ferroparticle to be perpendicular to the molecular director, a simplified relationship for the free energy density is obtained:

$$f_v\left(\theta,\theta_z\right)=\frac{1}{2}\theta_z^2\left(K_1\cos^2\theta+K_3\sin\theta\right)-\frac{K_2^2q_0^2}{2\left(K_3\sin^2\theta+K_2\cos^2\theta\right)}\cos^2\theta-$$
$$-M_s fB+\left(f\frac{W}{a}-\frac{1}{2}\mu_0^{-1}\chi_a B^2-\frac{1}{2}\varepsilon_0\varepsilon_a E^2\right)\sin^2\theta+\frac{K_2q_0^2}{2}+\frac{fk_BT}{V}\ln f \quad (20)$$

As before, in the vicinity of the transition point $\theta=\pi/2-\xi$, a new relationship, similar to Eq.7 is obtained. Its corresponding Euler-Lagrange is

$$K_3\frac{d^2\xi}{dz^2}-\left(2A+\frac{k_2^2q_0^2}{K_3}\right)\xi=0 \quad (21)$$

where

$$A=f\frac{W}{a}-\frac{1}{2}\mu_0^{-1}B^2\chi_a-\frac{1}{2}\varepsilon_0\varepsilon_a E^2\sin^2\theta \quad (22)$$

Introducing the solution (10) into Eq.22 a new relationship, similar to Eq.11 is obtained

$$\frac{r^2}{\left(\frac{K_3}{2K_2}\right)^2}-\frac{E^2}{E_c^2}-\frac{B^2}{B_c^2}=1-\frac{2fWd^2}{K_3\pi^2 a} \quad (23)$$

Noting with

$$s^2=1-\frac{2fWd^2}{K_3\pi^2 a} \quad (24)$$

eq. 23 becomes

$$\frac{r^2}{\left(s\frac{K_3}{2K_2}\right)^2}-\frac{E^2}{\left(sE_c\right)^2}-\frac{B^2}{\left(sB_c\right)^2}=1 \quad (25)$$

Eq. 25, similar to eq. 13 also represents a hyperboloid whose parameters are modified by a factor s defined by (24).

In this case the critical fields E_c and B_c are lower when compared to those obtained when the ferrocholesteric-ferronematic transition is involved.

3. Conclusions

The critical fields for (ferro)cholesteric-(ferro)nematic transitions are dependent on confinement ratios. When both elctric and magnetic fields are acted on a (ferro)cholesteric the (ferro)nematic phase is reached using electric and magnetic fields with strenghts lower than those needed in case of using only an unique field. These new critical values may be determined using phase diagrams similar to those given in Fig.3 or Fig.4.

R E F E R E N C E S

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