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AN APPROACH ON MISSION ANALYSIS

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During the early phases of aircraft design, there is the need to validate the configuration, based on intended missions and implications in operating cost.

The paper presents a method for the fuel planning type of mission analysis – to obtain the quantity of fuel, given the payload and distance, taking into account a mission profile, regulations and operational options. The reference vertical flight profile is presented, along with hypotheses.

Brief theoretical considerations regarding the main segments are presented showing the models used for obtaining the data and a work algorithm. Examples of analysis are presented for a small commercial aircraft theoretical model.

Keywords: fuel planning, mission analysis, fuel reserves.

Nomenclature

x – horizontal distance (m);
h – altitude (m);
V, Ve – true airspeed, equivalent airspeed (m/s);
m, mf – aircraft mass, fuel mass (kg);
T – thrust (N);
L – aerodynamic lift (N);
D – aerodynamic drag (N);
γ – flight path angle (rad);
α – angle of attack (rad);
θ – pitch angle (rad);
χ – engine setting (%);
SR, SE – specific range (m/kg), specific endurance (s/kg);
ISA – standard atmospheric conditions.

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1. Introduction

There are two types of mission analysis, range planning and fuel planning.

The range planning estimates the distance capability, given the fuel quantity and payload. The fuel planning estimates the fuel quantity, given the distance and payload. It is also called aircraft sizing, being based on a mission profile [4].

The reference mission is composed of several segments, with a set of theoretical models and hypotheses associated for each segment.

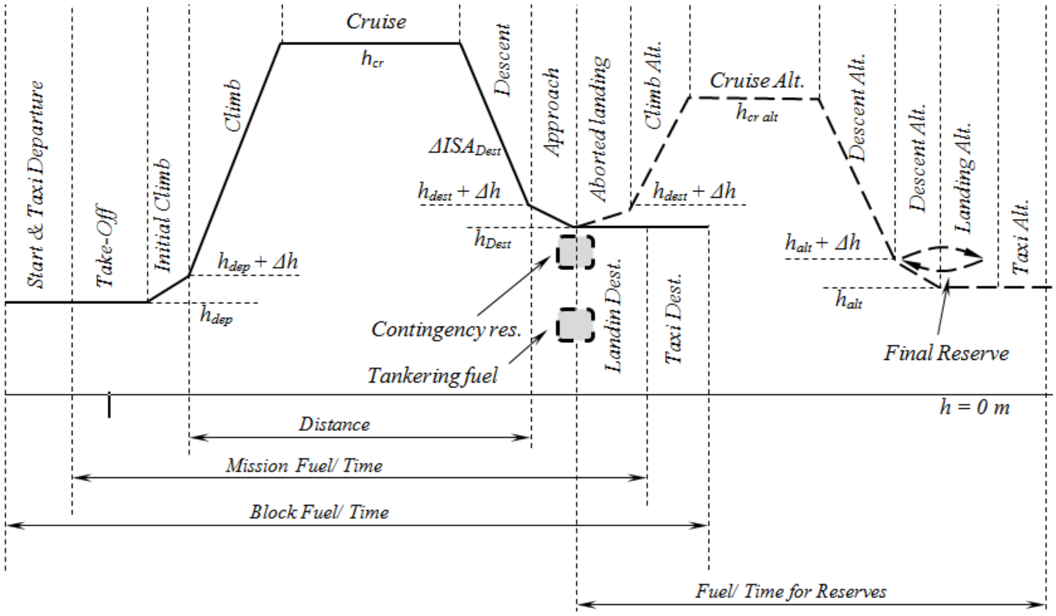

Fig. 1 The typical mission profile

The mission includes the take-off, an initial climb with transition to cruise configuration, the climb, cruise, descent, approach and landing. Additional segments account for engines start and taxi at departure and destination.

In the mission analysis, allowance for several types of reserves must be considered as follows: flight to an alternate airport after an aborted landing at destination, being treated as a secondary mission with its own segments definition; a time flown in specified condition (or hold time); percent of the mission fuel (or contingency).

Also, the carrying of fuel (tankering) for subsequent missions is sometimes justified economically [4,11,14,25]. Description of typical missions are found in [1,2,4,8,9,11,24]. The official definition of reserves is given in [18,19,20].

All the segments are taken into account for computation of fuel quantity and time. The segments for climb, cruise and descent are considered to have contribution for total distance to the destination airport [8]. A similar judgment is made for the flight to the alternate airport.

The paper presents a mission analysis approach based on the synthesis of the mission segments in an iterative algorithm, which can be configured for various mission profiles and fuel reserves structures. The mission segments are modelled in order to simulate common flight techniques. The mission analysis presented allows to determine the fuel needed for a specified mission and can be used also for other studies linked to mission performance.

2. The theoretical model

For trajectory analysis, the climb, cruise and descent phases are studied with the 3DoF model (point mass) for flight in a vertical plane in wind axes.

The assumption is for low rotation rates and negligible influence of the control surfaces on aerodynamic forces. Hence, the force (translation) equations can be separated from the moment (rotation) equations and used as 3DoF model.

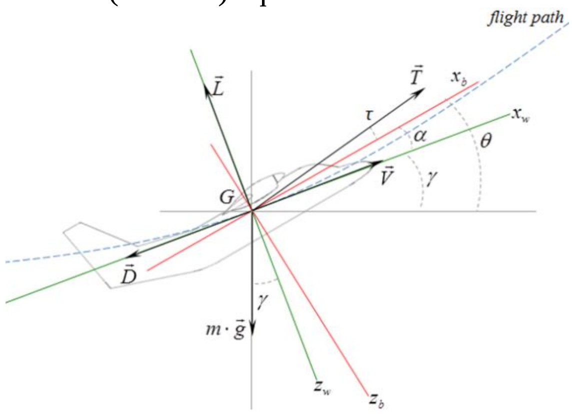

Fig. 2 Forces and angles definition – wind and body axes

The general form of the 3DoF equations for flight in the vertical plane is:

$$\begin{cases} \left.\begin{array}{l} \dot{x} = V \cdot \cos\gamma \\ \dot{h} = V \cdot \sin\gamma \end{array}\right\} \text{kinematic equations} \\ \left.\begin{array}{l} \dot{V} = \dfrac{1}{m} \cdot \left(T \cdot \cos(\alpha + \tau) - D - m \cdot g \cdot \sin\gamma\right) \\ \dot{\gamma} = \dfrac{1}{m \cdot V} \cdot \left(T \cdot \sin(\alpha + \tau) + L - m \cdot g \cdot \cos\gamma\right) \end{array}\right\} \text{dynamic equations} \\ \left.\dot{m} = -\dot{m}f\right\} \text{mass equation} \end{cases} \tag{1}$$

with the additional relationship $\theta = \gamma + \alpha$.

The equations of flight in the vertical plane for 6DoF and 3DoF are treated in [1,2,3,7,10,12,15,16,22,23,24]. The 3DoF model is used also in other mission or trajectory analysis models [23,24].

The aerodynamic forces are expressed as $L = L(h, V, \alpha)$, $D = D(h, V, \alpha)$ and the thrust and fuel flow are expressed as $T = T(h, V, \chi)$, $\dot{m}f = \dot{m}f(h, V, \chi)$.

2.1 The climb and descent with constant equivalent speed

In current operations the most common climb technique is with constant equivalent air speed (EAS), $V_e = ct$. The engine setting is set at maximum during the climb. The climb with constant EAS is a particular case of accelerated flight.

The true air speed (TAS) is variable with altitude, so we consider acceleration along the trajectory [2,3,15,16].

The true airspeed is [21] $V(h) = \dfrac{V_e}{\sqrt{\dfrac{\rho(h)}{\rho_0}}} = \dfrac{V_e}{\sqrt{\sigma(h)}}$. (2)

The hypotheses made are for small flight path angle, angle of attack and neglijible normal acceleration. For a constant temperature ($\Delta ISA = ct$) the 3DoF model (1) that describes the accelerated flight becomes:

$$\begin{cases} \dot{x} = V \\ \dot{h} = V \cdot \gamma \\ \dot{V} = \dfrac{1}{m} \cdot (T - D - m \cdot g \cdot \gamma) \\ 0 = L - m \cdot g \\ \dot{m} = -\dot{m}f \end{cases} \tag{3}$$

Obtaining α from the fourth algebraic equation and eliminating it from the third equation we have $D = D(h, V, m)$. Further, having $\dot{V} = \dfrac{dV}{dt} = \dfrac{dV}{dh} \cdot \dfrac{dh}{dt}$ the system (3) has the form:

$$\begin{cases} \dot{x} = V \\ \dot{h} = \dfrac{V}{m \cdot g} \cdot \dfrac{T - D}{1 + \dfrac{V}{g} \cdot \dfrac{dV}{dh}} \\ \dot{m} = -\dot{m}f \end{cases} \tag{4}$$

The system (3) can also be written with the altitude as integration variable:

$$\begin{cases} \dfrac{dx}{dh} = \dfrac{m \cdot g}{T - D} \cdot \left(1 + \dfrac{V}{g}\dfrac{dV}{dh}\right) \\ \dfrac{dt}{dh} = \dfrac{m \cdot g}{V} \cdot \dfrac{1 + \dfrac{V}{g}\dfrac{dV}{dh}}{T - D} \\ \dfrac{dm}{dh} = -\dot{m}f \cdot \dfrac{m \cdot g}{V} \cdot \dfrac{1 + \dfrac{V}{g}\dfrac{dV}{dh}}{T - D} \end{cases} \tag{5}$$

in which $V = V(h)$, $T = T(h, V, \chi_{\max})$, $D = D(h, V, m)$ and the derivative $\dfrac{dV}{dh}$ depends on *h*:

$$\frac{dV}{dh} = \frac{dV}{d\sigma} \cdot \frac{d\sigma}{dh} = -\frac{1}{2} \cdot \sigma(h)^{-\frac{3}{2}} \cdot V_e \cdot \frac{d\sigma}{dh}. \tag{6}$$

An example for climb is computed for the theoretical model of a small aircraft with constant EAS kept on altitude intervals. In practice it is common for small-medium commercial aircraft to use several constant EAS values on altitude intervals during climb.

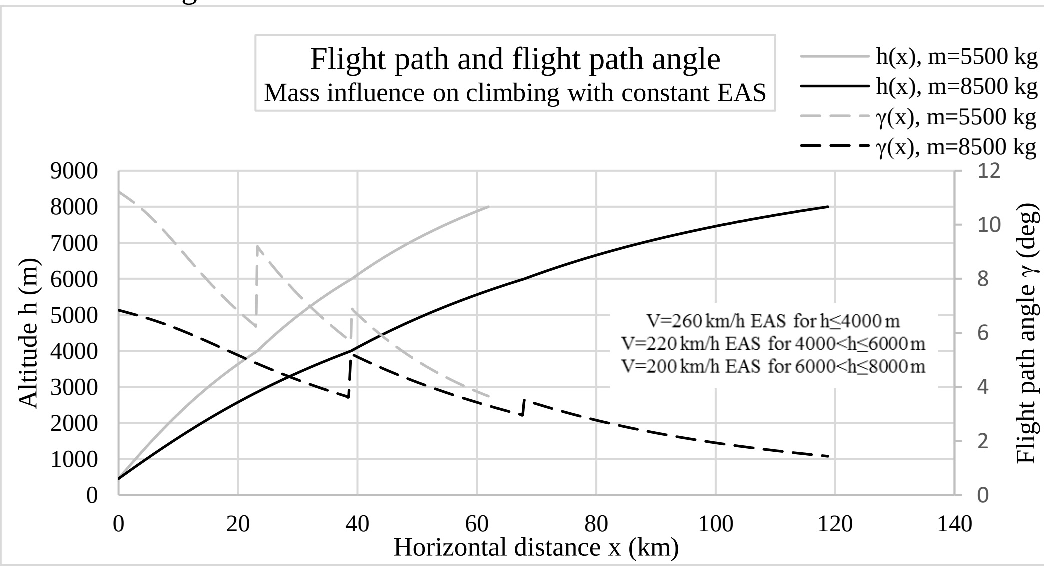

Fig. 3 The mass influence on climbing with constant EAS

The descent with constant EAS is treated in a similar way, the difference being the engine setting (a reduced χ is chosen for low traction).

2.2 The constant altitude cruise

For the quasi-steady level flight the equations of motion (1) become (for $\Delta ISA = ct$) [1,2,3,4,5,13]:

$$\begin{cases} \dot{x} = V \\ 0 = T - D \\ 0 = L - m \cdot g \\ \dot{m} = -\dot{m}f \end{cases} \tag{7}$$

The aerodynamic forces are expressed as $L = L(h, V, \alpha)$ and $D = D(h, V, \alpha)$ and the thrust and fuel flow are expressed as $T = T(h, V, \chi)$ and $\dot{m}_{fuel} = \dot{m}_{fuel}(h, V, \chi)$.

The distance and mass equations can be written with mass as integration variable:

$$\begin{cases} -\dfrac{dx}{dm} = \dfrac{V}{\dot{m}f} \\ -\dfrac{dt}{dm} = \dfrac{1}{\dot{m}f} \end{cases} \tag{8}$$

where $\dot{m}f = \dot{m}f(h, V, m)$ and $V = V(m)$.

It is seen that the speed schedule $V(m)$ must be known in order to integrate the above system.

The concepts of Specific Range and Specific Endurance are introduced [2,3,4,6,8,13,15]:

$$\begin{cases} SR = -\dfrac{dx}{dm} = \dfrac{V}{\dot{m}f} \\ SE = -\dfrac{dt}{dm} = \dfrac{1}{\dot{m}f} \end{cases} \tag{9}$$

The Specific Range (or Range Factor) $SR(h, V, m)$ is defined as the (air) distance flown per unit of fuel mass, and the Specific Endurance (or Endurance Factor) $SE(h, V, m)$ is defined as the time flown per unit of fuel mass.

From the other two equations, with a point performance analysis for a number of altitudes and masses, the speed and fuel flow are obtained for the available traction range [1,2,3,4] and examples are presented below.

Choosing the points of maximum *SR* on each $m = ct$ curve gives the speed profile *V(m)* for maximum range, hence the *SR(m)* function. The common cruise technique is the "long range" (at 99% maximum range) which offers speed benefit for a small loss of range. Also constant speed, constant engine setting or maximum cruise speed profiles are used. A similar plot for $SE(V,m)$ for $h = ct$ and $\Delta ISA = ct$ will lead to the maximum endurance technique, used when maximizing time is necessary.

Below an example is computed for the theoretical model of a small commercial aircraft at *h*=4000 m.

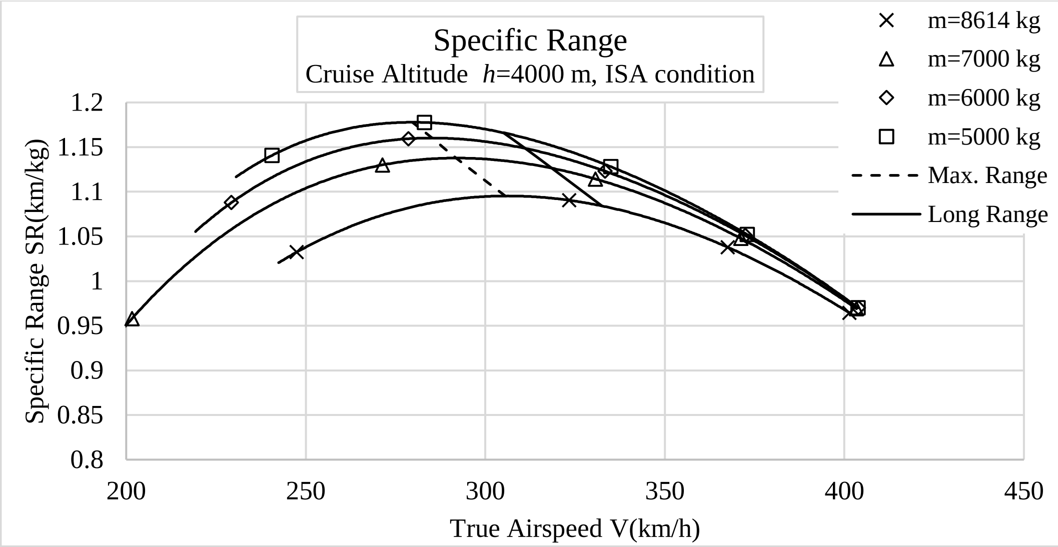

Fig. 3 The Specific Range *SR(V,m)*, *h*=4000 m, *ISA* condition

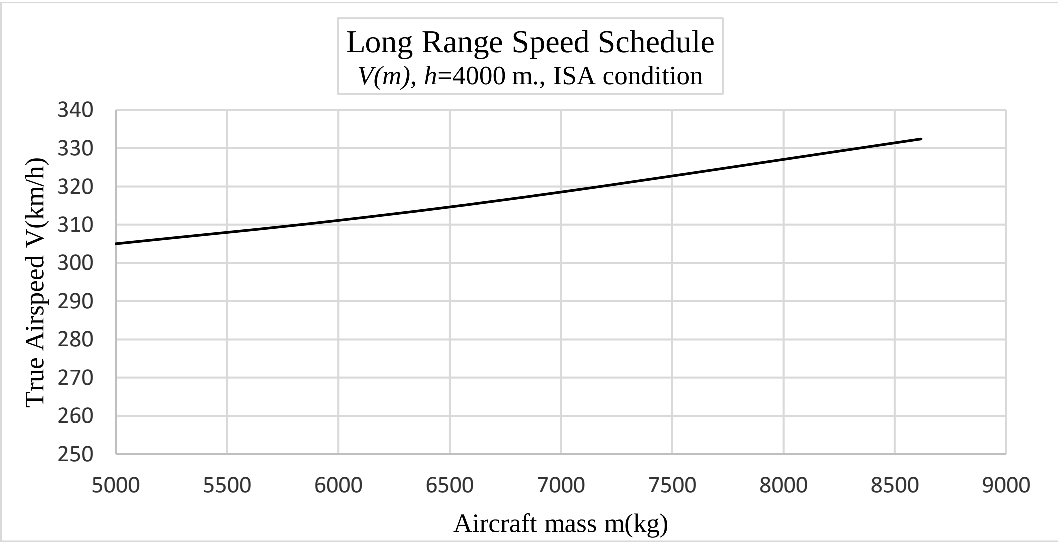

Fig. 4 The long Range speed profile at *h*=4000 m

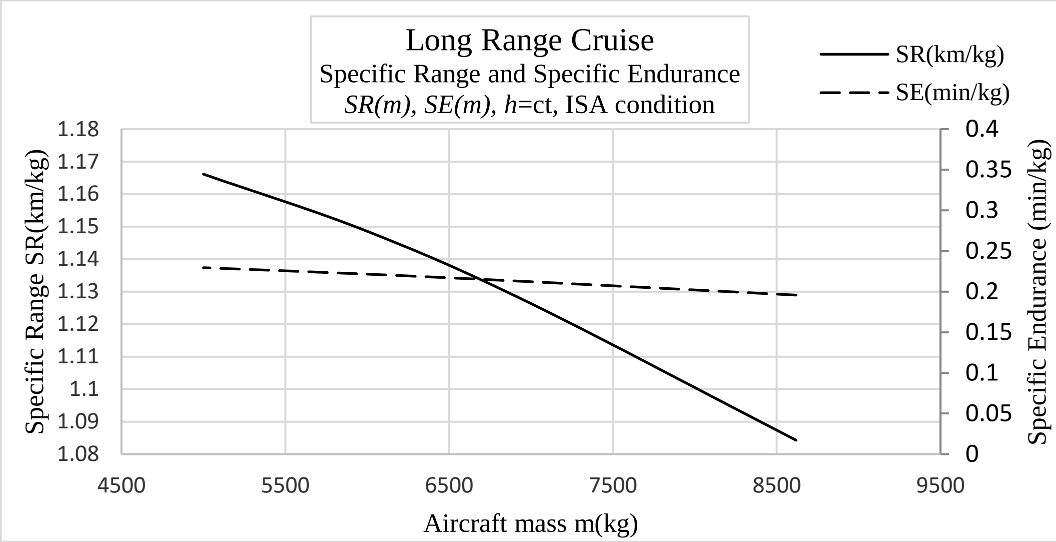

Fig. 5 The *SR* and *SE* for the Long Range speed profile, *h*=4000 m

For the known speed profile *V(m)* (Fig. 5), the distance and time can be obtained by integration [2,3,6,13,16]:

$$x=\int_{m_{fin}}^{m_{init}} SR(h,V,m)dm\,;\; t=\int_{m_{fin}}^{m_{init}} SE(h,V,m)dm\,;\; h=ct. \tag{10}$$

3. The mission analysis algorithm

An iterative algorythm is proposed for the fuel planning (aircraft sizing) problem, as a tool for mission analysis, taking into account the reserves structure.

The total fuel mass mf_{total} is initialized with a value up to maximum.

For the cruise there is the issue of obtaining a fuel mass so that, after the subsequent descent segment, to obtain the imposed distance x_{dest}. The fuel quantity for cruise is not known beforehand in the context of the whole mission, the following segments depending on this value. An initial value for the cruise fuel mass is allocated. The iterative cycle modifies the value mf_{cr} until convergence of the total distance is obtained, taking into account the segments that contribute to distance (climb, cruise, descent). The mass at the end of descent and the mission fuel determine the value for the contingency reserve $mf_{resCont}$.

The landing mass m_{land} at the Destination airport is the initializing parameter for the computations of segments for the secondary mission (flight to the alternate airport). The cruise fuel mass mf_{crAlt} is modified for the given distance convergence. After the convergence of fuel for the alternate flight is met, the sum of segment fuel masses gives the fuel for the flight to alternate airport mf_{resAlt}. The mass after descent determines the value of fuel mass for the final reserve $m_{resFinAlt}$.

The mission fuel for destination and the reserves give the total fuel mass in the current iteration. If the value of the total fuel mass computed is close to the previous computed value, then convergence is considered for mf_{total}.

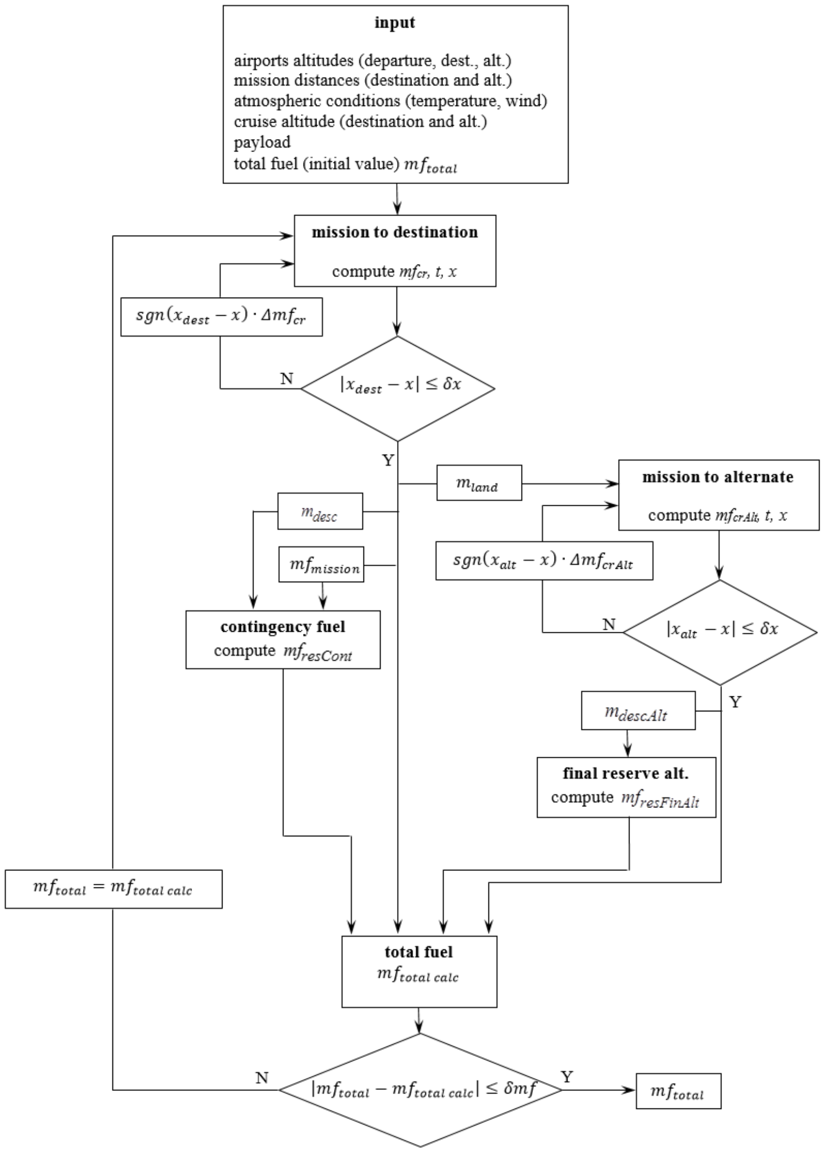

Fig. 6 The algorithm for fuel planning – simplified schematic

4. Examples

The theoretical model for a small commercial aircraft in CS-23 category [17] was used to demonstrate the method proposed for mission analysis.

The cruise altitude is investigated for short distance missions.

For each mission, the cruise altitude is chosen to minimize the mission fuel, while for long missions the cruise altitude is the maximum cruise altitude.

The cruise altitude for minimum mission fuel, for a given mission distance, is obtained from a parametric study. An example is shown for a distance of 150 km, where the cruise altitude for minimum mission fuel is 5500 m.

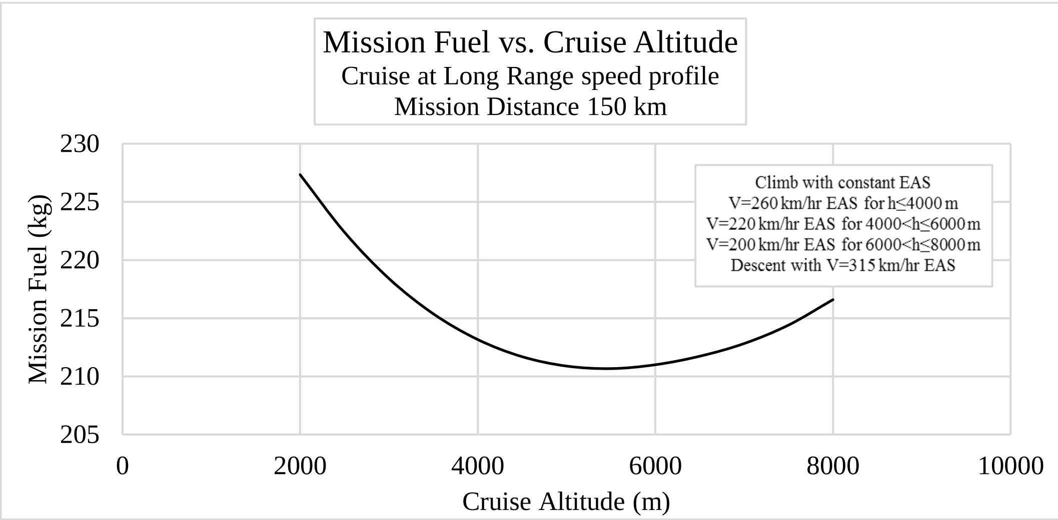

Fig. 7 Mission Fuel vs. Cruise Altitude for x=150 km

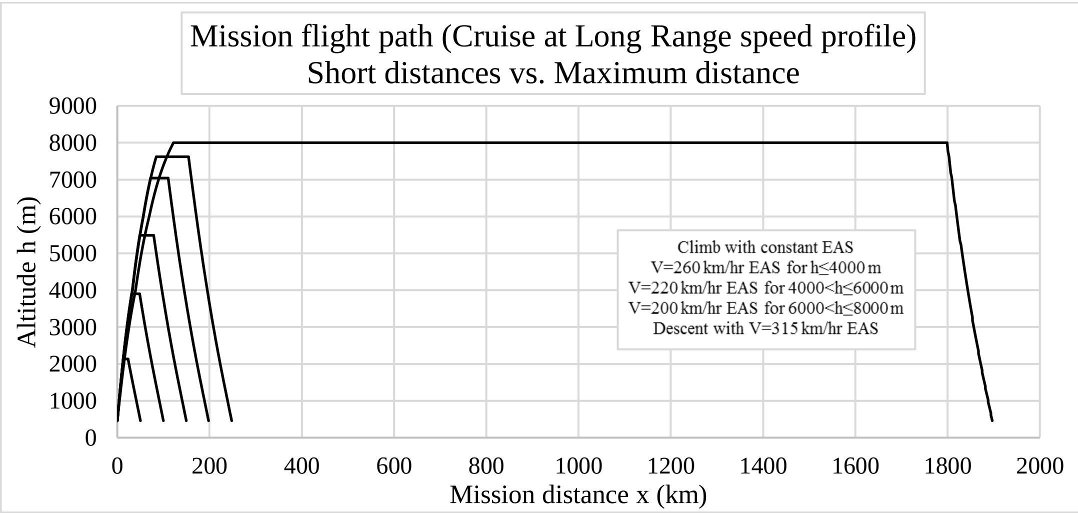

Fig. 8 Mission flight paths vs. distance

The influence of mission distance on the main flight segments fuel ratio as percentage of block fuel was investigated for the aircraft theoretical model.

On short distances, a relative large quantity is allocated for climb and descent, compared with the cruise fuel requirements, while on long distances the major part of block fuel is made up from cruise fuel.

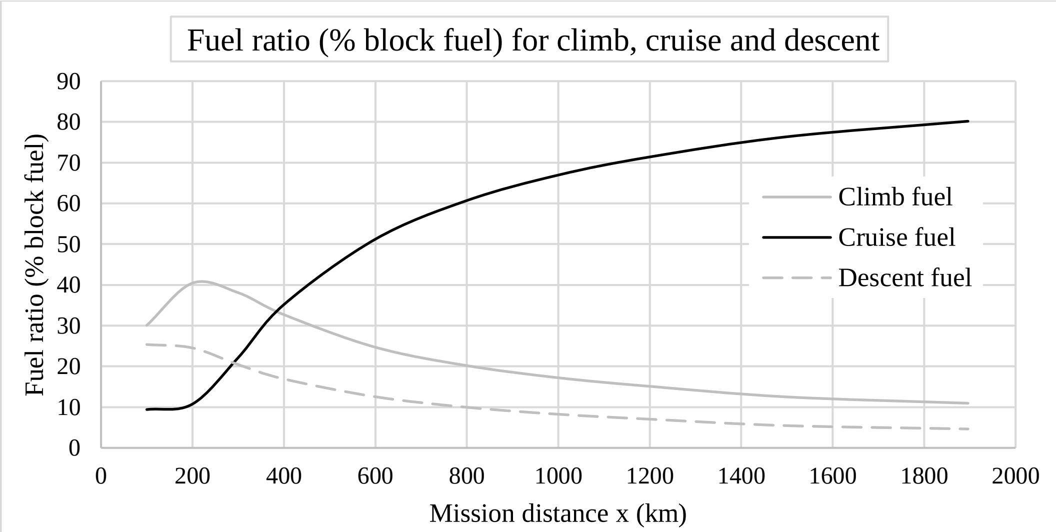

Fig. 9 Fuel ratios for flight segments as percentage of block fuel

The examples were computed with reserve for flight to alternate airport (50 km), final reserve above the alternate airport and contingency reserve. The cruise segments were computed at Long Range setting. The atmospheric condition are ISA, no wind.

5. Conclusion

The numerical simulations for typical flight techniques and a mission analysis algorithm are proposed, based on a standard mission configuration. The algorithm takes into account the different categories of fuel reserves.

Accurate data can be obtained for the whole mission and its segments - the fuel weight, time and distance - to assist in fuel planning, general sizing of the aircraft model and parametric studies.

In the implementation of the algorithm, the use of precomputed data can be used instead of actual computing for the flight segments. The precomputed performance data can be generated in similar format with the data that can be directly extracted from flight manuals of existing aircraft. The method presented is a tool for early critical evaluations in showing the strenghts and weaknesses of aircraft projects in the context of mission performance.

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