MODIFIED STATE ESTIMATION IN PRESENCE OF PMU MEASUREMENTS

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State estimation is a key function of the Energy Management System (EMS), providing a reliable and consistent data, by processing the information obtained from telemetering or remote units. Traditional power system state estimators are subject to important changes because of the extensive application of Phasor Measurement Unit (PMU).

In this paper a methodology of including PMU measurements in traditional state estimation process among SCADA measurements is proposed. A MATLAB application was developed and simulations were done on IEEE 30 bus system.

Keywords: State estimation, Phase measurement, PMU

1. Introduction

State estimation is a methodology that provides the best possible approximation for the state of a system by processing the available information [1]. In power systems as available information which is considered input there are provided:

- *network model*, implies topology, lines and transformers characteristics (resistance, reactance);

- *measurements* of power flows (real and reactive power flow through the transmission lines), power injections (real and reactive power injected at the buses), voltage magnitude (voltage magnitude measurements at the buses), current magnitude (current magnitude flowing through the transmission lines), etc. [2];

- *circuit breaker status* information, used also to establish the topology of the network.

Static or steady state operation of a power system can be fully determined with a minimal set of physical values called state variables which are the components of the state vector *X.* For a *n* bus power system the state vector is

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defined as a *2n-1* dimension real vector. Its component consists of voltage magnitudes from all buses and voltage phase angles from *n-1* buses. Until recently, this variable could not be obtained by direct measurement, it could only be determined from computation.

Before the real-time phase measurement unit (PMU) was introduced, power system state estimation had been only relying on the measurements provided from SCADA system. With the introduction of the PMU devices, now it can be considered that the power system state can be directly measured, because the voltage magnitude along with voltage angle can be determined directly, and this with high precision also. These advantages which affect and contribute on the traditional methods of power system state estimation are the research focus in the power system state estimation field [3]. Synchronized Phasor Measurements can be in form of voltage phasors and current phasors (angle and module).

Synchronized phasor measurements from PMU devices are provided with GPS-synchronized time-stamp, conventional SCADA measurements are only provided with a local time stamp.

For the power system state estimation, time-synchronized phasor measurements from PMU may be included in the traditional non-linear weighted least squares with performing some small adjustments or they may be taken into consideration after a preliminary system state has already been determined. It is recognized that a small number of these precise measurements can weigh heavily on the accuracy of the overall state of the system.

With both SCADA and PMU measurements, there are different methods to determine the power system state estimation. An example as presented in [3], taking the PMU data as true value to participate to the power system state estimation then solving the linear power system state estimation where PMU data makes the network completely observable. Another method is to use directly the PMU's linear measurement equations or to change the PMU data into power flow (or current) equations to accomplish the power system state estimation, this method is more advisable when the amount of PMU data is enough.

The application implemented in MATLAB follows the algorithm presented further in this paper. To test the application, simulations were done on IEEE 30 bus system and the results were compared with valid power flow results.

2. State estimation with only SCADA measurements

The measurement equation in the power system estimation can be formulated as follows [3, 4, 5]:

$$z = h(x) + \varepsilon \qquad (1)$$

where

z is the measurements vector (its elements usually contains measurements of the magnitude of bus voltage, bus injection of active power or reactive power, active or reactive power flow on transmission lines / power transformers);

x - the state variable vector (consists of the magnitude and angle of the bus voltage);

$h(x)$ - the vector of nonlinear functions relating measurements to the state vector x;

ε - the measurement error vector, which is assumed to have zero mean and constant variance σ^2, calculated to reflect the expected accuracy of the corresponding meter used.

Considering the WLS method, the objective function can be expressed as follows [5]:

$$J(x) = \left(z - h(x)\right)^T \cdot R^{-1} \cdot \left(z - h(x)\right) \tag{2}$$

where R^{-1} is the diagonal weighting matrix of measurement variance and it is given by

$$Cov(e) = E(e \cdot e^T) = R = \begin{bmatrix} \sigma_1^2 & & & \\ & \sigma_2^2 & & \\ & & \cdots & \\ & & & \sigma_m^2 \end{bmatrix} = diag\left(\sigma_i^2\right) \tag{3}$$

where m is the number of measurements.

At first an initial guess of the state variables is required, and then the final values of the state variables are determined by the Gauss Newton iteration. The iteration terminates when the state variances meet the convergence limit. The iteration equation is shown as follows [2, 5]:

$$\Delta x = \left(H^T R^{-1} H\right)^{-1} H^T R^{-1} \left(z - h\left(x^k\right)\right) \tag{4}$$

$$x^{k+1} = x^k + \Delta x$$

where k is the iteration number, $k = 0$ means the initial conditions, and $k = 1, 2, \ldots$ depends on the setting of the convergence limit.

H is the *Jacobian* matrix from the $h(x)$ given state variable values, H will change at each iteration:

$$H=\frac{\partial h(x)}{\partial x}=\begin{bmatrix}\frac{\partial h_1(x)}{\partial x_1} & \frac{\partial h_1(x)}{\partial x_2} & \cdots & \frac{\partial h_1(x)}{\partial x_n}\\ \frac{\partial h_2(x)}{\partial x_1} & \frac{\partial h_2(x)}{\partial x_2} & \cdots & \frac{\partial h_2(x)}{\partial x_n}\\ \vdots & \vdots & \cdots & \vdots\\ \frac{\partial h_m(x)}{\partial x_1} & \frac{\partial h_m(x)}{\partial x_2} & \cdots & \frac{\partial h_m(x)}{\partial x_n}\end{bmatrix} \tag{5}$$

where n is the number of the state variables.

If the power system has N buses, then $n=2N-1$, they are corresponding to the voltage magnitude and angle respectively, and the angle is zero as reference bus that is not included [3,4].

The structure of the measurement Jacobian H will be as follows [5]:

$$H=\begin{bmatrix}\frac{\partial P_{inj}}{\partial \theta} & \frac{\partial P_{inj}}{\partial V}\\ \frac{\partial P_{flow}}{\partial \theta} & \frac{\partial P_{flow}}{\partial V}\\ \frac{\partial Q_{inj}}{\partial \theta} & \frac{\partial Q_{inj}}{\partial V}\\ \frac{\partial Q_{flow}}{\partial \theta} & \frac{\partial Q_{flow}}{\partial V}\\ \frac{\partial I_{ij}}{\partial \theta} & \frac{\partial I_{ij}}{\partial V}\\ 0 & \frac{\partial V_i}{\partial V}\end{bmatrix} \tag{6}$$

The expressions for each Jacobian element are given below:

- elements corresponding to real power injection measurements:

$$\frac{\partial P_i}{\partial \theta_i};\qquad \frac{\partial P_i}{\partial \theta_j};\qquad \frac{\partial P_i}{\partial V_i};\qquad \frac{\partial P_i}{\partial V_j} \tag{7}$$

- elements corresponding to reactive power injection measurements:

$$\frac{\partial Q_i}{\partial \theta_i};\qquad \frac{\partial Q_i}{\partial \theta_j};\qquad \frac{\partial Q_i}{\partial V_i};\qquad \frac{\partial Q_i}{\partial V_j} \tag{8}$$

-elements corresponding to real power flow measurements:

$$\frac{\partial P_{ij}}{\partial \theta_i};\qquad \frac{\partial P_{ij}}{\partial \theta_j};\qquad \frac{\partial P_{ij}}{\partial V_i};\qquad \frac{\partial P_{ij}}{\partial V_j} \tag{9}$$

- elements corresponding to reactive power flow measurements:

$$\frac{\partial Q_{ij}}{\partial \theta_i}; \qquad \frac{\partial Q_{ij}}{\partial \theta_j}; \qquad \frac{\partial Q_{ij}}{\partial V_i}; \qquad \frac{\partial Q_{ij}}{\partial V_j} \tag{10}$$

- elements corresponding to voltage magnitude measurements:

$$\frac{\partial V_i}{\partial \theta_i} = 0 \qquad \frac{\partial V_i}{\partial \theta_j} = 0 \qquad \frac{\partial V_i}{\partial V_i} = 1 \qquad \frac{\partial V_i}{\partial V_j} = 0 \tag{11}$$

- elements corresponding to current magnitude measurements (ignoring the shunt admittance of the branch) :

$$\frac{\partial I_{ij}}{\partial \theta_i} = \frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} V_i \cdot V_j \cdot \sin\theta_{ij}; \quad \frac{\partial I_{ij}}{\partial \theta_j} = -\frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} V_i \cdot V_j \cdot \sin\theta_{ij} \quad \text{(12,a,b)}$$

$$\frac{\partial I_{ij}}{\partial V_i} = \frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} \left(V_i - V_j \cdot \cos\theta_{ij}\right); \quad \frac{\partial I_{ij}}{\partial V_j} = \frac{g_{ij}^2 + b_{ij}^2}{I_{ij}} \left(V_j - V_i \cdot \cos\theta_{ij}\right) \text{(12,c,d)}$$

where V_i is the voltage magnitude at bus *i*.

θ_i the phase angle at bus *i*.

$\theta_{ij} = \theta_i - \theta_j$

$G_{ij}+jB_{ij}$ the ij^{th} element of the complex bus admittance matrix.

$g_{ij}+jb_{ij}$ the admittance of the series branch connecting buses *i* and *j*.

$g_{si}+jb_{si}$ the admittance of the shunt branch connected at bus *i*.

Gain matrix is formed using the measurement Jacobian H and the measurement error covariance matrix, R, as follows [5]:

$$G = H^T R^{-1} H \tag{13}$$

The gain matrix G is very important for the power system state estimation algorithm, it is sparse, positive definite and symmetric provided if the system is fully observable.

3. Modified State Estimation with SCADA and PMU Measurements

A generalized formulation of a modified state estimator uses both conventional and synchronized phasor measurements for estimating the state of the system.

Therefore, the state estimation problem can be divided in a two step algorithm, within the first step there are processed only not-synchronized SCADA measurements, using the results of this step along with the time-synchronized PMU measurements in the second step the state estimation solution is much improved.

The considered state estimation model which has been implemented in *Matlab* environment is based on a linear measurement model composed of traditional state estimate augmented by PMU measurements of the following form:

$$Z_{aug} = H_{aug} \cdot \hat{V} + \varepsilon \tag{14}$$

where

H is the measurement Jacobian coefficient matrix;

$\hat{V}$ - state vector of real and imaginary components of bus voltages, $[V_{Re};V_{Im}]^T$;

ε - the vector of errors in the measurements.

The measurement vector **Z** is composed of:

- the state output from the classical state estimator calculated from the first step $[V_{Re};V_{Im}]_{SE}^T$;
- PMU voltage measurements $[V_{Re};V_{Im}]_{PMU}^T$;
- PMU current measurements $[I_{Re};I_{Im}]_{PMU}^T$.

The augmented measurement model is shown below with all voltages expressed in rectangular coordinates; the subscript *Re* and *Im* denotes the real and imaginary components of the voltages and current measurements [6].

$$Z_{aug} = \begin{bmatrix} \begin{bmatrix} V_{Re} \\ V_{Im} \end{bmatrix}_{SE} \\ \begin{bmatrix} V_{Re} \\ V_{Im} \end{bmatrix}_{PMU} \\ \begin{bmatrix} I_{Re} \\ I_{Im} \end{bmatrix}_{PMU} \end{bmatrix} = \begin{bmatrix} H_{11} & O_{12} \\ O_{21} & H_{22} \\ H_{31} & O_{32} \\ O_{41} & H_{42} \\ H_{51} & H_{52} \\ H_{61} & H_{62} \end{bmatrix}_{aug} \cdot \begin{bmatrix} V_{Re} \\ V_{Im} \end{bmatrix} + \begin{bmatrix} \varepsilon_{V_{Re}}^{SE} \\ \varepsilon_{V_{Im}}^{SE} \\ \varepsilon_{V_{Re}}^{PMU} \\ \varepsilon_{V_{Im}}^{PMU} \\ \varepsilon_{I_{Re}}^{PMU} \\ \varepsilon_{I_{Im}}^{PMU} \end{bmatrix} \tag{15}$$

where H_{11} and H_{22} are unit matrixes;

O_{12}, O_{21}, O_{32} and O_{41} are zero matrixes;

H_{31} and H_{42} have only one nonzero element that is 1 in every row;

H_{31} is a $P \times N$ matrix, P being the number of PMUs. Each row i corresponds to PMU i and has all zeros except at the j^{th} column corresponding to the index of $V = V_{Re,i}^{PMU}$ in the state vector.

$$H_{31,i} = \begin{bmatrix} 0 & \dots & 0 & \overset{j^{\text{th}}}{1} & 0 & \dots & 0 \end{bmatrix} \tag{16}$$

H_{42} is a $P \times N$ matrix, P being the number of PMUs. Each row i corresponds to PMU i and has all zeros except at the j^{th} column corresponding to the index of $V_{Im,i}^{PMU}$ in the state vector.

$$H_{42,i} = \begin{bmatrix} 0 & \dots & 0 & \overset{j^{\text{th}}}{1} & 0 & \dots & 0 \end{bmatrix} \tag{17}$$

The elements in H_{51} , H_{52} , H_{61} and H_{62} are relative to the admittance of the branches:

$$H_{51} = \frac{\partial I_{Re}^{PMU}}{\partial V_{Re}} \qquad H_{52} = \frac{\partial I_{Re}^{PMU}}{\partial V_{Im}}$$
$$H_{61} = \frac{\partial I_{Im}^{PMU}}{\partial V_{Re}} \qquad H_{62} = \frac{\partial I_{Im}^{PMU}}{\partial V_{Im}} \tag{18}$$

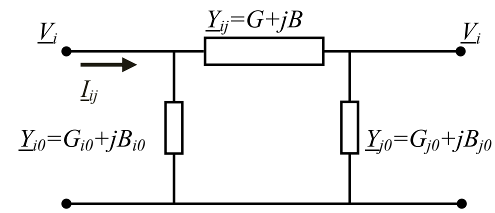

Fig. 1. Transmission branch π model

At this point we will develop the elements of H corresponding to the PMU current measurements. Given that we have the π model of a transmission branch in figure 1, the current $\underline{I}_{ij}$ is expressed as [7]:

$$\underline{I}_{ij} = \underline{V}_i \cdot \underline{Y}_{io} + \left(\underline{V}_i - \underline{V}_j\right)\underline{Y}_{ij} \tag{19}$$

Converting everything into rectangular components yields

$$I_{ij,Re} = \left(G + G_{i0}\right)V_{i,Re} - \left(B + B_{i0}\right)V_{i,Im} - GV_{j,Re} + BV_{j,Im} \tag{20}$$

$$I_{ij,Im} = \left(B + B_{i0}\right)V_{i,Re} + \left(G + G_{i0}\right)V_{i,Im} - BV_{j,Re} - GV_{j,Im} \tag{21}$$

the elements would be:

$$\frac{\partial I_{ij,Re}}{\partial V_{i,Re}} = G + G_{io} \qquad \frac{\partial I_{ij,Re}}{\partial V_{i,Im}} = -\left(B + B_{io}\right)$$
$$\frac{\partial I_{ij,Re}}{\partial V_{j,Re}} = -G \qquad \frac{\partial I_{ij,Re}}{\partial V_{j,Im}} = B \qquad \text{(22}$$

Similar expressions can be derived for

$$\frac{\partial I_{ij,Im}}{\partial V_{i,Re}} = B + B_{io} \qquad \frac{\partial I_{ij,Im}}{\partial V_{i,Im}} = G + G_{io}$$
$$\frac{\partial I_{ij,Im}}{\partial V_{j,Re}} = -B \qquad \frac{\partial I_{ij,Im}}{\partial V_{j,Im}} = -G \tag{23}$$

The elements of H_{51} and H_{62} are made up of real parts of the branch admittance, and the elements of H_{52} and H_{61} are made up of imaginary parts of the branch admittance.

So the linear state estimation is solved by the following equation [7]:

$$\hat{V} = \left(H_{aug}{}^{T} R^{-1} H_{aug}\right)^{-1} H_{aug}{}^{T} R^{-1} Z_{aug} \tag{24}$$

4. Study Case

The presented state estimation method was developed in the MATLAB environment (Fig. 2.). To test the developed application the simulations have been carried out on IEEE 30 bus system. The measurements source was considered a valid load flow solution of the test system and to simulate more realistic "noisy" measurements, noise (Gaussian random variable, zero mean unit variance) has been added to the perfect measurements. The measurement error variance σ^2, is assigned to each measurement type to reflect the expected accuracy of the meter used. These values are used as weights in the diagonal matrix R^{-1}.

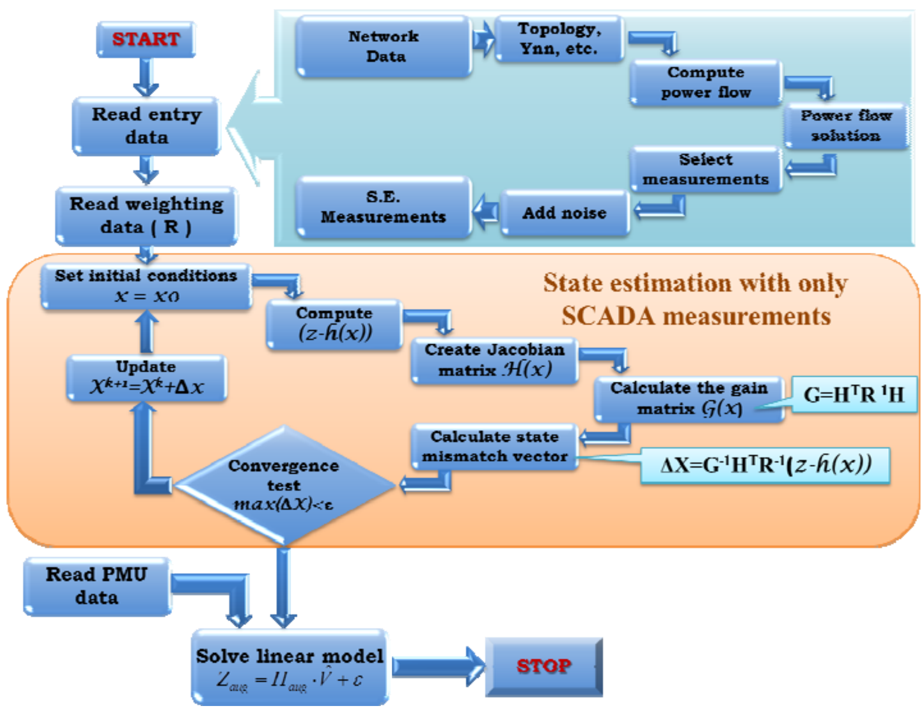

Fig. 2. Modified state estimation flow chart

Assumed values of the variance σ^2 depending on the measurement type are given in Table 1 [8].

Table 1

Measurements Variance

Type #	Measurement type	Variance σ^2
1	V magnitude	$1 \cdot 10^{-2}$
2	P injection	$3 \cdot 10^{-2}$
3	Q injection	$3 \cdot 10^{-2}$
4	P flow	$3 \cdot 10^{-2}$
5	Q flow	$3 \cdot 10^{-2}$
6	PMU measurements	$1 \cdot 10^{-4}$

For practical implementation, there should be enough redundancy in measurement throughout the network. Degree of redundancy is usually expressed in terms as ratio of number of meters to number of states and it is a very important quantity, more redundant measurements give more chances for bad data to be detected.

The one-line diagram of the IEEE 30-bus network with PMU placement is illustrated in Fig. 3; this system data was obtained from the Power Systems Test Case Archive – UWEE, (University of Washington, EE department) [9].

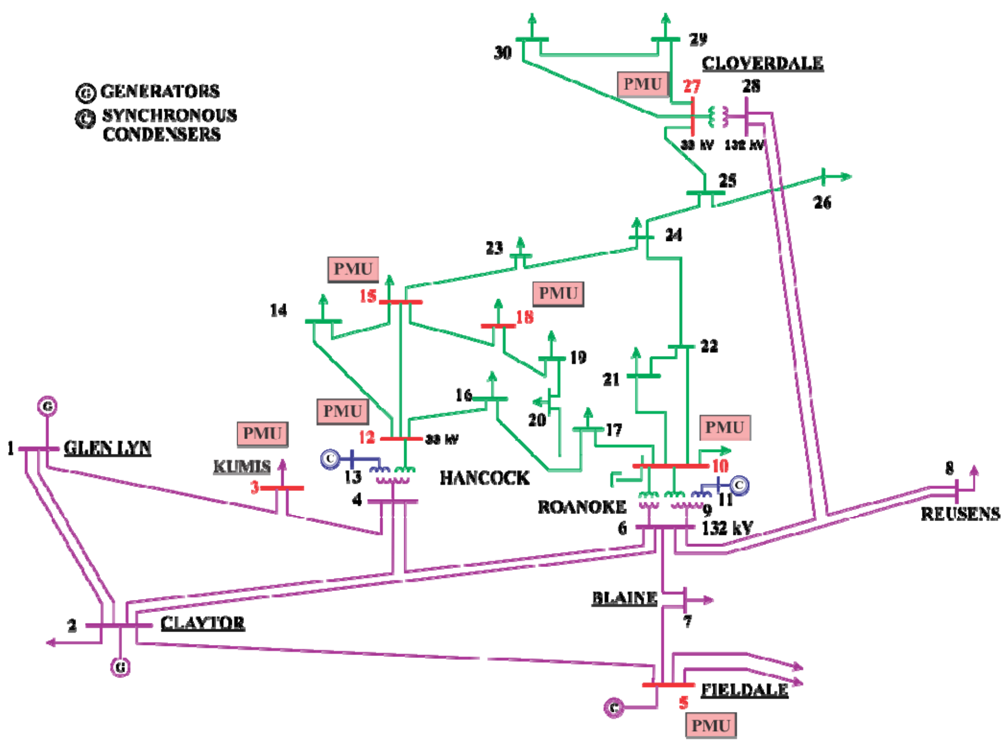

Fig. 3. IEEE 30 bus system with PMU placement

For the *IEEE 30* bus system the angle and voltage magnitudes errors are presented as differences from the nominal values, in figure 5 and 6. Comparing the results with the real state of the system from the power flow solution, the modified method is more accurate. In this case it was assumed that phasor measurement units are placed, according to bisecting search-simulated annealing method (Fig. 4.) which is proposed in [10] and [12], in the following 7 buses: 3, 5, 10, 12, 15, 18 and 27.

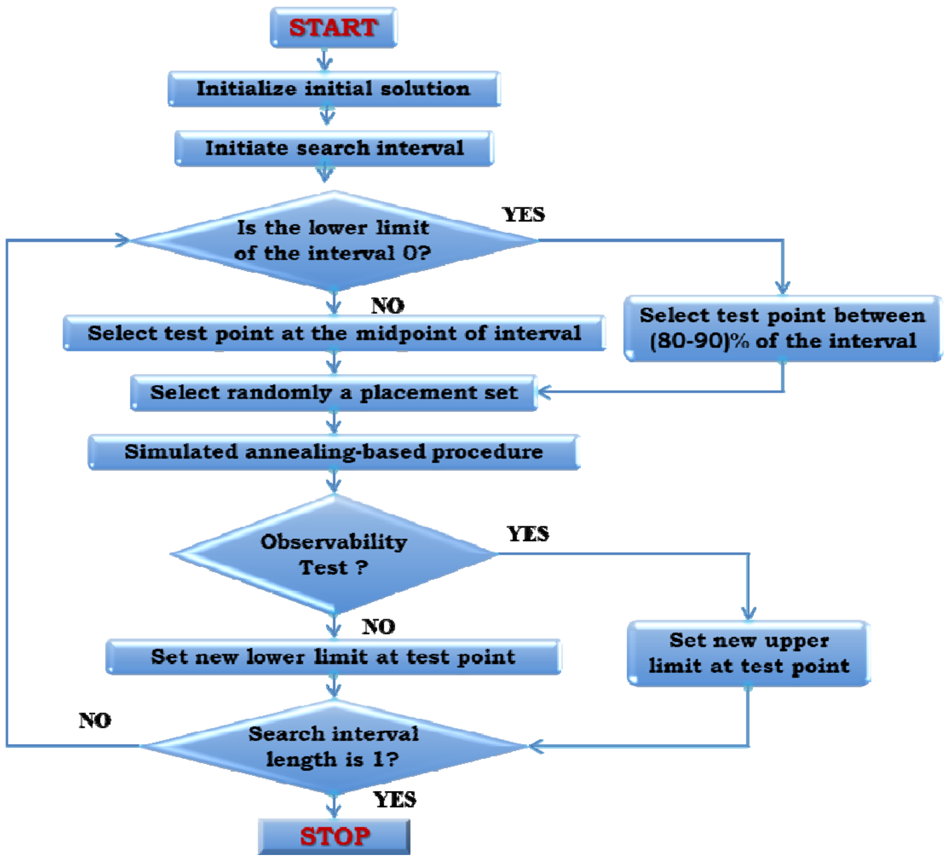

Fig. 4. Flowchart of the Bisecting Search

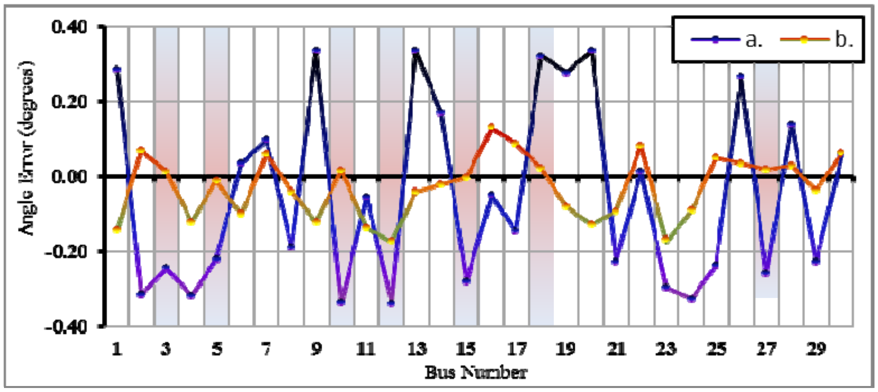

Fig. 5. Error in voltage angle estimation for the 30 bus network:
a. using traditional state estimation data; b. with phasor data added

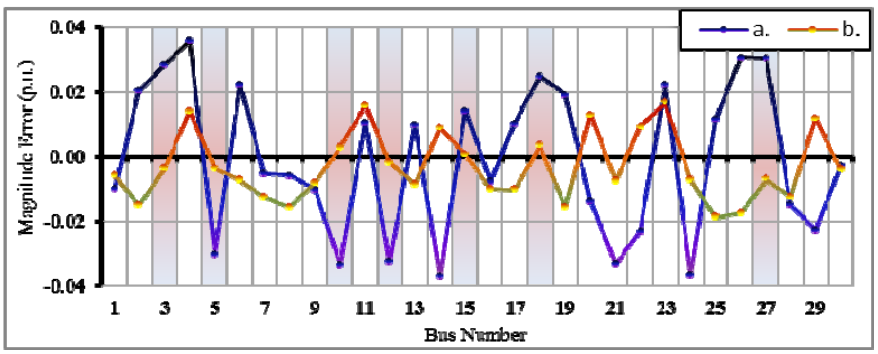

Fig. 6. Error in voltage magnitude estimation for the 30 bus network:
a. using traditional state estimation data; b. with phasor data added

5. Conclusions

An improved algorithm for integrating the PMU measurement in the classical state estimator was proposed in this paper. It has been shown that when synchronized phasor measurements are added to the other SCADA measurements in sufficient numbers, the efficiency/precision of the state estimate is improved.

From the simulation results carried out with developed MATLAB application, the hybrid state estimation results are better than traditional state estimator results. But the difference in accuracy of results may become significant if the amount of PMU measurements is increased. The flexibility offered by hybrid state estimator makes it more favorable over traditional state estimator as it does not require major changes to existing state estimators.

The hybrid state estimator is formulated in such a way that it can be easily modified in case of changes in the measurement configuration.

It should be highlighted that this paper handles only one specific aspect of PMU applications in state estimation, the inclusion of synchronized phasor measurements data in the state estimation process. The traditional state estimator is considered to be functioning normally in the absence of PMU data, it is considered that the existing SCADA system provides measurements in sufficient numbers with proper placement so that the state estimator is able to handle bad data and provide complete observability based on those measurements.

It is recognized that synchronized phasor measurements can be used to handle some deficiencies in the traditional measurement set, for example, to improve network observability, to aid in bad data processing, and in determining network topology.

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