FINITE TIME CONTROL OF NONLINEAR PERMANENT MAGNET SYNCHRONOUS MOTOR

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The finite time control of nonlinear permanent magnet synchronous motor is studied in this paper. Firstly, the nonlinear vibration of permanent magnet synchronous motor is analyzed. Then, on the basis of finite time stability theory, a novel terminal sliding mode control method is proposed for the vibration control of nonlinear permanent magnet synchronous motor. Furthermore, the controller designed only needs two terms, which is one less than the conventional sliding mode control method and accordingly easy for implementation. Finally, numerical simulations have verified the effectiveness and superiority when compared with the existing method.

Keywords: permanent magnet synchronous motor, vol. finite time stability, vol. terminal sliding mode, vol. nonlinear control

1. Introduction

Due to the direct applications in many areas especially for industrial applications in low-medium power range, since it has excellent features such as simple structure, high torque-to-inertia ratio, fast response and high efficiency [1-2], permanent magnet synchronous motor, no. PMSM) has been widely used in modern high-performance servo systems [3-5]. Especially in recent years, with the rapid development of permanent magnetic materials, power electronics technology and new motor control theory, PMSM will increasingly replace the conventional motor, which has great application potential. However, PMSM may exhibit nonlinear oscillation under certain conditions, which is harmful to the safe and stable operation of the system [6-8]. Recently, nonlinear analysis and control of PMSM has attracted many scholars' attention [9-11].

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There have been many results about the nonlinear analysis of PMSM until now. For PMSM nonlinear control, some schemes also have been proposed. For instance, on the basis of a novel sliding-mode observer, a sensor-less speed control scheme for the PMSM of ship propulsion interior is proposed in [12]. A fuzzy control method via linear matrix inequalities for fractional-order PMSM is presented in [13]. And effective predictive control scheme for PMSM which not only has the fast response of a direct torque control but also could reduce the torque ripples effectively is studied in [14]. An adaptive dynamic surface control method for the undesirable chaotic vibration of PMSM is first proposed in [15]. However, most of the mentioned control methods are based on the asymptotic stability theory, which need infinite time to realize the control objectives theoretically. Well known, the transition time and overshoot are essential for the control quality of practical engineering. From the perspective of optimizing the control time, finite time stability theory based control methods should be studied, which has good performance on improving the transition time, overshoot and oscillation frequency [16-19]. Well known, terminal sliding mode, no. TSM) control can stabilize the nonlinear systems in a finite time [20-21]. Could TSM be applied to the vibration control of PMSM? If possible, what are the application conditions and controller expressions? Relevant reports are quite few. It is challenging and worth studying.

On the basis of the above study, there are several advantages of our study. Firstly, based on the finite time stability theory, a novel TSM control method is proposed for the vibration control of nonlinear PMSM. Furthermore, it only needs two control terms, which is one less than the conventional sliding mode method and accordingly easy for implementation. Finally, simulation results are presented to verify the effectiveness and superiority compared with existing method.

The rest of the paper is organized as follows. In section 2, the PMSM system is shown. Section 3 presents the design of finite time terminal sliding mode controller. Simulation results are given in Section 4. Section 5 ends this paper.

2. System description

PMSM is a time varying, nonlinear and strong coupling system. By using affine transformation and time scale transformation, the dimensionless form can be described as [22]:

$$\begin{cases} \dot{x} = -x + yz \\ \dot{y} = -y - xz + az \\ \dot{z} = b \times (y - z) \end{cases} \tag{1}$$

where x and y are the stator currents, z is the rotor angular frequency, a=20, b=5.46. Fig. 1 shows the phase trajectory and time domain of PMSM, equation (1) with initial value $x(0)=0.1,\ \ y(0)=0.1,\ \ z(0)=0.1$. It is clear that the system exhibits nonlinear irregular oscillations, which would affect the stable operation of PMSM. Therefore, it is necessary to design controller for suppressing nonlinear even chaotic vibration of PMSM.

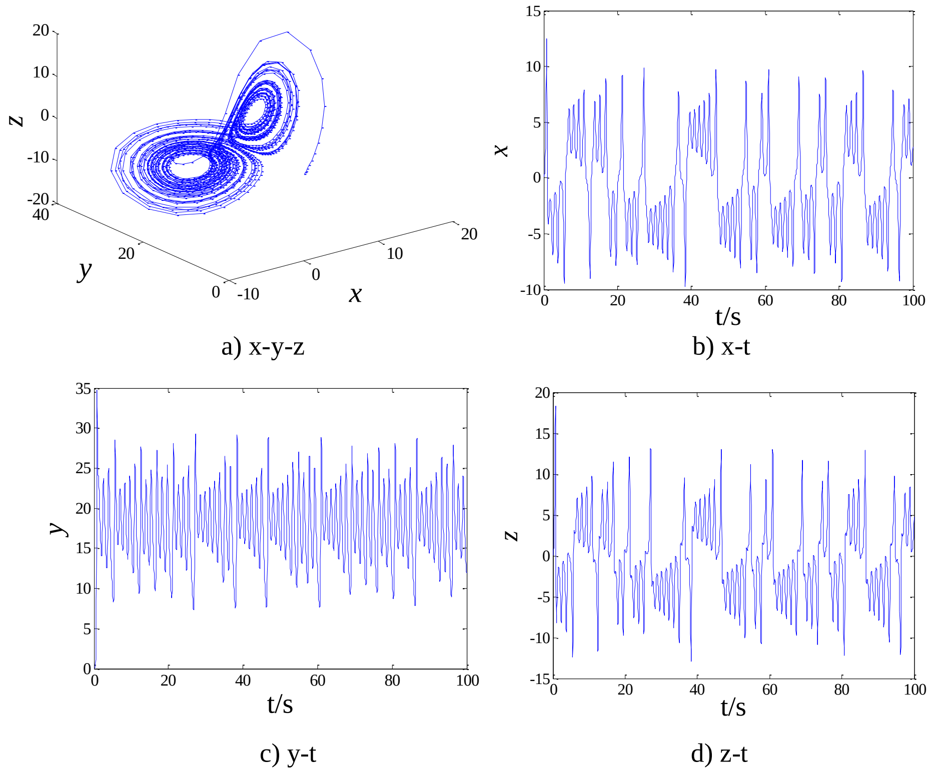


Fig. 1. Phase trajectory and time domain of PMSM, equation 1

3. Finite time terminal sliding mode controller design

Lemma 1 [23]: If there is a positive definite continuous function which satisfies the following differential inequality:

$$\dot{V}(t) \le -cV^{\eta}(t), \forall t \ge t_0, V(t_0) \ge 0 \tag{2}$$

where $c>0$, $0<\eta<1$. And, for any initial given time t_0, if $V(t)$ can satisfy the following inequality form:

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - c(1-\eta)(t-t_0), t_0 \le t \le t_1$$

Here $V(t) \equiv 0,\ \ \forall t \ge t_1$ and t_1 is given by:

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{c(1-\eta)}.$$

Then the system would be stabilized in a finite time t_1.

The PMSM system, equation (1) can be simplified as:

$$\begin{cases} \dot{x}_1 = A_{11}x_1 + A_{12}x_2 \\ \dot{x}_2 = A_{21}x_1 + A_{12}x_2 + f_2(x_1, x_2) + B_2 u \end{cases} \tag{3}$$

where x_1, x_2 are state variables of the system, *u* is control input which needs to be designed, B_2 is a full rank matrix, $f_2(x_1, x_2)$ is the nonlinearity. For convenience, system, equation (3) can be rewritten as the following compact form:

$$\dot{x} = Ax + Bu + f(x) \tag{4}$$

where:

$$x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},\ A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},\ B = \begin{pmatrix} 0 \\ B_2 \end{pmatrix},\ f(x) = \begin{pmatrix} 0 \\ f_2(x) \end{pmatrix}.$$

In order to make the system states reach the stable operation in a finite time, the fast terminal sliding mode switching surface is selected:

$$s = C_2 x_2 + C_1 x_1 + C_3 \left|\sin x_1\right| = Cx + C_3 \left|\sin x_1\right| \tag{5}$$

where $C_1 \in R^{(n-m)\times m}$, $C_2 \in R^{(n-m)\times(n-m)}$, $C_3 \in R^{(n-m)\times m}$, $C = \left|C_1, C_2\right|$.

Theorem 1: If the terminal sliding mode switching surface is selected as, equation (5), the terminal sliding mode controller can be designed as the following form:

$$U = -(CB)^{-1}\left[CAx + Cf(x) + C_3 sign(\sin x_1)\dot{x}_1 \cos x_1 + (\mu + \eta\|s\|^{\beta-1})s\right] \tag{6}$$

where $\mu > 0$, $\eta > 0$, $0 < \beta < 1$. The control law, equation (6) can make the state trajectories of PMSM system, equation (1) reaches to the sliding mode *S*=0 in a finite time.

Proof: We take the Lyapunov function as the following form:

$$V_1 = \frac{1}{2}s^T s \tag{7}$$

Taking the derivative of Lyapunov function, equation (7), one gets:

$$\begin{aligned} \dot{V}_1 &= \frac{1}{2}\dot{s} s^T + \frac{1}{2} s \dot{s}^T \\ &= s^T \dot{s} \end{aligned} \tag{8}$$

When $\sin x_1 \geq 0$, there is:

$$\begin{cases} s = Cx + C_3 \sin x_1 \\ \dot{s} = C\dot{x} + C_3 \dot{x}_1 \cos x_1 \end{cases} \quad (9)$$

When $\sin x_1 < 0$, one has:

$$\begin{cases} s = Cx - C_3 \sin x_1 \\ \dot{s} = C\dot{x} - C_3 \dot{x}_1 \cos x_1 \end{cases} \quad (10)$$

It can be concluded that

$$\dot{s} = C \cdot \dot{x} + C_3 \cdot \dot{x}_1 \cdot \text{sign}(\sin x_1) \cdot \cos x_1 \quad (11)$$

Here,

$$\text{sign}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \quad (12)$$

Thus,

$$\begin{aligned} \dot{V}_1 &= s^T \left(C\dot{x} + C_3 \cdot \dot{x}_1 \cdot \text{sign}(x_1) \cdot \cos x_1 \right) \\ &= s^T \left(CAx + CBu + Cf(x) + C_3 \cdot \dot{x}_1 \cdot \text{sign}(x_1) \cdot \cos x_1 \right) \\ &= -s^T \cdot \left(\mu + \eta \|s\|^{\beta-1} \right) \cdot s \\ &\le -\eta \|s\|^{\beta-1} \cdot \|s\|^2 \\ &= -\eta \left[\frac{1}{2} \|s\|^2 \right]^{\frac{\beta+1}{2}} \cdot 2^{\frac{\beta+1}{2}} \end{aligned}$$

According to lemma 1, it is clear that the trajectories of PMSM system, equation (1) will converge to the sliding mode S=0 in a finite time under the control law, equation (6). This completes the proof.

Theorem 2: If there exists an invertible matrix C_2 and undetermined parameter matrices C_1, C_3, which make the following inequality combined with matrix equation hold

$$\begin{cases} (A_{11} - A_{12}C_2^{-1}C_1) + (A_{11} - A_{12}C_2^{-1}C_1)^T \le 0 \\ A_{12}C_2^{-1}C_3 = \Lambda \end{cases}, \quad (13)$$

where $\Lambda = diag\{\Lambda_j (j = 1, \cdots, m)\}, \Lambda_j > 0$. Then, the designed controller, equation (6) can make the PMSM system, equation (1) stable in a finite time.

Proof: According to the variable structure theory, the system state will reach to S=0 in a finite time under the controller and can keep on moving on the sliding mode, which has the invariance. On the sliding surface, one has

$$s = C_2 x_2 + C_1 x_1 + C_3 \left|\sin x_1\right| = 0 \tag{14}$$

Thus,

$$x_2 = -C_2^{-1}\left[C_1 x_1 + C_3 \left|\sin x_1\right|\right] \tag{15}$$

Substituting equation (15) into equation (3), there is

$$\begin{aligned} \dot{x}_1 &= A_{11} x_1 + A_{12} x_2 \\ &= \left(A_{11} - A_{12} C_2^{-1} C_1\right) \cdot x_1 - A_{12} C_2^{-1} C_3 \left|\sin x_1\right| \end{aligned}, \tag{16}$$

The Lyapunov function is selected as:

$$V_2 = \frac{1}{2} x_1^T \cdot x_1 = \frac{1}{2}\left\|x_1\right\|^2 \tag{17}$$

Taking the derivative of equation (17), one gets:

$$\begin{aligned} \dot{V}_2 &= \frac{1}{2} x_1^T \cdot \dot{x}_1 + \frac{1}{2} \dot{x}_1^T \cdot x_1 \\ &= x^T \cdot \dot{x}_1 \\ &= x^T \left[\left(A_{11} - A_{12} C_2^{-1} C_1\right) x_1 + A_{12} C_2^{-1} C_3 \left|\sin x_1\right|\right] \\ &\le -x_1^T \cdot A_{12} C_2^{-1} C_3 \left|\sin x_1\right| \\ &\le -x_1^T \cdot A_{12} C_2^{-1} C_3 \\ &= -\left(\Lambda_1 \left|x_{11}\right| + \Lambda_2 \left|x_{12}\right| + \cdots + \Lambda_m \left|x_{1m}\right|\right) \\ &\le -\lambda\left(\left|x_{11}\right| + \left|x_{12}\right| + \cdots \left|x_{1m}\right|\right) \\ &\le -\lambda \sqrt{x_{11}^2 + x_{12}^2 + \cdots + x_{1m}^2} \\ &= -\lambda \left\|x_1\right\| \\ &= -\sqrt{2} \lambda V_2^{\frac{1}{2}} \end{aligned}$$

where $\lambda = \min\left\{\Lambda_j (j = 1, 2, \cdots, m)\right\}$. According to Lemma 1 and Theorem 1, it is clear that the system state x_1 will converge to the stabilization along the sliding mode S=0 in a finite time. Similarly, the system state x_2 can also converge to the origin in a finite time. Therefore, the PMSM system, equation (1) can be stabilized in a finite time under the designed controller, equation (6).

4. Numerical simulations

After adding the controller, the PMSM system, equation (1) can be represented as:

$$\begin{cases} \dot{z} = 5.46y - 5.46z \\ \dot{x} = -x + yz + u_1 \\ \dot{y} = -y - xz + 20z + u_2 \end{cases} \tag{18}$$

where u_1, u_2 are the control inputs determined by the controller, equation (6). Meanwhile, one can get:

$$A_{11} = (-5.46)\,,\ A_{12} = (0 \quad 5.46)\,,\ A_{21} = \begin{bmatrix} 0 \\ 20 \end{bmatrix},\ A_{22} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix},\ f(x) = \begin{bmatrix} 0 \\ zy \\ -xz \end{bmatrix}.$$

The parameter matrix of the selected finite time terminal sliding mode surface is given as:

$$C = |\,C_1 \quad C_2 \quad C_3\,| = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Thus, there is:

$$\begin{cases} (A_{11} - A_{12}C_2^{-1}C_1) + (A_{11} - A_{12}C_2^{-1}C_1)^T = -10.92 < 0 \\ A_{12}C_2^{-1}C_3 = 5.46 > 0 \end{cases} \tag{19}$$

The conditions of theorem 2 are satisfied. So, the finite time terminal sliding mode surface can be designed as the following form:

$$s = \begin{bmatrix} s1 \\ s2 \end{bmatrix} = \begin{bmatrix} z + x \\ y + |\sin z| \end{bmatrix} \tag{20}$$

According to Theorem 1, finite-time stabilization controllers are presented as follows:

$$\begin{cases} u_1 = -\left[-5.46z - x + 5.46y + zy - xz + \left(\mu + \eta \|s\|^{\beta - 1} \right) s \right] \\ u_2 = -\left[20z - y + 5.46y \cdot \mathrm{sign}(\sin z) \cdot \cos(z) - 5.46z \cdot sign(\sin z) \cdot \cos(z) + (\mu + \eta \|s\|^{\beta - 1}) \cdot s \right] \end{cases}, \tag{21}$$

The parameters are selected as $\mu = \eta = 1$, $\beta = 0.5$. Fig. 2 shows the simulation result with controller, equation (21). From Fig. 2, it is obvious that when the controller is applied to PMSM system, equation (18), the state variables are stable about the equilibrium point quickly, which shows the validity of the proposed method.

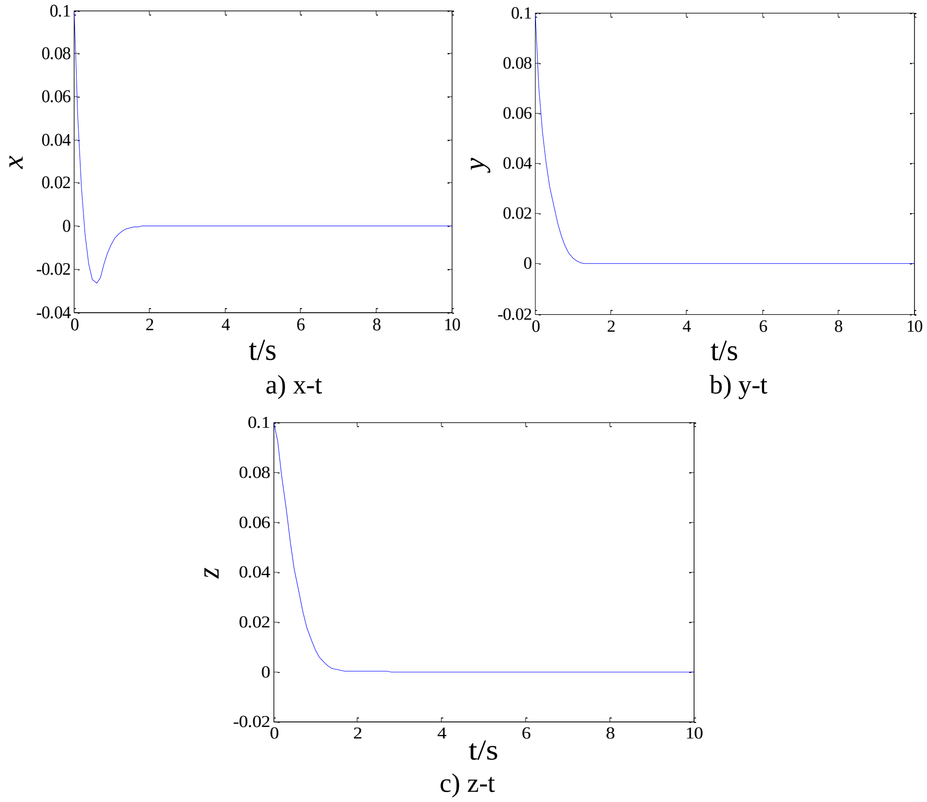
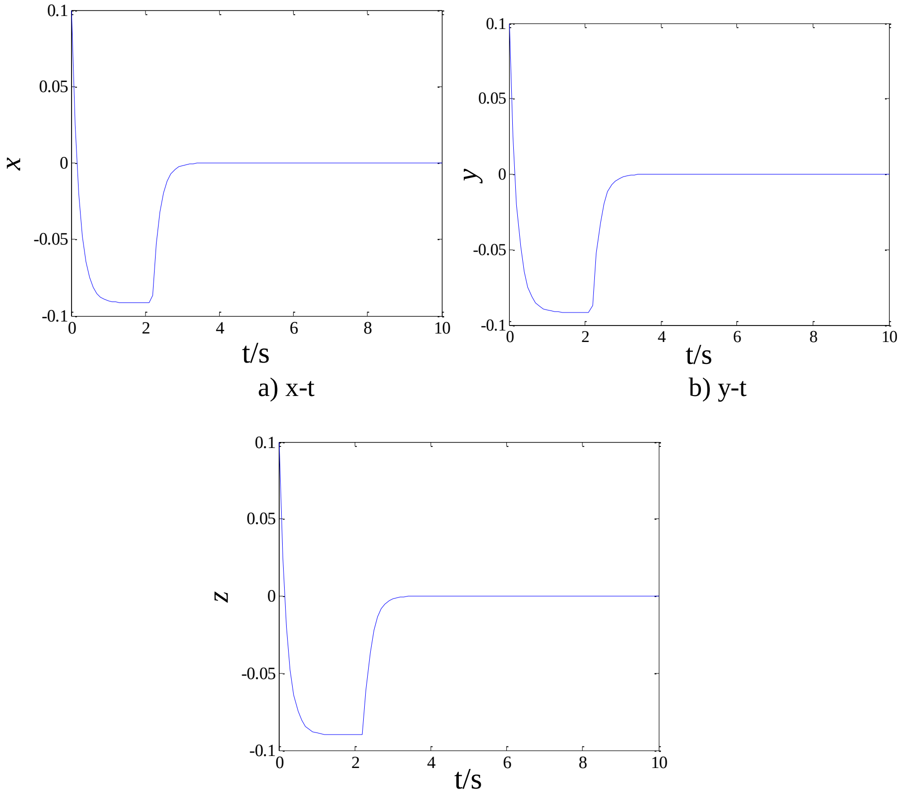


Fig. 2. Time domain of PMSM system, equation (18) with controller, equation (21)

When taking the existing method in Ref. [24], the following sliding mode, equation (22) and control law, equation (23) can be obtained. Fig. 3 shows the simulation result with controller, equation (23). As to the existing method, it takes long time for these state variables to stabilize to the equilibrium point. Accordingly, simulation result has demonstrated the effectiveness and superiority of the new scheme.

$$\begin{cases} S_1 = e_1 + \int_0^{\tau} 5e_1(\tau)\,d(\tau) \\ S_2 = e_2 + \int_0^{\tau} 5e_2(\tau)\,d(\tau) \\ S_3 = e_3 + \int_0^{\tau} 5e_3(\tau)\,d(\tau) \end{cases} \tag{22}$$

$$u=\begin{pmatrix}u_4\\u_5\\u_6\end{pmatrix}=\begin{pmatrix}-yz-4e_1-(0.45+\text{abs}(yz))\text{sign}(S_1)\\ xz-4e_2-20e_3-(0.45+\text{abs}(-xz))\text{sign}(S_2)\\ -5.46e_2+0.46e_3-0.45\text{sign}(S_3)\end{pmatrix} \tag{23}$$



a) x-t b) y-t c) z-t

Fig. 3. Time domain of PMSM system, equation (18) with controller, equation (23) in Ref. [24]

5. Conclusions

This paper studied the finite time control of nonlinear permanent magnet synchronous motor. A novel terminal sliding mode control method is proposed for the vibration control of nonlinear permanent magnet synchronous motor based on finite time stability theory. The designed scheme only needed two terms, which was one less than the conventional sliding mode control method and accordingly easy for implementation. Simulation result has verified the effectiveness and superiority when compared with the existing method.

The scheme designed is simple and easy to implement and could be applied to similar fractional order nonlinear PMSM systems such as in mechanical system, electrical system, and so on. In the future, to improve the transient process, more efficient finite time control methods should be studied for permanent magnet synchronous motor. There may be a focus on nonlinear control on the basis of finite time stability theory.

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