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PRIME (SEMIPRIME) BI-HYPERIDEALS OF SEMIHYPERGROUPS BASED ON INTUITIONISTIC FUZZY POINTS

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Using the concept of intuitionistic fuzzy point, the notion of $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of semihypergroups is introduced. Several characterizations of this notion are given and the behavior of this structure under homomorphisms of semihypergroups is discussed. Finally, the notion of prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of semihypergroups is introduced and some related properties are proved.

Keywords: semihypergroup, prime (semiprime) bi-hyperideal, intuitionistic fuzzy point, prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal

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1. Introduction

Hyperstructures, in particular hypergroups, were introduced in 1934 by a French mathematician, Marty, at the 8^{th} Congress of Scandinavian Mathematicians [27]. Now, they are widely studied from the theoretical point of view and for their applications to many subjects of pure and applied mathematics; for example, semihypergroups are the simplest algebraic hyperstructures which possess the properties of closure and associativity. A short review of the theory of hyperstructures appear in [11, 32]. A recent book [10] contains a wealth of applications of hyperstructures. There are applications to the following subjects: geometry, hypergraphs, binary relations, lattices, fuzzy sets and rough sets, automata, cryptography, combinatorics, codes, artificial intelligence, and probabilities.

Since the inception of the notion of a fuzzy set in [33] which laid the foundations of fuzzy set theory, the literature on fuzzy set theory and its applications has been growing rapidly amounting by now to several papers [3, 12, 13, 15, 21, 26, 28, 31]. These are widely scattered over many disciplines such as artificial intelligence, computer science, control engineering, expert systems, management science, operations research, pattern recognition, robotics, and others. After the introduction of fuzzy sets by Zadeh, reconsideration of some concepts of classical mathematics began. On the other hand, because of the importance of group theory in mathematics, as well as its many areas of applications, the notion of a fuzzy subgroup was defined

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and its structure was investigated by Rosenfeld [30]. This subject has been studied further by many mathematicians. A new type of fuzzy subgroup ($(\in, \in \vee q)$-fuzzy subgroup) was introduced in an earlier paper of Bhakat and Das [7] by using the combined notions of "belongingness" and "quasicoincidence" of fuzzy points and fuzzy sets. In fact, $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup. This concept has been studied further in [8]. Other important results related with this concept were given in [14, 20, 22, 23, 24, 25, 34].

Besides several generalizations of fuzzy sets, the intuitionistic fuzzy sets introduced by Atanassov [4, 5, 6] have been found to be highly useful to cope with imperfect and/or imprecise information. Atanassov's intuitionistic fuzzy sets are an intuitively straightforward extension of Zadeh's fuzzy sets: while a fuzzy set gives the degree of membership of an element in a given set, an Atanassov's intuitionistic fuzzy set gives both a degree of membership and a degree of non-membership. Intuitionistic fuzzy set theory has been successfully applied in decision analysis and pattern recognition, in logic programming and medical diagnosis. Recently, in [29], Mahmood introduced the notion of intuitionistic fuzzy bi-hyperideal of semihypergroups, also see [2] for crisp case. Coker and Demirci [9] introduced the notion of intuitionistic fuzzy point. Abdollah et al. [1] introduce the notion of (α, β)-intuitionistic fuzzy ideals of hemirings where α, β are any two of $\{\in, q, \in \vee q, \in \wedge q\}$ with $\alpha \neq \in \wedge q$ and related properties are investigated.

In this paper, using the concept of intuitionistic fuzzy point, the notion of $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of semihypergroups is introduced. Several characterizations of this notion are given and the behavior of this structure under homomorphisms of semihypergroups is discussed. Finally, the notion of prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of semihypergroups is introduced and some related properties are proved.

2. **Preliminaries and notations**

In this section, we give some notions and definitions of semihypergroups and intuitionistic fuzzy sets on which our research in this paper is based.

A *hypergroupoid* is a non-empty set S together with a map $\cdot : S \times S \longrightarrow \mathcal{P}^*(S)$ where $\mathcal{P}^*(S)$ denotes the set of all the non-empty subsets of S. The image of the pair (x, y) is denoted by $x \cdot y$. If $x \in S$ and A, B are non-empty subsets of S, then $A \cdot B$ is defined by $A \cdot B = \bigcup\limits_{a \in A, b \in B} a \cdot b$. Also $A \cdot x$ is used for $A \cdot \{x\}$ and $x \cdot A$ for $\{x\} \cdot A$. A hypergroupoid $(S, \cdot)$ is called a *semihypergroup* if $(x \cdot y) \cdot z = x \cdot (y \cdot z)$ for all $x, y, z \in S$. A non-empty subset X of a semihypergroup S is called a *subsemihypergroup* if $X \cdot X \subseteq X$. A subsemihypergroup X of S is called a *bi-hyperideal* if $X \cdot S \cdot X \subseteq X$. Let S and S' be semihypergroups. A function $f : S \longrightarrow S'$ is called a *homomorphism* if it satisfies $f(x \cdot y) = f(x) \cdot f(y)$ for all $x, y \in S$.

According to [4], a function $\mu : X \longrightarrow [0, 1]$ is called a *fuzzy set* in a set X. An *intuitionistic fuzzy set* (IFS for short) A in X is an object having the form $A = \{\langle x, \mu_A(x), \lambda_A(x)\rangle \mid x \in X\}$ where the functions $\mu_A : X \longrightarrow [0, 1]$ and $\lambda_A : X \longrightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of nonmembership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$ for all $x \in X$. For the sake of simplicity, we shall use the notation $A = (\mu_A, \lambda_A)$ instead of $A = \{\langle x, \mu_A(x), \lambda_A(x)\rangle \mid x \in X\}$. We refer the readers to [3, 17, 18, 19] to see some results on connections between intuitionistic fuzzy sets and algebraic structures.

Let f be a mapping from a set X to a set Y. If $A = (\mu_A, \lambda_A)$ and $B = (\mu_B, \lambda_B)$ are IFSs in X and Y, respectively, then the *preimage* of $B = (\mu_B, \lambda_B)$ under f is defined to be an intuitionistic fuzzy set $f^{-1}(B) = (\mu_{f^{-1}(B)}, \lambda_{f^{-1}(B)})$ where $\mu_{f^{-1}(B)}(x) = \mu_B(f(x))$ and $\lambda_{f^{-1}(B)}(x) = \lambda_B(f(x))$ for all $x \in X$, and the *image* of $A = (\mu_A, \lambda_A)$ under f is defined to be an intuitionistic fuzzy set $f(A) = (\mu_{f(A)}, \lambda_{f(A)})$, where

$$\mu_{f(A)}(y) := \begin{cases} \bigvee_{x \in f^{-1}(y)} \mu_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise,} \end{cases}$$

$$\lambda_{f(A)}(y) := \begin{cases} \bigwedge_{x \in f^{-1}(y)} \lambda_A(x) & \text{if } f^{-1}(y) \neq \emptyset, \\ 1 & \text{otherwise,} \end{cases}$$

for all $y \in Y$.

Definition 2.1. [13] *An IFS $A = (\mu_A, \lambda_A)$ in S is called intuitionistic fuzzy subsemihypergroup of S if*

(1) $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y)$,

(2) $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y)$,

for all $x, y \in S$ and $z \in x \cdot y$.

Definition 2.2. [29] *An intuitionistic fuzzy subsemihypergroup $A = (\mu_A, \lambda_A)$ of S is called an intuitionistic fuzzy bi-hyperideal of S if*

(1) $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y)$,

(2) $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y)$,

for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$.

Let X be a non-empty set and $c \in X$ a fixed element in X. A *fuzzy point* c_t (see [23]) for $t \in (0, 1]$ is a fuzzy subset of X such that

$$c_t(x) = \begin{cases} t & c = x, \\ 0 & \text{otherwise.} \end{cases}$$

If $t \in (0, 1]$ and $s \in [0, 1)$ are two fixed real numbers such that $0 \leq t+s \leq 1$, then the IFS $c(t, s) = \langle x, c_t, 1 - c_{1-s}\rangle$ is called an *intuitionistic fuzzy point* (IFP for short) (see [9] in X, where t (resp. s) is the degree of membership (resp. non-membership) of $c(t, s)$ and $c \in X$ is the support of $c(t, s)$.

An IFP $c(t, s)$ is said to *belong to* an IFS $A = (\mu_A, \lambda_A)$ of X, denoted by $c(t, s) \in A$, if $\mu_A(c) \geq t$ and $\lambda_A(c) \leq s$. We say that $c(t, s)$ is *quasi-coincident* with $A = (\mu_A, \lambda_A)$, denoted by $c(t, s)qA$, if $\mu_A(c) + t > 1$ and $\lambda_A(c) + s < 1$. To say that $c(t, s) \in \vee qA$ (resp. $c(t, s) \in \wedge qA$) means that $c(t, s) \in A$ or $c(t, s)qA$ (resp. $c(t, s) \in A$ and $c(t, s)qA$) and $c(t, s)\overline{\in \vee q}A$ means that $c(t, s) \in \vee qA$ does not hold.

For any $t \in [0, 1]$ and a fuzzy set μ of S, the set $U(\mu; t) = \{x \in S \mid \mu(x) \geq t\}$ (resp. $L(\mu; t) = \{x \in S \mid \mu(x) \leq t\}$) is called an *upper* (resp. *lower*) *t-level cut* of μ.

3. $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals

In what follows let S denote a semihypergroup. Now, we concentrate on $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals and give various characterizations.

Definition 3.1. *Let $A = \langle \mu_A, \lambda_A \rangle$ be IFS in S. Then, $A = (\mu_A, \lambda_A)$ is called an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S if for all $x, w, y \in S$, ($t_1, t_2 \in (0, 0.5]$ and $s_1, s_2 \in [0.5, 1)$) or ($t_1, t_2 \in (0.5, 1]$ and $s_1, s_2 \in [0, 0.5)$) the following conditions hold:*

$(I3)$ If $x(t_1, s_1) \in A$, $y(t_2, s_2) \in A \Longrightarrow z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qA$, for all $z \in x \cdot y$,

$(I4)$ If $x(t_1, s_1) \in A$, $y(t_2, s_2) \in A \Longrightarrow z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qA$, for all $z \in x \cdot w \cdot y$.

Example 3.1. *Let $\mathbb{N}$ be the set of all positive integers. Then, $(\mathbb{N}, \odot)$ is a semihypergroup, where $\odot$ is defined by $x \odot y = \{xty \mid t \in 4\mathbb{N}\}$ for all $x, y \in \mathbb{N}$. Now define*

$$\mu_A(x) = \begin{cases} 0.8, & if\ x \in 2\mathbb{N} \\ 0.9, & if\ x \notin 2\mathbb{N} \end{cases} \qquad \lambda_A(x) = \begin{cases} 0.2, & if\ x \in 2\mathbb{N} \\ 0.1, & if\ x \notin 2\mathbb{N}. \end{cases}$$

Once can easily check that $A = \langle \mu_A, \lambda_A \rangle$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of $(\mathbb{N}, \odot)$.

Lemma 3.1. *For an IFS $A = \langle \mu_A, \lambda_A \rangle$ in S, the following conditions are equivalent:*

$(I3)$ $x(t_1, s_1), y(t_2, s_2) \in A \Longrightarrow z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qA$,

$(I5)$ $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5$,

for all $x, y \in S$ and $z \in x \cdot y$, ($t_1, t_2 \in (0, 0.5]$ and $s_1, s_2 \in [0.5, 1)$) or ($t_1, t_2 \in (0.5, 1]$ and $s_1, s_2 \in [0, 0.5)$).

Proof. $(I3) \to (I5)$: Suppose that $(I5)$ does not hold. Then, there exist $x, y \in S$ and $z \in x \cdot y$ such that $\mu_A(z) < \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\lambda_A(z) > \lambda_A(x) \vee \lambda_A(y) \vee 0.5$. So, for some t and s, $\mu_A(z) < t \leq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\lambda_A(z) > s \geq \lambda_A(x) \vee \lambda_A(y) \vee 0.5$. We can consider the following cases:

(a) $\mu_A(x) \wedge \mu_A(y) < 0.5$ and $\lambda_A(x) \vee \lambda_A(y) > 0.5$. Then, $\mu_A(z) < t \leq \mu_A(x) \wedge \mu_A(y)$ and $\lambda_A(z) > s \geq \lambda_A(x) \vee \lambda_A(y)$. Thus, $x(t, s), y(t, s) \in A$ but $z(t, s) \bar{\in} A$. Also, $\mu_A(z) + t < 0.5 + 0.5 = 1$ and $\lambda_A(z) + s > 0.5 + 0.5 = 1$. Thus, $z(t, s) \bar{q} A$ and so $z(t, s) \overline{\in \vee q} A$, which is a contradiction. Therefore, $(I5)$ is valid.

(b) $\mu_A(x) \wedge \mu_A(y) \geq 0.5$ and $\lambda_A(x) \vee \lambda_A(y) \leq 0.5$. Then, $\mu_A(z) < 0.5$ and $\lambda_A(z) > 0.5$, which implies that $x(0.5, 0.5), y(0.5, 0.5) \in A$ but $z(0.5, 0.5) \bar{\in} A$. Also, $\mu_A(z) + 0.5 < 0.5 + 0.5 = 1$ and $\lambda_A(z) + 0.5 > 0.5 + 0.5 = 1$. Then, $z(0.5, 0.5) \bar{q} A$. Thus, $z(0.5, 0.5) \overline{\in \vee q} A$, which is a contradiction. Therefore, $(I5)$ is valid.

(c) $\mu_A(x) \wedge \mu_A(y) < 0.5$ and $\lambda_A(x) \vee \lambda_A(y) < 0.5$. Then, $\mu_A(z) < t \leq \mu_A(x) \wedge \mu_A(y)$ and $\lambda_A(z) > s \geq 0.5$, which implies that, $x(t, s), y(t, s) \in A$ but $z(t, s) \bar{\in} A$. Also, $\mu_A(z) + t < 0.5 + 0.5 = 1$ and $\lambda_A(z) + s > 0.5 + 0.5 = 1$. Thus, $z(t, s) \bar{q} A$ and so $z(t, s) \overline{\in \vee q} A$, which is a contradiction. Therefore, $(I5)$ is valid.

$(I5) \to (I3)$: Suppose that condition $(I5)$ holds. Let $x, y \in S$, ($t_1, t_2 \in (0, 0.5]$ and $s_1, s_2 \in [0.5, 1)$) or ($t_1, t_2 \in (0.5, 1]$ and $s_1, s_2 \in [0, 0.5)$), be such that $x(t_1, s_1), y(t_2, s_2) \in A$. Then, $\mu_A(x) \geq t_1$ and $\lambda_A(x) \leq s_1$, $\mu_A(y) \geq t_2$ and $\lambda_A(y) \leq$

s_2. For all $z \in x \cdot y$ we have $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 \geq t_1 \wedge t_2 \wedge 0.5$ and $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5 \leq s_1 \vee s_2 \vee 0.5$. There are two cases:

(a) $t_1 \wedge t_2 \leq 0.5$ and $s_1 \vee s_2 \geq 0.5$. Then, for all $z \in x \cdot y$, $\mu_A(z) \geq t_1 \wedge t_2$ and $\lambda_A(z) \leq s_1 \vee s_2$ and so $z(t_1 \wedge t_2, s_1 \vee s_2) \in A$.

(b) $t_1 \wedge t_2 > 0.5$ and $s_1 \vee s_2 < 0.5$. Then, for all $z \in x \cdot y$, $\mu_A(z) \geq 0.5$ and $\lambda_A(z) \leq 0.5$ which implies that $\mu_A(z) + t_1 \wedge t_2 > 1$ and $\lambda_A(z) + s_1 \vee s_2 < 1$ and so $z(t_1 \wedge t_2, s_1 \vee s_2)qA$.

Thus, $z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qA$. Therefore, $(I5)$ holds. □

Lemma 3.2. *For an IFS $A = \langle \mu_A, \lambda_A \rangle$ in S, the following conditions are equivalent:*

$(I4)$ $x(t_1, s_1), y(t_2, s_2) \in A \Longrightarrow z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qA$,

$(I6)$ $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ *and* $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5$,

for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$, $(t_1, t_2 \in (0, 0.5]$ and $s_1, s_2 \in [0.5, 1))$ or $(t_1, t_2 \in (0.5, 1]$ and $s_1, s_2 \in [0, 0.5))$.

Proof. The proof is similar to the proof of Lemma 3.1. □

In the next theorem, by using Lemmas 3.1 and 3.2, we obtain equivalent conditions for $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals.

Theorem 3.1. *An IFS $A = \langle \mu_A, \lambda_A \rangle$ in S is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S if and only if it satisfies $(I5)$ and $(I6)$.*

It is not difficult to see that every intuitionistic fuzzy bi-hyperideal of S is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal. The following example shows that the converse is not true in general.

Example 3.2. *Let $A = \langle \mu_A, \lambda_A \rangle$ be an IFS of $(\mathbb{N}, \odot)$ defined in Example 3.1. Then, it is easy verify $A = \langle \mu_A, \lambda_A \rangle$ in S is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of $(\mathbb{N}, \odot)$ but not an intuitionistic fuzzy bi-hyperideal of $(\mathbb{N}, \odot)$.*

Now, we discuss on the behavior of the intersection of a family of $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of a semihypergroup.

Theorem 3.2. *Let $\{A_i\}_{i \in I} = \{\langle \mu_{A_i}, \lambda_{A_i} \rangle\}_{i \in I}$ be a family of $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of S. Then, $\mathcal{A} = \langle \mu_A, \lambda_A \rangle = \bigcap\limits_{i \in I} A_i = \langle \bigwedge\limits_{i \in I} \mu_{A_i}, \bigvee\limits_{i \in I} \lambda_{Ai} \rangle$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S.*

Proof. Straightforward. □

Remark 3.1. *Let $\{A_i\}_{i \in I}$ be a family of $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of S. Is $\bigcup\limits_{i \in I} A_i = \langle \bigvee\limits_{i \in I} \mu_{A_i}, \bigwedge\limits_{i \in I} \lambda_{A_i} \rangle$ an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S?*

The following example gives a negative answer to the above question.

Example 3.3. *Let $S = \{a, b, c, d\}$ be a semihypergroup with the following table*

$\cdot$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a, d\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
d	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$

Let $A_1 = \langle\mu_{A_1}, \lambda_{A_1}\rangle$ and $A_2 = \langle\mu_{A_2}, \lambda_{A_2}\rangle$ be two IFSs in S such that $\mu_{A_1}(a) = \mu_{A_1}(b) = 0.4$, $\mu_{A_1}(c) = \mu_{A_1}(d) = 0$ and $\lambda_{A_1}(a) = \lambda_{A_1}(c) = 0.4$, $\lambda_{A_1}(b) = \lambda_{A_1}(d) = 0$; $\mu_{A_2}(a) = \mu_{A_2}(c) = 0.4$, $\mu_{A_2}(b) = \mu_{A_2}(d) = 0$ and $\lambda_{A_2}(a) = \lambda_{A_2}(b) = 0.4$, $\lambda_{A_2}(c) = \lambda_{A_2}(d) = 0$. Then, by Theorem 3.1 both A_1 and A_2 are $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of S but $A_1 \cup A_2$ is not an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S, since for $d \in b \cdot c$; $0 = \mu_{A_1}(d) \vee \mu_{A2}(d) = (\mu_{A_1} \vee \mu_{A2})(d) < 0.5 \wedge (\mu_{A_1} \vee \mu_{A2})(b) \wedge (\mu_{A_1} \vee \mu_{A2})(c) = 0.5 \wedge 0.4 \wedge 0.4 = 0.4$.

The following theorem for the union of $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals can be proved, if we present a sufficient condition.

Theorem 3.3. *Let $\{A_i\}_{i\in I} = \{\langle\mu_{A_i}, \lambda_{A_i}\rangle\}_{i\in I}$ be a family of $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of S and $A_i \subseteq A_j$ or $A_j \subseteq A_i$ for all $i, j \in I$ where $A_i \subseteq A_j$ means $\mu_{A_i} \subseteq \mu_{Aj}$ and $\lambda_{A_i} \supseteq \lambda_{Aj}$. Then $\mathcal{A} = \langle\mu_{\mathcal{A}}, \lambda_{\mathcal{A}}\rangle = \bigcup\limits_{i\in I} A_i$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S.*

Proof. Suppose that $x, w, y \in S$ and $z \in x \cdot w \cdot y$. By Theorem 3.1, we have

$$\begin{aligned} \mu_{\mathcal{A}}(z) &= (\bigvee_{i\in I} \mu_{A_i})(z) = \bigvee_{i\in I} \mu_{A_i}(z) \\ &\geq \bigvee_{i\in I} \mu_{A_i}(x) \wedge \mu_{A_i}(y) \wedge 0.5 \\ &= \bigvee_{i\in I} \mu_{A_i}(x) \wedge \bigvee_{i\in I} \mu_{A_i}(y) \wedge 0.5 \quad (1) \\ &= \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y) \wedge 0.5 \end{aligned}$$

Similarly, we have $\lambda_{\mathcal{A}}(z) \leq \lambda_{\mathcal{A}}(x) \vee \lambda_{\mathcal{A}}(y) \vee 0.5$. In the following we show that (1) holds. It is clear that

$$\bigvee_{i\in I} \mu_{A_i}(x) \wedge \mu_{A_i}(y) \wedge 0.5 \leq \bigvee_{i\in I} \mu_{A_i}(x) \wedge \bigvee_{i\in I} \mu_{A_i}(y) \wedge 0.5.$$

If possible, let $\bigvee_{i\in I} \mu_{A_i}(x) \wedge \mu_{A_i}(y) \wedge 0.5 \neq \bigvee_{i\in I} \mu_{A_i}(x) \wedge \bigvee_{i\in I} \mu_{A_i}(y) \wedge 0.5$. Then, there exists r such that $\bigvee\limits_{i\in I} \mu_{A_i}(x) \wedge \mu_{A_i}(y) \wedge 0.5 < r < \bigvee_{i\in I} \mu_{A_i}(x) \wedge \bigvee_{i\in I} \mu_{A_i}(y) \wedge 0.5$. Since$A_i \subseteq A_j$ or $A_j \subseteq A_i$ for all $i, j \in I$, there exists $k \in I$ such that $r < \mu_{A_k}(x) \wedge \mu_{A_k}(y) \wedge 0.5$. On the other hand, $\mu_{A_i}(x) \wedge \mu_{A_i}(y) \wedge 0.5 < r$ for all $i \in I$, a contradiction. Hence,$\bigvee_{i\in I} \mu_{A_i}(x) \wedge \mu_{A_i}(y) \wedge 0.5 = \bigvee_{i\in I} \mu_{A_i}(x) \wedge \bigvee_{i\in I} \mu_{A_i}(y) \wedge 0.5$. Similarly, we can prove that $\mu_{\mathcal{A}}(z) \geq \mu_{\mathcal{A}}(x) \wedge \mu_{\mathcal{A}}(y) \wedge 0.5$ and $\lambda_{\mathcal{A}}(z) \leq \lambda_{\mathcal{A}}(x) \vee \lambda_{\mathcal{A}}(y) \vee 0.5$, for all $x, y \in S$ and $z \in x \cdot y$. Therefore, $\mathcal{A} = \langle\mu_{\mathcal{A}}, \lambda_{\mathcal{A}}\rangle$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. □

In the next theorem, we investigate the behavior of $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals under the homomorphisms of semihypergroups.

Theorem 3.4. *Let $f : S_1 \longrightarrow S_2$ be a homomorphism of semihypergroups and $A = \langle \mu_A, \lambda_A \rangle$ and $B = \langle \mu_B, \lambda_B \rangle$ be $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of S_1 and S_2, respectively. Then, the following hold:*

(i) $f^{-1}(B)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S_1.

(ii) If $A = \langle \mu_A, \lambda_A \rangle$ satisfies the "sup-property", i.e., for any subset T of S_1 there exist $x_0 \in T$ such that $\mu_A(x_0) = \bigvee \{\mu_A(x) \mid x \in T\}$ and $\lambda_A(x_0) = \bigwedge \{\lambda_A(x) \mid x \in T\}$; Then, $f(A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S_2, when f is onto.

Proof. (i) Let $x, w, y \in S_1$ and $z \in x \cdot w \cdot y$, then $f(z) \in f(x) \cdot f(w) \cdot f(y)$ and we have $\mu_B(f(z)) \geq \mu_B(f(x)) \wedge \mu_B(f(y)) \wedge 0.5$ and $\lambda_B(f(z)) \leq \lambda_B(f(x)) \vee \lambda_B(f(y)) \vee 0.5$, or equivalently, for all $z \in x \cdot w \cdot y$ we have $\mu_{f^{-1}(B)}(z) \geq \mu_{f^{-1}(B)}(x) \wedge \mu_{f^{-1}(B)}(y) \wedge 0.5$ and $\lambda_{f^{-1}(B)}(z) \leq \lambda_{f^{-1}(B)}(x) \vee \lambda_{f^{-1}(B)}(y) \vee 0.5$. Similarly, we can prove that $\mu_{f^{-1}(B)}(z) \geq \mu_{f^{-1}(B)}(x) \wedge \mu_{f^{-1}(B)}(y) \wedge 0.5$ and $\lambda_{f^{-1}(B)}(z) \leq \lambda_{f^{-1}(B)}(x) \vee \lambda_{f^{-1}(B)}(y) \vee 0.5$ for all $z \in x \cdot y$. Therefore, $f^{-1}(B)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S_1.
(ii) Let $a, c, b \in S_2$ and $t_1, t_2 \in (0, 1]$ and $s_1, s_2 \in [0, 1)$ be such that $a(t_1, s_1)$, $b(t_2, s_2) \in f(A)$. Then, $\mu_{f(A)}(a) \geq t_1$ and $\lambda_{f(A)}(a) \leq s_1$, $\mu_{f(A)}(b) \geq t_2$ and $\lambda_{f(A)}(b) \leq s_2$. Since A has the sup-property, there exist $x \in f^{-1}(a)$ and $y \in f^{-1}(b)$ such that $\mu_A(x) = \bigvee\{\mu_A(\alpha) \mid \alpha \in f^{-1}(a)\}$ and $\lambda_A(x) = \bigwedge\{\lambda_A(\alpha) \mid \alpha \in f^{-1}(a)\}$, $\mu_A(y) = \bigvee\{\mu_A(\beta) \mid \beta \in f^{-1}(b)\}$ and $\lambda_A(y) = \bigwedge\{\lambda_A(\beta) \mid \beta \in f^{-1}(b)\}$. Then, $x(t_1, s_1) \in A$ and $y(t_2, s_2) \in A$. Since A is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S_1, we have $z'(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qA$, for all $z' \in x \cdot w \cdot y$ and $w \in f^{-1}(c)$. Now, $z' \in f^{-1}(z)$, for all $z \in a \cdot c \cdot b$ and so $\mu_{f(A)}(z) \geq \mu_A(z')$ and $\lambda_{f(A)}(z) \leq \lambda_A(z')$. Thus, $\mu_{f(A)}(z) \geq (t_1 \wedge t_2)$ and $\lambda_{f(A)}(z) \leq (s_1 \vee s_2)$ or $\mu_{f(A)}(z) + (t_1 \wedge t_2) > 1$ and $\lambda_{f(A)}(z) + (s_1 \vee s_2) < 1$, which means that $z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qf(A)$, for all $z \in a \cdot c \cdot b$. Similarly, we can prove that $z(t_1 \wedge t_2, s_1 \vee s_2) \in \vee qf(A)$, for all $z \in a \cdot b$. Consequently, $f(A)$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S_2. □

For any intuitionistic fuzzy set $A = \langle \mu_A, \lambda_A \rangle$ in S and $t \in (0, 1], s \in [0, 1)$, we denote $A_{(t,s)} = \{x \in S \mid x(t, s)qA\}$, $U_{(t,s)} = \{x \in S \mid x(t, s) \in A\}$ and $[A]_{(t,s)} = \{x \in S \mid x(t, s) \in \vee qA\}$. Obviously, $[A]_{(t,s)} = A_{(t,s)} \cup U_{(t,s)}$. $U_{(t,s)}$, $A_{(t,s)}$ and $[A]_{(t,s)}$ are called $\in$-*level set*, q-*level set* and $\in \vee q$-*level set* of A, respectively.

In the next theorem, we characterize $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals based on $\in$-level sets.

Theorem 3.5. *Let $A = \langle \mu_A, \lambda_A \rangle$ be an IFS on S. Then, A is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S if and only if $U_{(t,s)} \neq \emptyset$ is a bi-hyperideal of S for all $t \in (0, 0.5]$ and $s \in [0.5, 1)$.*

Proof. Let $A = \langle \mu_A, \lambda_A \rangle$ be an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S and $t \in (0, 0.5]$ and $s \in [0.5, 1)$. If $x, y \in U_{(t,s)}$ and $w \in S$, then $\mu_A(x) \geq t$, $\lambda_A(x) \leq s$, $\mu_A(y) \geq t$ and $\lambda_A(y) \leq s$. Thus, we have $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 \geq t \wedge 0.5 = t$ and $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5 \leq s \vee 0.5 = s$, for all $z \in x \cdot w \cdot y$, which implies that $z \in U_{(t,s)}$. Similarly, $z \in U_{(t,s)}$, for all $z \in x \cdot y$. Therefore, $U_{(t,s)}$ is a bi-hyperideal of S.

Conversely, let for every $0 < t \leq 0.5$ and $0.5 \leq s < 1$ each non-empty $U_{(t,s)}$ be a bi-hyperideal of S. If $x, w, y \in S$, we can say that $\mu_A(x) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 = t_0$, $\mu_A(y) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 = t_0$, $\lambda_A(x) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5 = s_0$

and $\lambda_A(y) \le \lambda_A(x) \vee \lambda_A(y) \vee 0.5 = s_0$. Then, $x, y \in U_{(t_0,s_0)}$ and so $z \in U_{(t_0,s_0)}$, for all $z \in x \cdot w \cdot y$, which implies that $\mu_A(z) \ge t_0 = \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\lambda_A(z) \le s_0 = \lambda_A(x) \vee \lambda_A(y) \vee 0.5$, for all $z \in x \cdot w \cdot y$. In a similar way, we can show that $\mu_A(z) \ge \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\lambda_A(z) \le \lambda_A(x) \vee \lambda_A(y) \vee 0.5$, for all $z \in x \cdot y$. Therefore, $A = \langle \mu_A, \lambda_A \rangle$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. □

In the next theorem, we investigate some equivalent conditions for $U_{(t,s)}$ as a bi-hyperideal.

Theorem 3.6. *Let $A = \langle \mu_A, \lambda_A \rangle$ be an IFS in S. Then, $U_{(t,s)}(\neq \emptyset)$ is a bi-hyperideal of S for all $t \in (0.5, 1]$ and $s \in [0, 0.5)$ if and only if*

(i) $\mu_A(x) \wedge \mu_A(y) \le \mu_A(z) \vee 0.5$ and $\lambda_A(x) \vee \lambda_A(y) \ge \lambda_A(z) \wedge 0.5$, for all $x, y \in S$ and $z \in x \cdot y$.

(ii) $\mu_A(x) \wedge \mu_A(y) \le \mu_A(z) \vee 0.5$ and $\lambda_A(x) \vee \lambda_A(y) \ge \lambda_A(z) \wedge 0.5$, for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$.

Proof. Let $U_{(t,s)}(\neq \emptyset)$ be a bi-hyperideal of S for all $t \in (0.5, 1]$ and $s \in [0, 0.5)$. If there exist $x, w, y \in S$ with $z \in x \cdot w \cdot y$ such that $\mu_A(z) \vee 0.5 < \mu_A(x) \wedge \mu_A(y) = t$ and $\lambda_A(z) \wedge 0.5 > \lambda_A(x) \vee \lambda_A(y) = s$, then $t \in (0.5, 1]$ and $s \in [0, 0.5)$, $\mu_A(z) < t$, $\lambda_A(z) > s$ and $x, y \in U_{(t,s)}$. Since $U_{(t,s)}$ is a bi-hyperideal, thus $z \in U_{(t,s)}$ and so $\mu_A(z) \ge t$ and $\lambda_A(z) \le s$ for all $z \in x \cdot w \cdot y$, which is a contradiction. Therefore, for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$ we have $\mu_A(x) \wedge \mu_A(y) \le \mu_A(z) \vee 0.5$ and $\lambda_A(x) \vee \lambda_A(y) \ge \lambda_A(z) \wedge 0.5$. The proof of (i) is similar and omitted.

Conversely, let (i) and (ii) hold. Assume that $t \in (0.5, 1]$, $s \in [0, 0.5)$ and $x, y \in U_{(t,s)}$. Then, by (i) we have $0.5 < t \le \mu_A(x) \wedge \mu_A(y) \le \mu_A(z) \vee 0.5$ and $0.5 > s \ge \lambda_A(x) \vee \lambda_A(y) \ge \lambda_A(z) \wedge 0.5$, and so $\mu_A(z) \ge t$ and $\lambda_A(z) \le s$ for all $z \in x \cdot y$, which means that $z \in U_{(t,s)}$. Hence, $x \cdot y \subseteq U_{(t,s)}$. Also, suppose that $t \in (0.5, 1]$, $s \in [0, 0.5)$ and $x, y \in U_{(t,s)}$ and $w \in S$. Then by (ii) we have $0.5 < t \le \mu_A(x) \wedge \mu_A(y) \le \mu_A(z) \vee 0.5$ and $0.5 > s \ge \lambda_A(x) \vee \lambda_A(y) \ge \lambda_A(z) \wedge 0.5$, and so $\mu_A(z) \ge t$ and $\lambda_A(z) \le s$ for all $z \in x \cdot w \cdot y$, which means that $z \in U_{(t,s)}$. Hence, $x \cdot w \cdot y \subseteq U_{(t,s)}$. Therefore, $U_{(t,s)}$ is a bi-hyperideal of S. □

Now, we characterize $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals based on $\in \vee q$-level subsets.

Theorem 3.7. *Let $A = \langle \mu_A, \lambda_A \rangle$ be an IFS in S. Then, A is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S if and only if $[A]_{(t,s)} \neq \emptyset$ is a bi-hyperideal of S for all $(t_1, t_2 \in (0, 0.5]$ and $s_1, s_2 \in [0.5, 1))$ or $(t_1, t_2 \in (0.5, 1]$ and $s_1, s_2 \in [0, 0.5))$.*

Proof. Assume that $A = \langle \mu_A, \lambda_A \rangle$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S and $(t_1, t_2 \in (0, 0.5]$ and $s_1, s_2 \in [0.5, 1))$ or $(t_1, t_2 \in (0.5, 1]$ and $s_1, s_2 \in [0, 0.5))$ such that $[A]_{(t,s)} \neq \emptyset$. Let $x, y \in [A]_{(t,s)}$ and $w \in S$. Then, $(\mu_A(x) \ge t$ and $\lambda_A(x) \le s)$ or $(\mu_A(x) + t > 1$ and $\lambda_A(x) + s < 1)$, $(\mu_A(y) \ge t$ and $\lambda_A(y) \le s)$ or $(\mu_A(y) + t > 1$ and $\lambda_A(y) + s < 1)$. We can consider four cases:

(i) $\mu_A(x) \ge t$ and $\lambda_A(x) \le s$, $\mu_A(y) \ge t$ and $\lambda_A(y) \le s$,
(ii) $\mu_A(x) \ge t$ and $\lambda_A(x) \le s$, $\mu_A(y) + t > 1$ and $\lambda_A(y) + s < 1$,
(iii) $\mu_A(x) + t > 1$ and $\lambda_A(x) + s < 1$, $\mu_A(y) \ge t$ and $\lambda_A(y) \le s$,
(iv) $\mu_A(x) + t > 1$ and $\lambda_A(x) + s < 1$, $\mu_A(y) + t > 1$ and $\lambda_A(y) + s < 1$.

For the first case, we have

$$\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 \geq t \wedge 0.5 = \begin{cases} 0.5, & t > 0.5 \\ t, & t \leq 0.5 \end{cases}$$

and

$$\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5 \leq s \vee 0.5 = \begin{cases} 0.5, & s < 0.5 \\ s, & s \geq 0.5. \end{cases}$$

for all $z \in x \cdot w \cdot y$. If $t \leq 0.5$ and $s \geq 0.5$, then $z \in U_{(t,s)}$. If $t > 0.5$ and $s < 0.5$, then, $\mu_A(z) + t > 0.5 + 0.5 = 1$ and $\lambda_A(z) + s < 0.5 + 0.5 = 1$. Hence, $z(t,s)qA$. Therefore, $z \in U_{(t,s)} \cup A_{(t,s)} = [A]_{(t,s)}$, for all $z \in x \cdot w \cdot y$. For the case (ii), assume that $t > 0.5$ and $s < 0.5$. Then, $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 = \mu_A(y) \wedge 0.5 > 1 - t \wedge 0.5 = 1 - t$ and $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5 = \lambda_A(y) \vee 0.5 < 1 - s \vee 0.5 = 1 - s$. This means that $\mu_A(z) + t > 1$ and $\lambda_A(z) + s < 1$ and so $z(t,s)qA$. If $t \leq 0.5$ and $s \geq 0.5$, then $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 \geq t \wedge 1 - t \wedge 0.5 = t$ and $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5 \leq s \vee 1 - s \vee 0.5 = s$, and so $z(t,s) \in A$. Therefore, $z \in A_{(t,s)} \cup U_{(t,s)} = [A]_{(t,s)}$, for all $z \in x \cdot w \cdot y$. We have similar way in the case (iii). For the final case, if $t > 0.5$ and $s < 0.5$, then $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 > 1 - t \wedge 0.5 = 1 - t$ and $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5 < 1 - s \vee 0.5 = 1 - s$. Hence, $\mu_A(z) + t > 1$ and $\lambda_A(z) + s < 1$ and thus $z(t,s)qA$. If $t \leq 0.5$ and $s \geq 0.5$, then $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5 \geq 1 - t \wedge 0.5 = 0.5 \geq t$ and $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5 \leq 1 - s \vee 0.5 = 0.5 \leq s$. Hence, $z(t,s) \in A$. Therefore, $z \in A_{(t,s)} \cup U_{(t,s)} = [A]_{(t,s)}$, for all $z \in x \cdot w \cdot y$. Thus, in any case, we have $z \in [A]_{(t,s)}$, for all $z \in x \cdot w \cdot y$. Similarly, for all $x, y \in S$ and $z \in x \cdot y$, we can show that $z \in [A]_{(t,s)}$. Therefore, $[A]_{(t,s)}$ is a bi-hyperideal of S for all ($t_1, t_2 \in (0, 0.5]$ and $s_1, s_2 \in [0.5, 1)$) or ($t_1, t_2 \in (0.5, 1]$ and $s_1, s_2 \in [0, 0.5)$).

Conversely, suppose that $A = \langle \mu_A, \lambda_A \rangle$ be an IFS in S such that $[A]_{(t,s)}$ is a bi-hyperideal of S. Let A is not an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. Then, there exist $x, w, y \in S$ and $z \in x \cdot w \cdot y$ such that $\mu_A(z) < \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\lambda_A(z) > \lambda_A(x) \vee \lambda_A(y) \vee 0.5$. Choose t and s such that $\mu_A(z) < t < \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\lambda_A(z) > s > \lambda_A(x) \vee \lambda_A(y) \vee 0.5$. This implies that $x, y \in [A]_{(t,s)}$ and so $z \in [A]_{(t,s)}$. Hence, $\mu_A(z) \geq t$ and $\lambda_A(z) \leq s$ which is contradiction. Therefore, we have $\mu_A(z) \geq \mu_A(x) \wedge \mu_A(y) \wedge 0.5$ and $\lambda_A(z) \leq \lambda_A(x) \vee \lambda_A(y) \vee 0.5$, for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$. The same result holds for all $x, y \in S$ and $z \in x \cdot y$. Hence, A is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. □

4. Prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals

In this section, we describe semiprime and prime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals of semihypergroups and investigate some properties of these structures. Results formulated for prime and semiprime ($(\in, \in \vee q)$-intuitionistic fuzzy) bi-hyperideals will be proved only for prime ($(\in, \in \vee q)$-intuitionistic fuzzy) bi-hyperideals. The proofs for semiprime ($(\in, \in \vee q)$-intuitionistic fuzzy) bi-hyperideals can be obtained from the proofs for prime ($(\in, \in \vee q)$-intuitionistic fuzzy) bi-hyperideals by putting $x = y$.

Definition 4.1. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S is called semiprime if for all $x, w \in S$, ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$);*

$z(t,s) \in A$ implies that $x(t,s) \in \vee qA$, for all $z \in x \cdot w \cdot x$. An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S is called prime if for all $x, w, y \in S$, ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$); $z(t,s) \in A$ implies that $x(t,s) \in \vee qA$ or $y(t,s) \in \vee qA$, for all $z \in x \cdot w \cdot y$.

Now, we have a characterization of prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals.

Theorem 4.1. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal $A = \langle \mu_A, \lambda_A \rangle$ of S is prime if and only if, for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$, $\mu_A(x) \vee \mu_A(y) \geq \mu_A(z) \wedge 0.5$, and $\lambda_A(x) \wedge \lambda_A(y) \leq \lambda_A(z) \vee 0.5$.*

Proof. Let $A = \langle \mu_A, \lambda_A \rangle$ be a prime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. Suppose that for some $x, w, y \in S$ and $z \in x \cdot w \cdot y$ we have $\mu_A(x) \vee \mu_A(y) < \mu_A(z) \wedge 0.5$ and $\lambda_A(x) \wedge \lambda_A(y) > \lambda_A(z) \vee 0.5$. Then, $\mu_A(x) \vee \mu_A(y) < t < \mu_A(z) \wedge 0.5$ and $\lambda_A(x) \wedge \lambda_A(y) > s > \lambda_A(z) \vee 0.5$, for some t and s. This means that $z(t,s) \in A$ but $x(t,s) \overline{\in \vee q} A$ and $y(t,s) \overline{\in \vee q} A$, a contradict. Hence, $\mu_A(x) \vee \mu_A(y) \geq \mu_A(z) \wedge 0.5$ and $\lambda_A(x) \wedge \lambda_A(y) \leq \lambda_A(z) \vee 0.5$, for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$.

Conversely, assume that $\mu_A(x) \vee \mu_A(y) \geq \mu_A(z) \wedge 0.5$, and $\lambda_A(x) \wedge \lambda_A(y) \leq \lambda_A(z) \vee 0.5$ hold for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$ and let ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$). Then, $z(t,s) \in A$ implies that $\mu_A(x) \vee \mu_A(y) \geq t \wedge 0.5$ and $\lambda_A(x) \wedge \lambda_A(y) \leq s \vee 0.5$. Form this, for $t \leq 0.5$ and $s \geq 0.5$, we conclude that either $\mu_A(x) \geq t$ and $\lambda_A(x) \leq s$ or $\mu_A(y) \geq t$ and $\lambda_A(y) \leq s$. Thus, either $x(t,s) \in A$ or $y(t,s) \in A$. For $t > 0.5$ and $s < 0.5$ we obtain $\mu_A(x) \vee \mu_A(y) \geq 0.5$ and $\lambda_A(x) \wedge \lambda_A(y) \leq 0.5$, i.e., either $\mu_A(x) + t > 0.5 + 0.5 = 1$ and $\lambda_A(x) + s < 0.5 + 0.5 = 1$ or $\mu_A(y) + t > 0.5 + 0.5 = 1$ and $\lambda_A(y) + s < 0.5 + 0.5 = 1$. Hence, either $x(t,s)qA$ or $y(t,s)qA$. Therefore, $x(t,s) \in \vee qA$ or $y(t,s) \in \vee qA$. This completes the proof. □

Corollary 4.1. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S is semiprime if and only if $\mu_A(x) \geq \mu_A(z) \wedge 0.5$ and $\lambda_A(x) \leq \lambda_A(z) \vee 0.5$, for all $x, w \in S$ and $z \in x \cdot w \cdot x$.*

Example 4.1. *(i) Let $A = \langle \mu_A, \lambda_A \rangle$ be an IFS of $(\mathbb{N}, \odot)$ defined in Example 3.1. By Theorem 4.1, it is easy to check that $A = \langle \mu_A, \lambda_A \rangle$ is a prime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of $(\mathbb{N}, \odot)$.*
(ii) Let $S = \{a, b, c, d\}$. Then, $(S, \cdot)$ is a semihypergroup where "$\cdot$" is defined by the following table

$\cdot$	a	b	c	d
a	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
b	$\{a\}$	$\{a\}$	$\{a\}$	$\{a\}$
c	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a\}$
d	$\{a\}$	$\{a\}$	$\{a, b\}$	$\{a, b\}$

Now, define an intuitionistic fuzzy set $A = \langle \mu_A, \lambda_A \rangle$ on S by $\mu_A(a) = \mu_A(c) = 0.7$, $\mu_A(b) = 0.6$, $\mu_A(d) = 0.8$ and $\lambda_A(a) = 0.2$, $\lambda_A(b) = 0.3$, $\lambda_A(c) = \lambda_A(d) = 0.1$. By Corollary 4.1, it is routine to see that $A = \langle \mu_A, \lambda_A \rangle$ is a semiprime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S.

Theorem 4.2. *The intersection of any family of prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S is a prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S.*

Proof. It follows from Theorem 3.2. □

Definition 4.2. *A bi-hyperideal P of S is said to be semiprime if for all $x, w \in S$; $x \cdot w \cdot x \subseteq P$ implies that $x \in P$. A bi-hyperideal P of S is said to be prime if for all $x, w, y \in S$; $x \cdot w \cdot y \subseteq P$ implies that $x \in P$ or $y \in P$.*

Example 4.2. *Let $S = \{x, y\}$ be a semihypergroup with the following table*

$\cdot$	x	y
x	$\{x\}$	S
y	$\{y\}$	$\{y\}$

Then, it is not difficult to see that the set $\{y\}$ is a prime (semiprime) bi-hyperideal of S.

Finally, we characterize prime (semiprime) $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideals based on $\in$-level sets and $\in \vee q$-level sets.

Theorem 4.3. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal $A = \langle \mu_A, \lambda_A \rangle$ of S is prime (semiprime) if and only if for $0 < t \leq 0.5$ and $0.5 \leq s < 1$, each non-empty $U_{(t,s)}$is a prime (semiprime) bi-hyperideal of S.*

Proof. Let $A = \langle \mu_A, \lambda_A \rangle$ be a prime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S and $t \in (0, 0.5]$ and $s \in [0.5, 1)$. Then, by Theorem 3.5, each non-empty $U_{(t,s)}$ is a bi-hyperideal of S. Let $x, w, y \in S$ and $z \in x \cdot w \cdot y$. By Theorem 4.1, for each $z \in U_{(t,s)}$ we have $\mu_A(x) \vee \mu_A(y) \geq \mu_A(z) \wedge 0.5 \geq t \wedge 0.5 = t$. and $\lambda_A(x) \wedge \lambda_A(y) \leq \lambda_A(z) \vee 0.5 \leq s \vee 0.5 = s$. So, $\mu_A(x) \geq t$ and $\lambda_A(x) \leq s$ or $\mu_A(y) \geq t$ and $\lambda_A(y) \leq s$. Thus $x \in U_{(t,s)}$ or $y \in U_{(t,s)}$. Hence $U_{(t,s)}$ is a prime bi-hyperideal of S.

Conversely, assume that $U_{(t,s)} \neq \emptyset$ is a prime bi-hyperideal of S, for $t \in (0, 0.5]$ and $s \in [0, 0.5)$. Then, by Theorem 3.5, $A = \langle \mu_A, \lambda_A \rangle$ is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. Let $x, w, y \in S$ and $z \in x \cdot w \cdot y$ such that $z(t, s) \in A$. Then, $z \in U_{(t,s)}$, so either $x \in U_{(t,s)}$ or $y \in U_{(t,s)}$. That is $x(t, s) \in A$ or $y(t, s) \in A$. Thus, $x(t, s) \in \vee qA$ or $y(t, s) \in \vee qA$. Therefore, $A = \langle \mu_A, \lambda_A \rangle$ is a prime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. □

Theorem 4.4. *An $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal $A = \langle \mu_A, \lambda_A \rangle$ of S is prime (semiprime) if and only if for all ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$) each $[A]_{(t,s)}$ is a prime (semiprime) bi-hyperideal of S.*

Proof. If $A = \langle \mu_A, \lambda_A \rangle$ is a prime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S, then for all ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$) each $[A]_{(t,s)}$ is non-empty and according to Theorem 3.7, each $[A]_{(t,s)}$ is a bi-hyperideal of S. To prove that it is prime let $x, w, y \in S$ be such that $z \in [A]_{(t,s)}$, for all $z \in x \cdot w \cdot y$. Since, $[A]_{(t,s)} = A_{(t,s)} \cup U_{(t,s)}$, we have $z \in A_{(t,s)}$ or $z \in U_{(t,s)}$. At first, we consider the case when $z \in A_{(t,s)} \setminus U_{(t,s)}$. In this case $\mu_A(z) + t > 1$, $\lambda_A(z) + s < 1$ and $\mu_A(z) < t$, $\lambda_A(z) > s$, for $\mu_A(z) \leq 0.5$ and $\lambda_A(z) \geq 0.5$, we obtain $\mu_A(x) \vee \mu_A(y) + t \geq \mu_A(z) \wedge 0.5 + t = \mu_A(z) + t > 1$ and $\lambda_A(x) \wedge \lambda_A(y) + s \leq \lambda_A(z) \vee 0.5 + s = \lambda_A(z) + s < 1$. This proves that $x \in A_{(t,s)} \subseteq [A]_{(t,s)}$ or $y \in A_{(t,s)} \subseteq [A]_{(t,s)}$. For $\mu_A(z) > 0.5$ and $\lambda_A(z) < 0.5$ we have, $0.5 < \mu_A(z) < t$ and $s < \lambda_A(z) < 0.5$. Consequently, $\mu_A(x) \vee \mu_A(y) + t \geq \mu_A(z) \wedge 0.5 + t = 0.5 + t > 1$ and $\lambda_A(x) \wedge \lambda_A(y) + s \leq \lambda_A(z) \vee 0.5 + s = 0.5 + s < 1$. Thus, $x \in A_{(t,s)} \subseteq [A]_{(t,s)}$ or $y \in A_{(t,s)} \subseteq [A]_{(t,s)}$. For $\mu_A(z) < 0.5$ and $\lambda_A(z) < 0.5$ we have, $\mu_A(x) \vee \mu_A(y) + t \geq \mu_A(z) \wedge 0.5 + t = \mu_A(z) + t > 1$ and $s < \lambda_A(z) < 0.5$ and so, $\lambda_A(x) \wedge \lambda_A(y) + s \leq \lambda_A(z) \vee 0.5 + s = 0.5 + s < 1$. Thus, $x \in A_{(t,s)} \subseteq [A]_{(t,s)}$ or $y \in A_{(t,s)} \subseteq [A]_{(t,s)}$. So, $z \in A_{(t,s)} \setminus U_{(t,s)}$ implies that $x \in [A]_{(t,s)}$ or $y \in [A]_{(t,s)}$. Now, let $z \in U_{(t,s)}$. In this case $\mu_A(z) \geq t$ and $\lambda_A(z) \leq s$. Hence, for $t \leq 0.5$ and $s \geq 0.5$, we obtain $\mu_A(x) \vee \mu_A(y) \geq \mu_A(z) \wedge 0.5 \geq t$ and $\lambda_A(x) \wedge \lambda_A(y) \leq \lambda_A(z) \vee 0.5 \leq s$. Thus, $x \in U_{(t,s)} \subseteq [A]_{(t,s)}$ or $y \in U_{(t,s)} \subseteq [A]_{(t,s)}$. If $t > 0.5$ and $s < 0.5$, then $\mu_A(x) \vee \mu_A(y) \geq t \wedge 0.5 > 0.5$ and $\lambda_A(x) \wedge \lambda_A(y) \leq s \vee 0.5 < 0.5$, and consequently $\mu_A(x) \vee \mu_A(y) + t > 1$ and $\lambda_A(x) \wedge \lambda_A(y) + s < 1$. Therefore, $x \in A_{(t,s)} \subseteq [A]_{(t,s)}$ or $y \in A_{(t,s)} \subseteq [A]_{(t,s)}$. So, in any case $z \in [A]_{(t,s)}$ implies that $x \in [A]_{(t,s)}$ or $y \in [A]_{(t,s)}$. Hence, $[A]_{(t,s)}$ is a prime bi-hyperideal of S.

Conversely, if $A = \langle \mu_A, \lambda_A \rangle$ be an IFS of S such that $[A]_{(t,s)}$ is a prime bi-hyperideal of S for each ($t \in (0, 0.5]$ and $s \in [0.5, 1)$) or ($t \in (0.5, 1]$ and $s \in [0, 0.5)$), then according to Theorem 3.7, A is an $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. Since $[A]_{(t,s)}$ is a prime bi-hyperideal, from $z(t,s) \in A$, for all $x, w, y \in S$ and $z \in x \cdot w \cdot y$, it follows that $z \in U_{(t,s)} \subseteq [A]_{(t,s)}$, whence we obtain, $x \in [A]_{(t,s)}$ or $y \in [A]_{(t,s)}$. This implies that $x(t,s) \in \vee qA$ or $y(t,s) \in \vee qA$. Therefore, A is a prime $(\in, \in \vee q)$-intuitionistic fuzzy bi-hyperideal of S. □

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