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DYNAMICS ANALYSIS OF THE 3-DOF PARALLEL ROBOT WITH PRISMATIC ACTUATORS

D. ZHANG[1], Şt. STAICU[2]

Recursive matrix relations for kinematics and dynamic analysis of a parallel robot with prismatic actuators are established in the paper. Three active links of the robot, hydraulically or pneumatically actuated posses three independent degrees of freedom. Given the platform's motion, the inverse kinematical problem is developed, to determine the positions, velocities and accelerations of the mechanism. Using the method of virtual powers, the inverse dynamics problem is solved. Finally, matrix relations and graphs of the powers of three actuators are determined.

Lucrarea prezentă stabileşte relaţii matriceale pentru cinematica şi dinamica unui robot paralel cu acţionare prismatică. Cele trei elemente active ale robotului, acţionate hidraulic sau pneumatic posedă trei grade de libertate independente. Fiind dată mişcarea platformei, se dezvoltă o problemă cinematică inversă pentru a determina poziţiile, vitezele şi acceleraţiile mecanismului. Utilizând metoda puterilor virtuale, se rezolvă problema de dinamică inversă a manipulatorului. În partea finală a lucrării se determină relaţii matriceale şi grafice pentru puterile celor trei sisteme active.

Keywords: dynamics, parallel robot, virtual power

1. Introduction

Generally, a parallel manipulator consists of two platforms: one is attached to the fixed reference frame and the other one may exhibit arbitrary motions into its workspace. Several mobile legs or limbs, structured as serial robots, connect the end-effectors, which is interposed between the moving platform and the fixed

[1] Prof., Faculty of Engineering and Applied Science, University of Ontario Institute of Technology, CANADA
[2] Prof.; Department of Mechanics, University "Politehnica" of Bucharest, ROMANIA

base. Typically, the number of actuating legs is equal to the number of degrees of freedom of the system such that one actuator controls one leg and all the actuator can be mounted at or near the fixed base. For that reason, parallel manipulators are sometimes called *platform manipulators*. Because the actuators share the external load, parallel manipulators are able to manifest a large load-carrying capacity.

A parallel manipulator is said to be symmetrical if it satisfies two conditions. First, the number of actuating limbs is equal to the number of degrees of freedom of the moving platform and, second, the number and locations of actuated joints in all the limbs are the same (Tsai, 1999 [1]). The links of the robot are connected one to another by ball-joints, helical joints, and revolute or prismatic joints.

Compared with serial robots, parallel mechanisms possess some specific characteristics: greater structural rigidity, potentially higher precision, stabile functioning, larger capacity and a suitable position of actuating systems. Parallel manipulators had found many technical applications, where it is desired to position a body with high speed. Accuracy and precision of the task are essential since the robot is intended to operate on fragile objects and any errors in the tool positioning could lead to expensive damages.

Many efforts have been devoted recently to the kinematics and dynamic analysis of fully parallel manipulators and a series of companies have developed them as high precision machine tools. Such a typical platform is the 6-DOF aircraft simulator, a Gough-Stewart platform in fact (Stewart, 1965 [2]; Merlet, 2000 [3]). The parallel Star manipulator (Hervé and Sparacino, 1992 [4]; Tremblay and Baron, 1999 [5]) and the parallel Delta robot (Clavel, 1988 [6]; Staicu and Carp-Ciocardia, 2003 [7]; Tsai and Stamper, 1996 [8]) equipped with three motors in parallel setting, train on the effector in a 3-DOF translation motion. Angeles (1997 [9]), Wang (1998 [10]) and Gosselin (1989 [11]) developed the direct dynamic analysis of a prototype Agile Wrist spherical manipulator, which has three concurrent rotations. Kane and Levinson (1985 [12]) obtained some vector recursive relations concerning the equilibrium of generalized forces that are applied to a serial manipulator.

2. Inverse kinematics of the robot

A spatial 3-DOF parallel mechanism (Zhang, 2001 [13]), which can be used in several applications, including machine tools is proposed in this paper and the geometric, kinematical and dynamic analysis is conducted.

Let $Ox_0y_0z_0\,(T_0)$ be a fixed Cartesian frame. A spatial manipulator of three degrees of freedom is moving with respect to this frame; the mechanism consists of four kinematical chains, including three variable length legs with identical topology and one passive constraining leg, all connecting the fixed base to a moving platform (Fig. 1). All the dimensions and masses are known. In this 3-DOF parallel mechanism, a kinematical chain, associated with one of the three identical active legs, was introduced between the base and the moving platform. It consists of a fixed Hook joint, characterized by the angular velocity $\omega_{10}^{A}=\dot{\varphi}_{10}^{A}$ and the angular acceleration $\varepsilon_{10}^{A}=\ddot{\varphi}_{10}^{A}$, and a moving link of length l_2, mass m_2 and tensor of inertia $\hat{J}_2$, which has a relative rotation with the angle φ_{21}^{A}, so that $\omega_{21}^{A}=\dot{\varphi}_{21}^{A}$, $\varepsilon_{21}^{A}=\ddot{\varphi}_{21}^{A}$. A first actuated prismatic joint establishes a moving link to the $A_3x_3^Ay_3^Az_3^A\,(T_3^A)$ frame, having a relative displacement λ_{32}^{A}, velocity $v_{32}^{A}=\dot{\lambda}_{32}^{A}$ and acceleration $\gamma_{32}^{A}=\ddot{\lambda}_{32}^{A}$. It has the length l_3, mass m_3 and tensor of inertia $\hat{J}_3$. Finally, a ball-joint is attached to the platform. The fourth constraining chain has a different architecture. It consists of a prismatic joint attached to the base and a moving link of length l_1 and mass m_1, having a purely vertical displacement λ_{10}^{D}, velocity $v_{10}^{D}=\dot{\lambda}_{10}^{D}$ and acceleration $\gamma_{10}^{D}=\ddot{\lambda}_{10}^{D}$. A Hook joint is attached to the moving platform. This last platform is equivalent to an equilateral triangle with the edge l, mass m_4 and tensor of inertia $\hat{J}_4$.

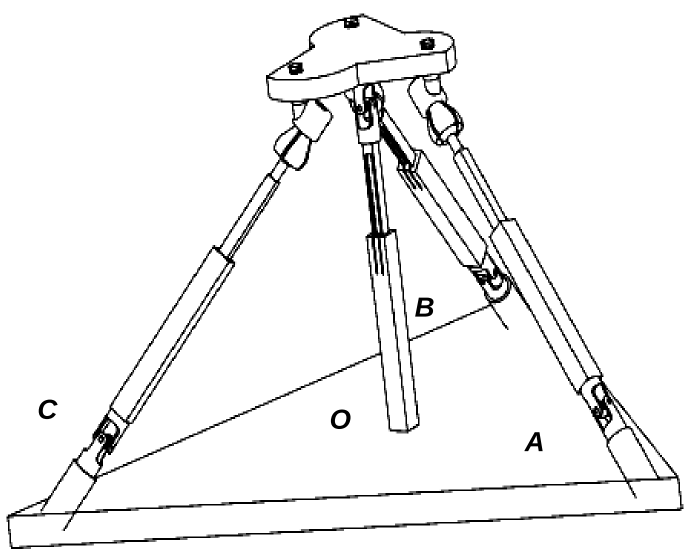

Fig. 1 Parallel manipulator with prismatic actuators

Rotations are defined by the angles φ_{21}^{D} and φ_{32}^{D} in the local coordinates (Fig. 2). The orientation of the joints *A*, *B*, *C* on the fixed platform (Fig. 1) is given by corresponding angles:

$$\alpha_A = 0,\ \alpha_B = \frac{2\pi}{3}\ \alpha_C = -\frac{2\pi}{3}. \tag{1}$$

Let us consider the displacements $\lambda_{10}^{A}, \lambda_{10}^{B}, \lambda_{10}^{C}$ of the prismatic actuators A_3, B_3, C_3, three parameters, which give the instantaneous position of the mechanism. In the inverse kinematic problem, it is convenient to consider that the independent coordinates of the platform have been chosen as $z_0^G, \alpha_1, \alpha_2$, where z_0^G is the height of the platform and α_1, α_2 are the angles of the Hook joint, giving the absolute position of the moving platform.

Pursuing the passive constraining leg D on the $OD_1D_2D_3$ way, the passing matrices are derived:

$$d_{10} = \hat{e},\ d_{21} = d_{21}^{\varphi} a_1,\ d_{32} = d_{32}^{\varphi} a_2. \tag{2}$$

Now, considering the $OA_1A_2A_3A_4$ track of the limb A, the transfer matrices are given by

$$a_{10} = a_{10}^{\varphi} a_1 a_{\alpha}^{A},\ a_{21} = a_{21}^{\varphi} a_{\beta} a_2,\ a_{32} = a_1\ , \tag{3}$$

where (Staicu, 1998 [14]):

$$a_1 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, a_2 = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, a_{\alpha}^{A} = \begin{bmatrix} \cos\alpha_A & \sin\alpha_A & 0 \\ -\sin\alpha_A & \cos\alpha_A & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a_{\beta} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}, a_{k,k-1}^{\varphi} = \begin{bmatrix} \cos\varphi_{k,k-1}^{A} & \sin\varphi_{k,k-1}^{A} & 0 \\ -\sin\varphi_{k,k-1}^{A} & \cos\varphi_{k,k-1}^{A} & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$a_{k0} = \prod_{j=1}^{k} a_{k-j+1,k-j}\ ,\ \ (k = 1,2,3).$$

Analogous relations can be written for the other two loops O-B and O-C of the mechanism (Fig. 2).

Suppose the absolute motion of the platform is given by

$$z_0^G = l_1 + l_6 + z_0^{G*}(1 - \cos\frac{2\pi}{3}t)$$

$$\alpha_i = \alpha_i^{*}(1 - \cos\frac{2\pi}{3}t) \quad (i = 1,2). \tag{5}$$

In order to solve the inverse kinematic problem, consider first the passive constraining leg as a serial 3-DOF mechanism with the coordinates determined by the conditions

$$d_{30}^{\circ T} d_{30} = a,\ \vec{u}_3^{\,T}\{\vec{r}_{10}^{\,D} + \lambda_{10}^{D}\vec{u}_3 + d_{10}^{T}\vec{r}_{21}^{\,D}\} = z_0^G \tag{6}$$

where $a = \theta_2\theta_1$ is a rotation matrix from the frame $Ox_0y_0z_0$ to $Gx_Gy_Gz_G$, and

$$\vec{r}_{10}^{\,D} = l_6 \vec{u}_3, \;\; \vec{r}_{21}^{\,D} = l_1 \vec{u}_3$$

$$d_{30}^{\circ T} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, \;\; \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \;\; \widetilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\theta_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha_1 & \sin\alpha_1 \\ 0 & -\sin\alpha_1 & \cos\alpha_1 \end{bmatrix}, \theta_2 = \begin{bmatrix} \cos\alpha_2 & 0 & -\sin\alpha_2 \\ 0 & 1 & 0 \\ \sin\alpha_2 & 0 & \cos\alpha_2 \end{bmatrix}; \tag{7}$$

it results

$$\lambda_{10}^{D} = z_0^G - l_1 - l_6, \;\; \varphi_{21}^D = \alpha_1, \;\; \varphi_{32}^D = \alpha_2 . \tag{8}$$

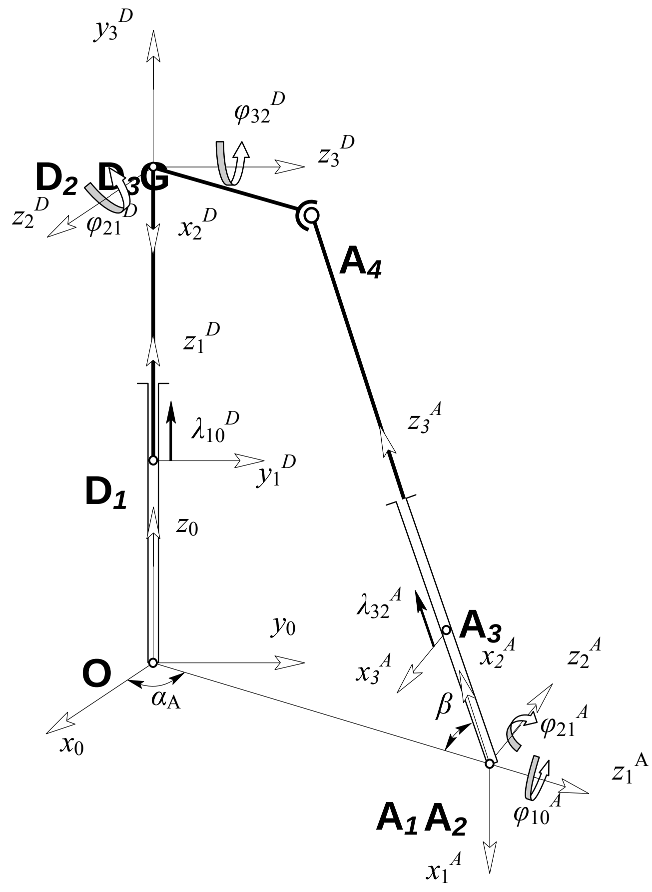

Fig. 2 Kinematical scheme of the manipulator

Once the solution of the inverse kinematic of this 3-DOF serial chain found, the complete position and orientation of the limbs A, B, C is determined by the following geometric conditions

$$\vec{r}_{10}^{A} + \sum_{k=1}^{3} a_{k0}^{T} \vec{r}_{k+1,k}^{A} - d_{30}^{T} \vec{r}_{3}^{A_4} = \vec{r}_{10}^{B} + \sum_{k=1}^{3} b_{k0}^{T} \vec{r}_{k+1,k}^{B} - d_{30}^{T} \vec{r}_{3}^{B_4} =$$

$$= \vec{r}_{10}^{C} + \sum_{k=0}^{3} c_{k0}^{T} \vec{r}_{k+1,k}^{C} - d_{30}^{T} \vec{r}_{3}^{C_4} = z_0^G \vec{u}_3, \tag{9}$$

where

$$\vec{r}_{10}^{A} = l_0 a_{\alpha}^{AT} \vec{u}_1, \ \vec{r}_{21}^{A} = \vec{0}, \ \vec{r}_{32}^{A} = l_5 \vec{u}_1 + \lambda_{32}^{A} a_{32}^{T} \vec{u}_3, \ \vec{r}_{43}^{A} = l_3 \vec{u}_3. \tag{10}$$

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \ \vec{r}_3^{A_4} = \begin{bmatrix} -l_4 \cos\alpha_A \\ 0 \\ l_4 \sin\alpha_A \end{bmatrix}.$$

3. Velocities and accelerations

The motion of all links (for example of leg A) are characterized by the following skew-symmetric matrices (Staicu, 1999 [15]:

$$\tilde{\omega}_{k0}^{A} = a_{k,k-1} \tilde{\omega}_{k-1,0}^{A} a_{k,k-1}^{T} + \omega_{k,k-1}^{A} \tilde{u}_3. \tag{11}$$

These matrices are *associated* to the absolute angular velocities, given by the recurrence relations

$$\vec{\omega}_{k0}^{A} = a_{k,k-1} \vec{\omega}_{k-1,0}^{A} + \omega_{k,k-1}^{A} \vec{u}_3. \tag{12}$$

Relation expresses the velocity $\vec{v}_{k0}^{A}$ of the joint A_k

$$\vec{v}_{k0}^{A} = a_{k,k-1} \{ \vec{v}_{k-1,0}^{A} + \tilde{\omega}_{k-1,0}^{A} \vec{r}_{k,k-1}^{A} \} + v_{k,k-1}^{A} \vec{u}_3,$$
$$\vec{v}_{\sigma,\sigma-1}^{A} = \vec{0} \qquad (\sigma = 1, 2, 4). \tag{13}$$

The following *matrix conditions of connectivity* constitute the inverse kinematical model

$$\omega_{10}^{A} \vec{u}_i^T a_{10}^T \tilde{u}_3 a_{21}^T \{ \vec{r}_{32}^{A} + a_{32}^T \vec{r}_{43}^{A} \} + \omega_{21}^{A} \vec{u}_i^T a_{20}^T \tilde{u}_3 \{ \vec{r}_{32}^{A} + a_{32}^T \vec{r}_{43}^{A} \} +$$
$$+ v_{32}^{A} \vec{u}_i^T a_{30}^T \vec{u}_3 = v_{10}^{D} \vec{u}_i^T \vec{u}_3 + \vec{u}_i^T d_{30}^T \tilde{\omega}_{30}^{D} \vec{r}_3^{A_4}. \tag{14}$$

These relations generate Jacobi matrix of the mechanism, defining the robot workspace. From equation (14), relative velocities $\omega_{10}^{A}, \omega_{21}^{A}, v_{32}^{A}$ result as functions of platform's characteristic velocities

$$v_{10}^{D} = \dot{z}_{0}^{G}\,,\;\; \omega_{21}^{D} = \dot{\alpha}_{1}\,,\;\; \omega_{32}^{D} = \dot{\alpha}_{2} \tag{15}$$

Let us assume that the robot has a virtual motion determined by the angular velocities $v_{32a}^{Av} = 1, v_{32a}^{Bv} = 0, v_{32a}^{Cv} = 0$. The characteristic virtual velocities, expressed as function of manipulator's position, are given by new conditions of connectivity of relative velocities

$$\vec{u}_i^{\,T} a_{30}^{T} \{\vec{v}_{30a}^{\,Av} + \widetilde{\omega}_{30a}^{Av} \vec{r}_{43}^{\,A}\} = \vec{u}_i^{\,T} d_{30}^{T} \{\vec{v}_{30a}^{\,Dv} + \widetilde{\omega}_{30a}^{Dv} \vec{r}_{3}^{\,A_4}\}\,. \tag{16}$$

Other two compatibility relations of the loops *O-B* and *O-C* can be obtained, if one considers successively that $v_{32b}^{Bv} = 1, v_{32b}^{Cv} = 0, v_{32b}^{Av} = 0$ and $v_{32c}^{Cv} = 1, v_{32c}^{Av} = 0, v_{32c}^{Bv} = 0$.

New relations of connectivity give the angular accelerations $\varepsilon_{10}^{A}, \varepsilon_{21}^{A}, \gamma_{32}^{A}$:

$$\begin{aligned}
&\varepsilon_{10}^{A} \vec{u}_i^{\,T} a_{10}^{T} \widetilde{u}_3 a_{21}^{T} \{\vec{r}_{32}^{\,A} + a_{32}^{T} \vec{r}_{43}^{\,A}\} + \varepsilon_{21}^{A} \vec{u}_i^{\,T} a_{20}^{T} \widetilde{u}_3 \{\vec{r}_{32}^{\,A} + a_{32}^{T} \vec{r}_{43}^{\,A}\} + \\
&+ \gamma_{32}^{A} \vec{u}_i^{\,T} a_{30}^{T} \vec{u}_3 = \gamma_{10}^{D} \vec{u}_i^{\,T} \vec{u}_3 + \vec{u}_i^{\,T} d_{30}^{T} \{\widetilde{\omega}_{30}^{D} \widetilde{\omega}_{30}^{D} + \widetilde{\varepsilon}_{30}^{D}\} \vec{r}_{3}^{\,A_4} - \\
&- \omega_{10}^{A} \omega_{10}^{A} \vec{u}_i^{\,T} a_{10}^{T} \widetilde{u}_3 \widetilde{u}_3 a_{21}^{T} \{\vec{r}_{32}^{\,A} + a_{32}^{T} \vec{r}_{43}^{\,A}\} - \\
&- \omega_{21}^{A} \omega_{21}^{A} \vec{u}_i^{\,T} a_{20}^{T} \widetilde{u}_3 \widetilde{u}_3 \{\vec{r}_{32}^{\,A} + a_{32}^{T} \vec{r}_{43}^{\,A}\} - \\
&- 2\omega_{10}^{A} \omega_{21}^{A} \vec{u}_i^{\,T} a_{10}^{T} \widetilde{u}_3 a_{21}^{T} \widetilde{u}_3 \{\vec{r}_{21}^{\,A} + a_{32}^{T} \vec{r}_{43}^{\,A}\} - \\
&- 2\omega_{10}^{A} v_{32}^{A} \vec{u}_i^{\,T} a_{10}^{T} \widetilde{u}_3 a_{21}^{T} a_{32}^{T} \vec{u}_3 - 2\omega_{21}^{A} v_{32}^{A} \vec{u}_i^{\,T} a_{20}^{T} \widetilde{u}_3 a_{32}^{T} \vec{u}_3 .
\end{aligned} \tag{17}$$

When the other two kinematical chains are pursued, analogous relations can be easily obtained.

The following relations give the angular accelerations $\vec{\varepsilon}_{k0}^{\,A}$ and the linear once $\vec{\gamma}_{k0}^{\,A}$ of the joints

$$\begin{aligned}
&\vec{\varepsilon}_{k0}^{\,A} = a_{k,k-1}\, \vec{\varepsilon}_{k-1,0}^{\,A} + \varepsilon_{k,k-1}^{A}\, \vec{u}_3 + \omega_{k,k-1}^{A}\, a_{k,k-1}\, \widetilde{\omega}_{k-1,0}^{A}\, a_{k,k-1}^{T}\, \vec{u}_3 \\
&\widetilde{\omega}_{k0}^{A}\, \widetilde{\omega}_{k0}^{A} + \widetilde{\varepsilon}_{k0}^{A} = a_{k,k-1}\, \{\widetilde{\omega}_{k-1,0}^{A}\, \widetilde{\omega}_{k-1,0}^{A} + \widetilde{\varepsilon}_{k-1,0}^{A}\}\, a_{k,k-1}^{T} + \\
&\qquad + \omega_{k,k-1}^{A}\, \omega_{k,k-1}^{A}\, \widetilde{u}_3\, \widetilde{u}_3 + \varepsilon_{k,k-1}^{A}\, \widetilde{u}_3 + 2\omega_{k,k-1}^{A}\, a_{k,k-1}\, \widetilde{\omega}_{k-1,0}^{A}\, a_{k,k-1}^{T}\, \widetilde{u}_3 \\
&\vec{\gamma}_{k0}^{\,A} = a_{k,k-1}\, \vec{\gamma}_{k-1,0}^{\,A} + a_{k,k-1}\, \{\widetilde{\omega}_{k-1,0}^{A}\, \widetilde{\omega}_{k-1,0}^{A} + \widetilde{\varepsilon}_{k-1,0}^{A}\}\, \vec{r}_{k,k-1}^{\,A}\,, \\
&\vec{\gamma}_{\sigma,\sigma-1}^{\,A} = \vec{0} \qquad (\sigma = 1, 2, 4)\,.
\end{aligned} \tag{18}$$

The equation (14), (17) represent the *inverse kinematical model* of the robot.

4. Dynamic simulation of the motion

For the inverse dynamic problem we apply here the virtual powers method. Graphs and recursive matrix relations for the forces of the three actuators are obtained, which eliminate the joint reaction forces.

Three independent pneumatic systems *A*, *B*, *C* generate the following three active forces

$$\vec{f}_{32}^{A} = f_{32}^{A}\vec{u}_3,\ \vec{f}_{32}^{B} = f_{32}^{B}\vec{u}_3,\ \vec{f}_{32}^{C} = f_{32}^{C}\vec{u}_3. \quad (19)$$

These forces have the directions of the axes $A_3z_3^A, B_3z_3^B, C_3z_3^C$ and control the motion of the manipulator's limbs. The force of inertia and the resultant moment of the forces of inertia of the rigid body T_k are determined in respect to the centre of the joint A_k, On the other hand the characteristic vectors $\vec{f}_k^*$ and $\vec{m}_k^*$ evaluate the influence of the weight action $m_k\vec{g}$ and of all other external and internal forces applied to the same T_k manipulator link.

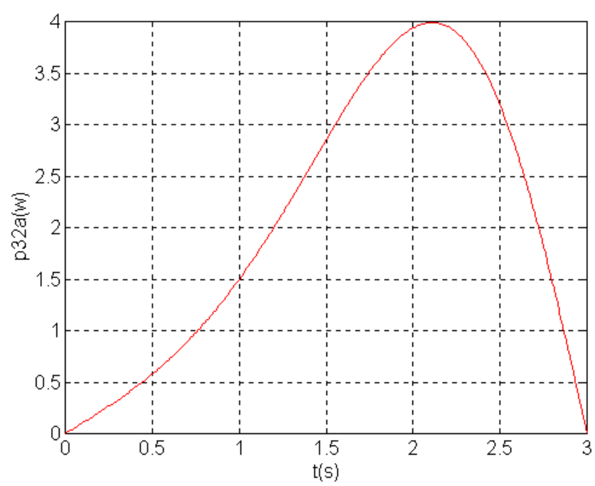

Fig. 3 Power p_{32}^{A} of first actuator

Given the absolute motion of the platform by equations (5), first the position, velocity and acceleration of each joint are determined. Consequently the pending wrench about A_k can also be determined. Assuming that negligible frictional forces act at the joints, we can then calculate the actuating forces. There are three methods that provide the same results regarding the actuator forces, namely the Newton-Euler classic procedure, the Lagrange equations and his multipliers formalism, and the principle of the virtual powers.

The fundamental principle of the virtual powers states that a mechanism is under dynamic equilibrium if and only if the virtual power developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed to the mechanism. Assuming that frictional forces at the joints are negligible, the virtual power produced by the forces of constraint at the joints is zero. Applying *the fundamental equations of the parallel robots dynamics* established by Staicu [16], the following compact matrix relation results

$$\begin{aligned} f_{32}^{A} = \vec{u}_3^{T} (\vec{F}_3^{A} + \omega_{21a}^{Av} \vec{M}_2^{A} + \omega_{10a}^{Av} \vec{M}_1^{A} + \\ + \omega_{21a}^{Bv} \vec{M}_2^{B} + \omega_{10a}^{Bv} \vec{M}_1^{B} + \omega_{21a}^{Cv} \vec{M}_2^{C} + \omega_{10a}^{Cv} \vec{M}_1^{C} + \\ + v_{10a}^{Dv} \vec{F}_1^{D} + \omega_{21a}^{Dv} \vec{M}_2^{D} + \omega_{32a}^{Dv} \vec{M}_3^{D}), \end{aligned} \tag{20}$$

where, for example, we denoted

$$\begin{aligned} \vec{F}_{k0}^{A} &= m_k^A \vec{\gamma}_{k0}^{A} + m_k^A \{ \tilde{\omega}_{k0}^{A} \tilde{\omega}_{k0}^{A} + \tilde{\varepsilon}_{k0}^{A} \} \vec{r}_k^{CA} + 9.81\, m_k^A\, a_{k0}\, \vec{u}_3 \\ \vec{M}_{k0}^{A} &= m_k^A \tilde{r}_k^{CA} \vec{\gamma}_{k0}^{A} + \hat{J}_k^A \vec{\varepsilon}_{k0}^{A} + \tilde{\omega}_{k0}^{A} \hat{J}_k^A \vec{\omega}_{k0}^{A} + 9.81\, m_k^A \tilde{r}_k^{CA} a_{k0}\, \vec{u}_3 \\ \vec{F}_k^{A} &= \vec{F}_{k0}^{A} + a_{k+1,k}^{T} \vec{F}_{k+1}^{A} \\ \vec{M}_k^{A} &= \vec{M}_{k0}^{A} + a_{k+1,k}^{T} \vec{M}_{k+1}^{A} + \tilde{r}_{k+1,k}^{A} a_{k+1,k}^{T} \vec{F}_{k+1}^{A}, \qquad (k = 1, 2, 3) \end{aligned} \tag{21}$$

The equations (20) and (21) represent the *inverse dynamic model* of the-3-DOF parallel manipulator with prismatic actuators.

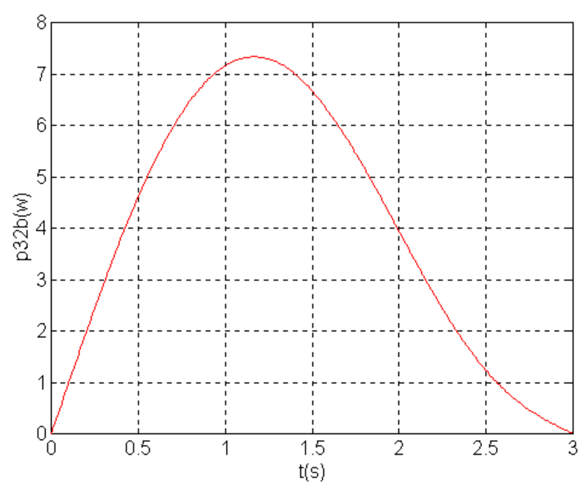

Fig. 4 Power p_{32}^{B} of first actuator

An example is given here with the following parameters

$$\alpha_1^* = \pi/12, \alpha_2^* = \pi/18, \; z_0^{G*} = 0.1\,\text{m}; \; \Delta t = 3\,\text{s}$$

$$l = 0.75\,\text{m}, D_1D_2 = l_1 = 0.65\,\text{m}, l_2 = 1.2\,\text{m}$$

$$GA_4 = l/\sqrt{3}, \; A_3A_4 = l_3 = 1.25\,\text{m}$$

$$A_2A_3 = l_5 = 0.25\,\text{m}, \; OD_1 = l_6 = 0.1\,\text{m}$$

$$\sin\beta = (l_1 + l_6)/(l_3 + l_5), l_0 = (l_3 + l_5)\cos\beta + l_4$$

$$m_1 = 1\,\text{kg}, \; m_2 = 2.5\,\text{kg}, \; m_3 = 1.5\,\text{kg}, \; m_4 = 5\,\text{kg}.$$

Finally, one obtains the graphs of the powers s $p_{32}^A = \omega_{32}^A f_{32}^A$ (Fig. 3), $p_{32}^B = \omega_{32}^B f_{32}^B$ (Fig. 4), $p_{32}^C = \omega_{32}^C f_{32}^C$ (Fig. 5) developed by the three prismatic actuators.

5. Conclusions

1. Within the inverse positional and kinematic analysis exact matrix relations for the real time computation of position, velocity and acceleration of each link of a 3-DOF parallel manipulator have been established.

2. Using the Newton-Euler classic method, which takes separately into account each of the rigid bodies of the kinematical chain since the mechanism is over constrained, the required equations of equilibrium are written for all moving links that had to be solved. All the reaction forces at the joints and the actuator forces are friendly computed from this system.

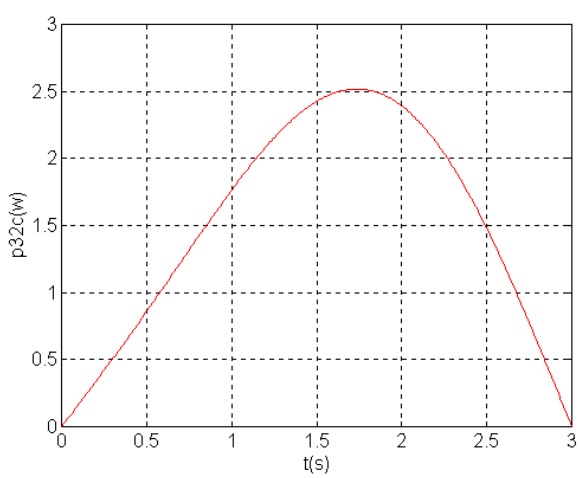

Fig. 5 Power p_{32}^C of first actuator

3. The analytical calculus involved in the Lagrange formalism is very tedious, thus presenting an elevated risk for errors. Furthermore, the duration of numerical computations grows larger when the number of bodies of the device is large. In that respect the method here developed is really advantageous and especially suited for a computerised design.

4. Based on the virtual work principle, the newly described approach renders a direct, recursive avenue to determine the real time variation of the forces and powers of all the actuators of a parallel robot. In a context of automatic control, the iterative matrix relations (20) and (21) given in this dynamic model can be easily transformed into a robust model for the computerised control of the most general parallel manipulators.

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