STUDY OF FLOW AT THE AIR – WATER INTERFACE

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În această lucrare se urmăreşte analizarea creşterii presiunii şi a vitezei în regimurile de curgere nepermanente, provocată de prezența unei pungi de aer în fața coloanei de apă, într-un sitem conservativ închis sau deschis. Analiza undei de presiune atât în mediul lichid cât şi în mediul gazos se face unidimensional, urmărindu-se efectele propagării undelor, în ipoteza separării coloanei de apă de punga de aer printr-o interfață verticală. Aceasta ipoteză simplificatoare duce la limitarea capacității predictive a modelului matematic.

The object of this study is to determine the pressure and velocity oscillation in transitory flows, caused by the presence of an air pocket in front of the water column. The hydraulic system is conservative, with the downstream end closed or opened. The pressure wave analyze is one-dimensional both in water or air phase. The mathematical model is based on the hypothesis that the water column is separated from air by a vertical interface. This simplifying hypotheses leads to limited predictive capacity of the model.

Keywords: air – water, air pocket, interface, pressure wave.

1. Introduction

When air gets inside of a pressurized water system, the two fluids move together. The air release can be useful sometimes, because the pressure wave velocity can significantly decrease. But the entrapped air can cause undesirable and consistent pressure oscillations, depending on air quantity and on its location in the pipe. When air is trapped between two water columns it can cause pressure rise over the supplying pressure [1].

The expulsion through orifices or vanes, followed by the water column, can cause water hammer pressure. If the orifice is relatively small, the flow speed is decelerated. In this situation, the pressure magnitude depends strongly on orifice size. For relatively large size of the orifice, the flow speed increases, but, the impact can be minim, because the pressure wave is decreased. In case of intermediate sizes of the orifice, the air is expulsed fast, and the water can flow with high velocity, affecting the pipe at the impact.

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The entrapped air in hydraulic systems is important in transient flow study. The mathematical models can consider, or not, the dynamic behavior of the liquid phase and of the pressure wave in gas mass. Some models consider only the inertial effect of the water column, while most models consider the elasticity of the air pocket.

Usually, the mathematical models used to study the effects of air pockets over the transient regimes have simplified configurations. This is mainly caused by reasons such as:

- pressure wave speed, because in some systems it can vary from 100 m/s to 1200 m/s, depending on air quantity, which is difficult to be determined;
- air pockets implosion, which can happen suddenly or slowly. This makes difficult the mathematical determination of air pockets behavior;
- the structure of a pipe system is very complicate with a lot of branches, slope changes, aeration shafts and other discontinuities;
- rapid transition from free to pressurized flow.

The equations used in transient flow study are: Reynolds transport theorem, mass and momentum conservation equation and water hammer equations.

For the mathematical model is considered a pipe supplied from a tank with constant pressure head. At the downstream end of the pipe, in front of the water column, there is an air pocket. The water column and the air pocket are separated by a closed vane (SV), which is opened in a very short time. The assumptions made for the mathematical model of the flow are:

- the pipe is horizontal. The difference between the model which considers the deformation of water column and the rigid water column model is smaller than 2% for rapid filling horizontal pipes [2];
- the air pocket fills the whole section of the pipe;
- the air-water interface is vertical (it is assumed that the air pocket is keeping the cylindrical shape);
- the evolution of gas mass is polytrophic;
- rigid or elastic liquid phase;
- cvasistatic or dynamic evolution of gas mass;
- one-dimensional study;
- the Darcy coefficient is the same as in uniform flow. The difference between maximum pressure in steady flow model and the maximum pressure in the unsteady flow model is less that 5% [3].

2. Pipe closed at downstream end

Figure 1 presents the scheme for the case of a rapid filling horizontal pipe closed at the downstream end which contains air. At the moment t = 0s, the separation vane and the downstream end are closed. In order to obtain the air pocket, the water downstream the separation vane is drained off (through vane DV) and the air gets inside at atmospheric pressure (through AV). The flow parameters are: H_0, upstream pressure head, v, water column velocity during rapid filling, V_0 air volume, l water column length, D pipe diameter and l_a air column length. The pipe length is $L = 10$ m and the pipe diameter is $D = 37$ mm.

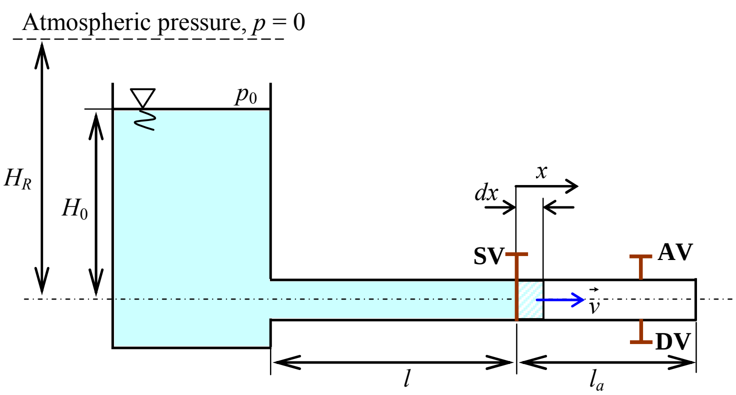

Fig. 1. Scheme for the mathematical model with the pipe closed at downstream end.

Considering the displacement of the water column at moment $t \le 0$ s the parameters are,

$$V_0 = l_a A\ ,\ p = p_0\, ,\ m_0 = m = \text{const.} \Leftrightarrow \rho_0 l_a A = \rho(l_a - x)A\, , \tag{1}$$

so the variation of the air density can be determined considering the length of the air column, l_a

$$\rho = \rho_0 \frac{l_a}{l_a - x} = \frac{\rho_0}{1 - \dfrac{x}{l_a}}\, ,\ \text{or}\ \ \frac{\rho}{\rho_0} = \frac{1}{1 - \dfrac{x}{l_a}}\, . \tag{2}$$

The pressure variation results as,

$$\frac{p}{\rho^n} = \frac{p_0}{\rho_0^n} \Rightarrow p = \frac{p_0}{\left(1 - \dfrac{x}{l_a}\right)^n} = p_0\left(1 + \frac{x}{l_a - x}\right)^n \tag{3}$$

where n is the polytrophic exponent, x is the displacement of water column, A is the flow section area, p_0 is the atmospheric pressure.

Because the phenomenon is very fast, the gas mass evolution is considered without heat exchange, so the polytrophic exponent is equal to adiabatic exponent, k. In engineering computations good quality results are obtained for $k = 1.4$ [4].

Bernoulli equation between a point at the free surface of the supply tank and a point at the air – water interface is

$$\frac{v_0^2}{2}+\frac{p_0}{\rho_w}+gH_0=\frac{v^2}{2}+\frac{p}{\rho_w}+\lambda\frac{l+x}{2D}v|v|+(l+x)\frac{dv}{dt}, \qquad (4)$$

with initial values at $t=0$, $v_0\cong 0$ and $\frac{p_0}{\rho_w g}+H_0=H_R$,

The total pressure head in supply tank H_R, is

$$gH_R=\frac{v^2}{2}+(l+x)\frac{dv}{dt}+\frac{\lambda}{2D}(l+x)v|v|+\frac{p}{\rho_w}. \qquad (5)$$

Thus, the governing equation system becomes

$$\begin{cases}\dfrac{dv}{dt}=\dfrac{1}{l+x}\left[gH_R-\dfrac{v^2}{2}-\dfrac{p}{\rho_w}-\dfrac{\lambda}{2D}(l+x)v\ |v|\right]\\ \dfrac{dx}{dt}=v\\ p=p_0\left(1+\dfrac{x}{l_a-x}\right)^n\end{cases} \qquad (6)$$

The system is solved using a program made in MatLAB, with a Runge – Kutta method.

The numerical simulations are made for different values of the supply head and of the water column lengths. They correspond to an experimental set-up, which in future works will be used to validate the mathematical model. The Darcy coefficient is considered $\lambda = 0.020$.

Figures 2 and 3 present the pressure oscillations for the minimum and maximum supply head available at the experimental set-up, H_R, and for each supply head three different lengths of water column, l.

If the displacement of water column, x, regarding to water column length, l, is neglected and the water column length is considered constant [2], the mathematical model is simplified and the equation (5) can be written as follows

$$gH_R=\frac{v^2}{2}+l\frac{dv}{dt}+\lambda\frac{l}{2D}v|v|+\frac{p}{\rho_w}. \qquad (7)$$

In this case the governing equations system becomes

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{1}{l}\left[gH_R - \dfrac{v^2}{2} - \dfrac{p}{\rho_w} - \dfrac{\lambda l}{2D} v \, |v| \right] \\ \dfrac{dx}{dt} = v \\ p = p_0 \left(\dfrac{l_a}{l_a - x} \right)^n \end{cases} \tag{8}$$

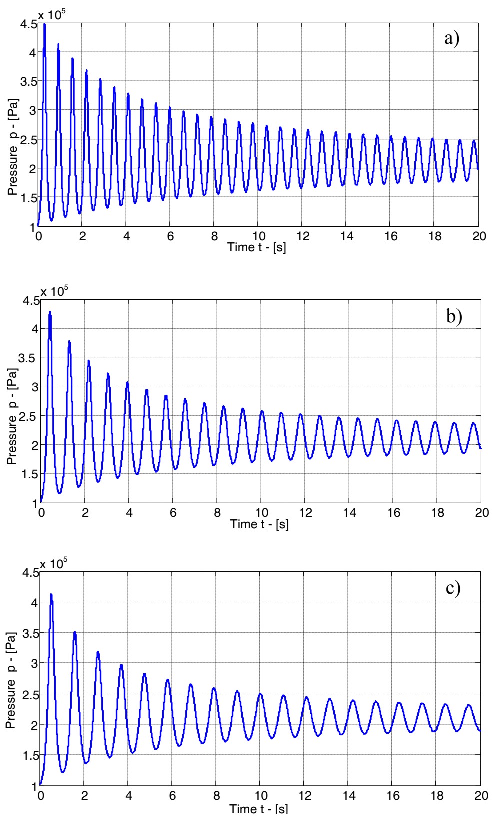

Fig. 2. Pressure oscillations for H_R = 21.32 m and l – a) 9.5 m, b) 9 m, c) 8.5 m.

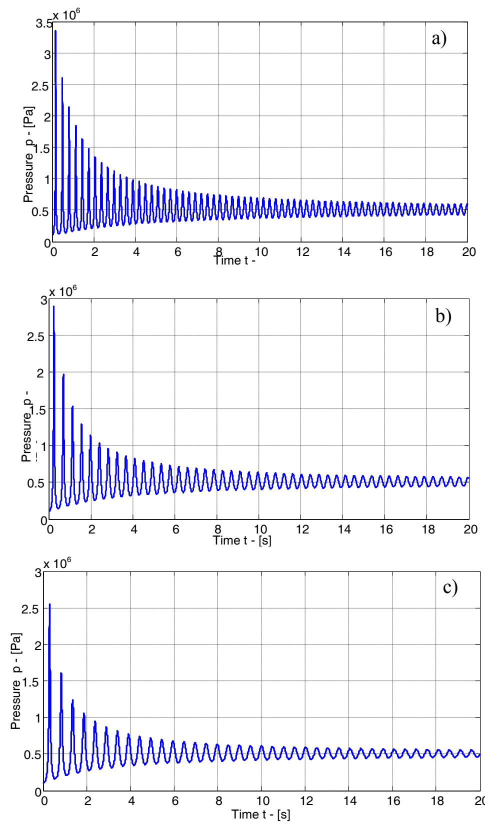

Fig. 3. Pressure oscillations for H_R = 51.32 m and l – a) 9.5 m, b) 9 m, c) 8.5 m.

Figs. 4 and 5 present a comparison between the two governing equations systems (6) and (8), in order to determine the influence of water length displacement, x, over the pressure and velocity oscillations. In the first seconds of the process both models have similar results, so the model can be simplified by considering constant the water column length.

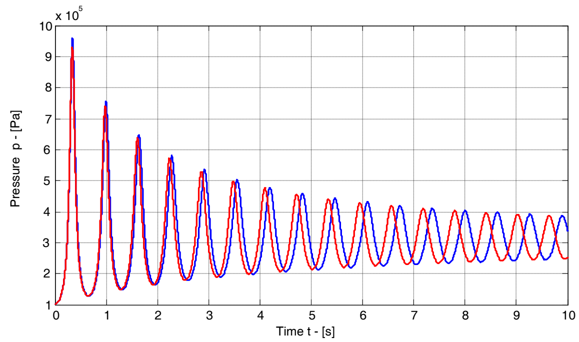

Fig. 4. Pressure oscillation for H_R = 31.32 m and water column length l = 9 m, determined with both models

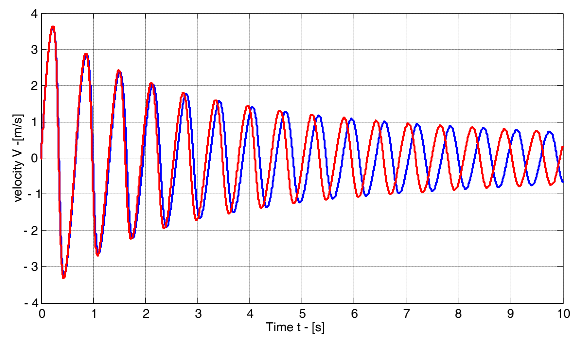

Fig. 5. Velocity oscillations of water column, for H_R = 31.32 m and water column length l = 9 m, determined with both models

3. Pipe opened at downstream end

The equations system is determined for pressure and air-water interface velocity, in case the downstream end of the pipe is opened and has diaphragm with the cross section A_0 (Fig. 6).

At the initial moment t = 0s, the separation vane (SV) is closed. The phenomenon is initiated when the separation vane is suddenly opened (considering a very short time).

The flow equations are the same as in the closed pipe case

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{1}{l+x}\left(gH_0 + \dfrac{p_0}{\rho_w g}\right) - \dfrac{v^2}{2(l+x)} - \dfrac{1}{l+x}\dfrac{p_0}{\rho_w} - \dfrac{\lambda}{2D} v\,|v| \\ \dfrac{dx}{dt} = v \end{cases} \tag{9}$$

Because the space occupied by air has the volume, $V = A(l_a - x)$, the variation in time of the air volume is

$$\frac{dV}{dt} = -A\,\frac{dx}{dt}\,. \tag{10}$$

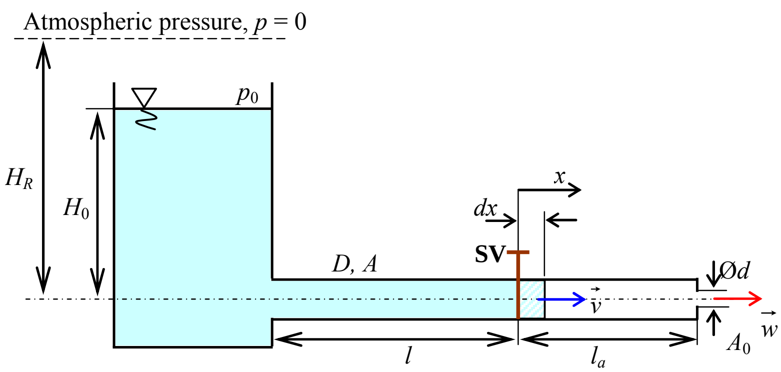

Fig. 6. Scheme for the mathematical model with the pipe opened at the downstream end.

The air discharge through the orifice can be expressed as a function of the air pressure p in the pipe

$$Q = \mu A_0 w = \mu A_0 \sqrt{\frac{2n}{n-1}\,\frac{p}{\rho_0}\left(\frac{p_0}{p}\right)^{\frac{2}{n}}\left[1-\left(\frac{p_0}{p}\right)^{\frac{n-1}{n}}\right]}\,, \tag{11}$$

where μ is the discharge coefficient, estimated at $\mu = 0.6$, n is the polytrophic exponent, and w is the air velocity.

The evolution of the phenomenon is considered without heat exchange because it is very rapid. So the polytrophic exponent is equal to the adiabatic exponent, $n = k = 1.4$ [4].

The continuity equation of the gas mass can be written as

$$\frac{dp}{dt} = -k\,\frac{p}{V}\left(\frac{dV}{dt} + Q\right) \tag{12}$$

The governing equations system in this case is

$$\begin{cases} \dfrac{dv}{dt} = \dfrac{1}{l+x}\left[gH_R - \dfrac{v^2}{2} - \dfrac{p}{\rho_w} - \dfrac{\lambda}{2D}(l+x)v\,|v|\right] \\ \dfrac{dx}{dt} = v \\ \dfrac{dp}{dt} = -k\,\dfrac{p}{A(l_a - x)}\left(-A\,\dfrac{dx}{dt} + Q\right) \end{cases} \tag{13}$$

The system is also numerically solved with a program made in MatLAB, using a Runge – Kutta method.

The initial conditions are at the moment $t = 0$, when the water column velocity is $v = 0$, the displacement $x = 0$ and the pressure in the pipe, $p = p_0$.

Numerical computations were made for cases that correspond to the parameters of the experimental set-up, for supply head of H_R = 21.32 m, water column length, l = 5 m and different orifice diameter, d_0 (1, 2 and 3 mm) (Fig. 7); for supply head of H_R = 41.32 m, orifice diameter d_0 = 1 mm and water column lengths l of 5 m, 6 m, 7 m and 8 m (Fig. 8).

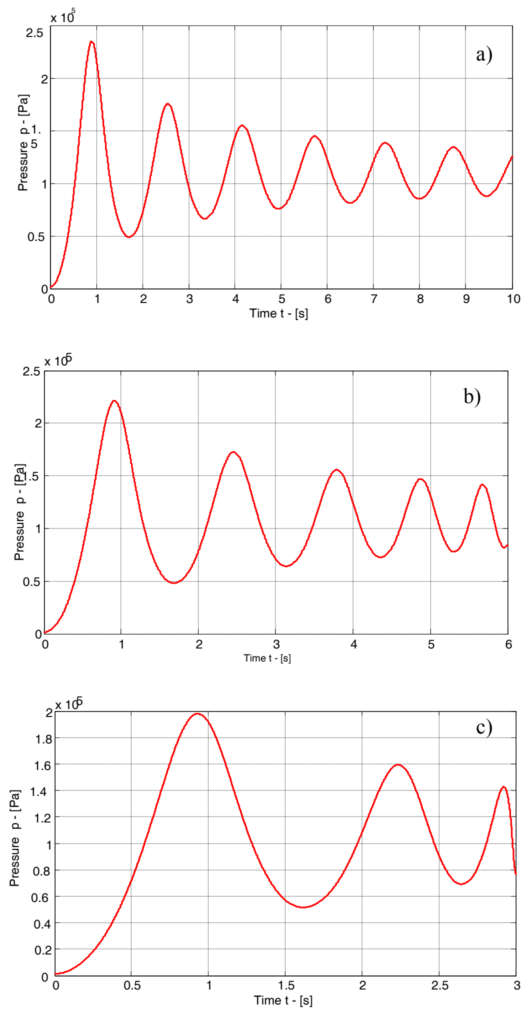

Fig. 7. Pressure oscillations for supply head of H_R = 21.32 m, water column length, l = 5 m and orifice diameter d_0 – a) 1 mm, b) 2mm, c) 3mm

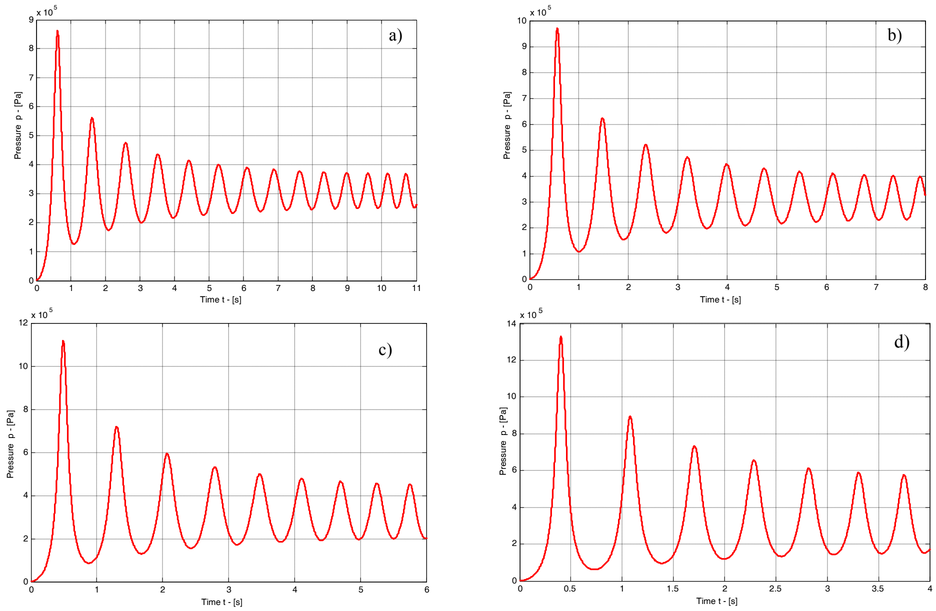

Fig. 8. Pressure oscillations for supply head H_R = 41.32 m, orifice diameter d_0 = 1mm and water column length, l – a) 5 m, b) 6 m, c) 7 m, d) 8m

To verify the solutions gave by the two elaborated programs, a comparison between the solution obtained in the first case (with downstream end closed) and the solution obtained in the second case, with open downstream end, for which the orifice was considered A_0 = 0. The results obtained with the two mathematical models are very close as showed in Fig. 9.

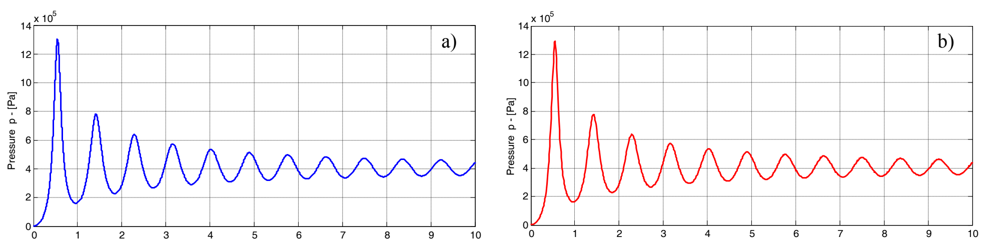

Fig. 9. Comparison for a supply head of H_R = 41.32 m and water column length, l = 5 m, between a) the case with downstream end completely closed and b) the case with downstream end opened with a diaphragm diameter equal to zero.

6. Conclusions

Two mathematical models were developed to study the pressure oscillation in case of rapid opening of the intake vane. The first model considers the pipe closed at the downstream end and the second one considers the pipe opened with a diaphragm of area A_0.

For both models numerical solving programs were elaborated in MatLAB.

Very close results (the same pressure oscillations) were obtained with both programs for a case with common conditions. This confirms the two solutions.

The two programs were run with parameters corresponding to those of the experimental set-up. In future works the mathematical models will be compared with the experimental results. The experimental set-up is located in the laboratory of the Hydraulics and Hydraulic Machinery Dpt, University Politehnica of Bucharest and will be operational from September 2010.

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