COOPERATIVE COMBINATORIAL MODEL BASED ON DISCRETE PULSES GENERATED WITHIN LIMITED TIME INTERVALS

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The article presents a mathematical formalism describing the cooperative action of a finite set of n individual entities, through a multiplicative interaction, characterized by a certain delay time. For this purpose, a binomial expression including a delay operator is set up. This is associated to time varying amplitudes, proportional to the combinations of k elements from a set of n elements. Thus, similar to wave-packet model, discrete pulses are generated in a finite time interval. On this basis, the behavior of some different number of participants, that can be changed during the started process, is analyzed in detail. This formalism is applicable to team-building procedures so as to organize efficient and perfectible work teams, within the framework of quality management in industrial units.

Keywords: combinatorial model, cooperative behavior, delay time, discrete pulses, quality management, organization development

1. Introduction

Managing human resource, establishing a work team, brain-storming methods etc. are important parts of the Q*uality Management* (QM) strategies, for accounting on expected outcomes, avoiding surprises and quantifying risks [1, 2]. The concept of QM is continuously enriched with new methods, asked by practical reasons. In agreement with the overall recognized PDCA model [3] of the Deming Cycle (Plan-Do-Control-Act), these methods can establish new measures to be implemented during the next steps of a process.

The concept of Organization Development (OD), meaning implement practices and techniques for organizational change and modify performances, is nowadays included in QM. Organizational change process, evidenced by the

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Lewin's change model (Unfreeze, Change, Refreeze) [4], was developed by many others, that showed the importance of changes in organization and the role of leadership in organizational outcomes, as for example VUCA (Volatility, Unpredictability, Complexity, Ambiguity) [5]. This approach is important for the adaptive capability, flexible organization of the activites, management of costs, in agreement with the market exigencies and opportunities.

In the economico-social domain, many theories have been given by different researchers [6, 7], in order to control the activity and obtain the best results. Quantitative models [8, 9] come to help in a more accurate description of real systems, as a completion of the qualitative analysis of the specific activities.

In this work, we take into account that, in economical organizations, a certain time interval is in general required so as significant results generated through cooperative behavior to be noticed. Since a limited time is given for ending any activity, specific strategies must be developed. The model presented in the paper elaborates such methods.

In our *combinatorial model*, a deterministic interaction is launched at an initial time moment. We describe this interaction by a *binomial* expression containing a *delay operator* τ, passing through all n interacting individuals, in a multiplicative manner. Thus, a distribution in time of the response, similar to an alternating *pulse* defined on a limited time interval, is generated.

This new approach is necessary since the classical system theory can justify the time interval required for significant results, but not changes in time due to cooperative behavior. One must mention that binomial methods were initially applied in probabilities as Cox-Ross-Rubinstein method, were generalized using stretch parameter for fine-tuning [10] or random walk method studied via scale functions [11], but were not applied to economic activities. The same for the combinatoric analysis [12].

Our proposal uses together these methods for modeling the *cooperative* work process of many individuals in terms of the necessary time interval and the effect of a change made *during* the started proces (health problems, security, environment considerations etc). When new individuals are added to the initial ensemble, acting within the same limited time-interval, some supplementary terms occur, strongly dependent on the addition time moment.

This situation is similar with many phenomena in *Physics*, exhibiting a pulse-like interaction. In the case of a suddenly emerging phenomenon on a limited space-time interval it is difficult to justify its propagation at long distances. Due to distortion, since the velocities corresponding to waves with different frequencies are not the same, the original shape of the pulse is altered. Attempts for coherent structural trapping were studied [13]. Waves considered from the very beginning with effectively limited duration (wavelets) were used to study propagating phenomena (see [14]). In the case of complex systems modeled by equations of evolution, a specific set of delayed differential equations is usually used [15].

These considerations offer a scientific basis to our model that, with adequate and flexible methods, can be integrated in an efficiently structured system of QM, health and security assessment, risks estimation and combative management.

2. Cooperative combinatorial model

2.1.Generating pulses based on delay-time operator and binomial distribution

The *delayed time response* of a system is well-known in electronics and control engineering [16]. Let's consider a set of n individuals starting to interact in a cooperative manner at an initial time moment as a linear system with a constant time delay τ. The delay time (also called dead-time) is usually defined as the time duration by which the arrival of a signal is retarded after transmission through a physical system. We can describe the interaction of the *n* individuals by using this delay time as $(1\pm \tau)^n$ acting upon an initial state S_i of the system. The monomial τ^k is considered as a delay operator with a certain time interval τ applied k times. The cooperative evolution of this system can be characterized by a state S:

$$S = (1+\tau)^n S_i \tag{1}$$

interpreted as an interaction described by $(1+\tau)$ passing through all the n interacting individuals in a multiplicative manner.

For simplicity, the initial time moment is set to zero and the initial state to a scalar value $S_i = 1$. A sum of *n* terms for the n interacting individuals, each term becoming visible at a different time moment, is obtained:

$$S = (1+\tau)^n S_i = \{C_n^0\tau^0 + C_n^1\tau^1 + \cdots + C_n^k\tau^k + \cdots + C_n^n\tau^n\} \tag{2}$$

which can be written in a compact form:

$$S = (1+\tau)^n = \sum_{k=0}^{n} C_n^k \tau^k \tag{3}$$

The coefficient C_n^k being connected to a certain amplitude at the time moment $k\tau$, the generated function in MATLAB is a pulse with a maximum in the central zone, represented in figure 1 for *n=50*. For a better qualitative understanding of the physical sense, the binomial coefficients are normalized by dividing them by 2^n (the sum of all binomial coefficients for a binomial expansion), and the time interval to unity. This means that the maximum delay interval $n\tau$ equals unity and $\tau = 1/n$.

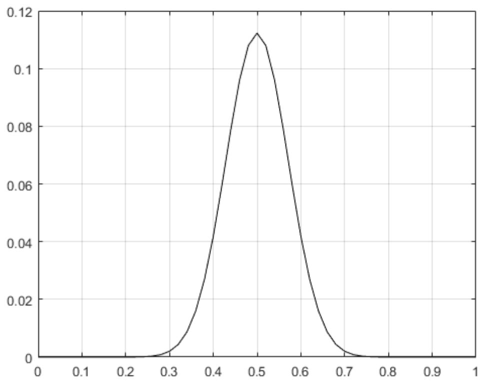

Fig 1. Amplitude *S* versus time for n=50 (normalized values)

The figure represents the cooperative behavior of the set of *n* individuals, and is a distribution in time of the response, similar to a *pulse* defined on a limited time interval.

2.2 Effect of changes during the launched process

In many situations, the system (for example a work team) must be changed during the work process. In our model, a change of the number n within the time interval of the pulse, is illustrated by the recurrence relation

$$C_n^k = C_{n-1}^k + C_{n-1}^{k-1} \tag{4}$$

where C_n^k is corresponding to a time moment $k\tau$, and C_{n-1}^{k-1} is the binomial coefficient of τ^{k-1} in the expansion of $(1+\tau)^{n-1}$. For taking into account this change, we use the method exposed below.

The addition of *a new individual* is considered at $(k-1)\tau$ from the moment. Then a *supplementary amplitude* C_{n-1}^k should be added to the previously amplitude C_{n-1}^{k-1}. It appears so as the supplementary term corresponds to the amplitude obtained from a group of $(n-1)$ individuals, and the time origin is shifted "to the left" by τ. (In reality, this term corresponds to the delay operator τ applied k times, and the resulting amplitude C_n^k correspond to the cooperative behavior of n individuals increased by 1, at time moment $k\tau$.) Starting from this shifted time origin, at the addition of a new individual, the total time interval for the cooperative behavior of the n individuals changes from $(n-1)\tau$ to $n\tau$.

Therefore, the *shift i*n the time origin by τ to the left means that the increase in the total time interval is *compensated* by the shift of the time origin. The end of the total time interval is the same, since the remaining time interval until the end:

$$\Delta t_r(n-1, k-1) = (n-1)\tau - (k-1)\tau = (n-k)\tau \tag{5}$$

is the same as the remaining time interval:

$$\Delta t_r(n,k) = n\tau - k\tau = (n-k)\tau \tag{6}$$

from the time moment $k\tau$ until the end of the total interval $n\tau$.

By using these considerations, in figure 2 is represented the evolution of the pulse amplitude *S* in a system (for example a industrial unit) of $n = 69$ individuals (work team), after a new individual is added at *t*=37τ moment (dotted line). The continuous line represents the evolution of the same system in the same time interval without adding this individual. A necessary transition of the system from a curve to the other is evident. The amplitude and time were represented as unnormalized values.

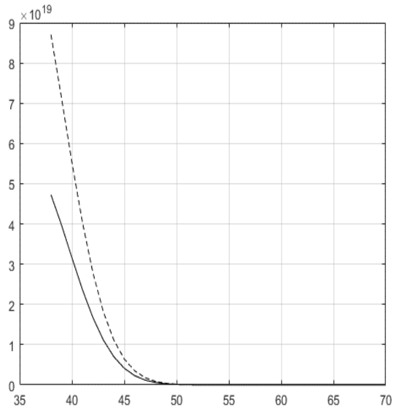

Fig.2. Pulse amplitude *S* versus time *t* for *n=69* (continuous line); dotted line: a new individual is added at *t*=37τ

Let's analyze this result for a work team involved in a project on a limited time interval.

We consider a sum *Z* of all amplitudes C_n^k at time moments $k\tau, k = 0, 1, .. n$ as a global quantity for characterizing the entire pulse. If the number of interacting individuals is constant on the total time interval:

$$Z(n) = \sum_{k=0}^{n} C_n^k = \{C_n^0 + C_n^1 + \cdots + C_n^k + \cdots + C_n^n\} = 2^n \quad (7)$$

If n varies within the time interval from n_i to n_f, the sum Z will experience a transition from 2^{n_i} to 2^{n_f}, hence

$$2^{n_i} < Z(n_i \to n_f) < 2^{n_f} \quad (8)$$

depending on the time moments when new individuals have joined the team. Since $Z \to 2^{n_f}$, it results that the effect of adding new individuals is more *significant when the addition is closer to the time origin.*

According to OD, we can apply these results to an industrial unit, for making *efficient* changes in the work team. Supposing that, at an initial time moment, the individuals are grouped in teams of few members, the number of combinations at a certain time moment is low. Considering the obtained result depending on this number, this result will be also weak. If after a certain time interval, the individuals will be grouped in teams consisting of half of total number, this corresponds to a *maximum number* of possible combinations and, consequently, to maximum results to be achieved. Further on, if in time the individuals join in groups of larger numbers, the number of possible combinations at the same moment decreases and, consequently, a time-decreasing quality of cooperative behavior extended on large time intervals appears [17]. These considerations were applied for students (future engineers) in organizing their practical activities; the outcomes and participants satisfaction were analyzed, confirming the considerations exposed in the model.

2.3. Alternating wave-trains generated by the delay operator

Many times, some formalisms from mathematics and physics were used in other domains of the human knowledge, for obtaining a quantitative treatment of problems evidenced by the practical activities [18]. We extend the previous analysis by using the analogy with propagating phenomena characterized by *fast alternating* functions of a central dominant frequency, defined on a limited time interval [19].

Exchanges within the group of *n* individuals, when an amount of energy flows between the members of the work team, corresponds with a minus sign in front of the delay operator τ. In advanced mathematical formalism in modern physics, this alternating flow is suggested by the coefficient $(-i)$ in front of the time-dependent Hamiltonian $H(t)$ within Taylor series of the evolution operator $1 - iH(t)/\hbar$.

The time-function generated by the binomial expression which contains the delay effects corresponds to the envelope of an alternating sinusoidal function, similar to a wave-packet with the same amplitude and with slightly different frequencies within a certain interval equal to $n\tau$. (A pulse generated by the delay operator through binomial expansion can be defined only for $t < n\tau$). The number *n* of individuals acting in a cooperative manner will be associated to *frequency*. Passing from a time moment $k\tau$ to $(k+1)\tau$, the amplitude changes sign (polarity).

In this way, the considerations exposed in 2.2 can be extended to pulse generated for higher *n* values, $n = 100$ or $n = 150$, in the case of companies with a big number of employees. It results:

$$(1-\tau)^n = C_n^0\tau^0 - C_n^1\tau^1 + C_n^2\tau^2 - C_n^3\tau^3 + \cdots = \sum_{k=0}^{n}(-1)^k C_n^k \tau^k \quad (9)$$

Graphs of normalized amplitude *S=y(t)* versus time *t* are represented for *n=50*, *n=100* and *n=150* in figure 3 and figure 4, respectively. The amplitude has been normalized by dividing it to 2^n and the time - by dividing it to $n\tau$.

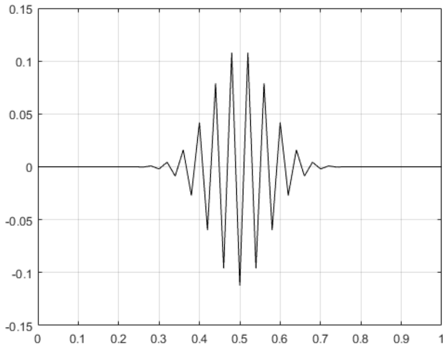

Fig.3. Amplitude versus time for *n=50* (normalized values)-alternating function

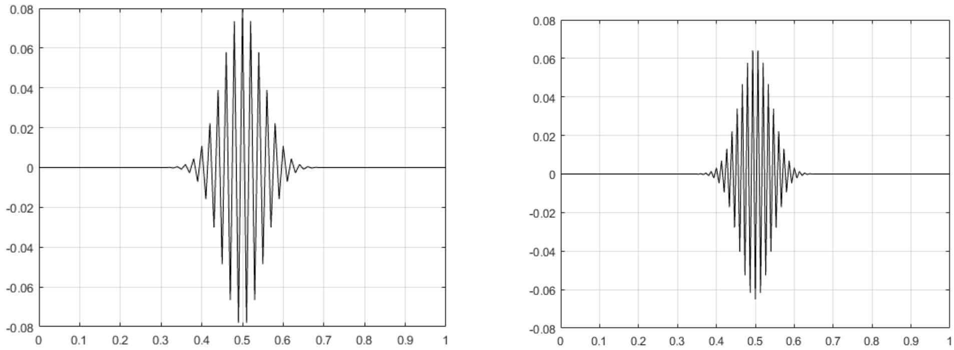

Fig.4. Amplitude versus time *t* for *n=100* and *n=150* (normalized values)

The generated functions have n changes in polarity on the entire time interval, and a certain frequency $\tilde{\nu}=n/2T_{total}$, where $T_{total} = n\tau$, can be associated to these alternating functions. Corresponding to displacement along a spatial axis with the velocity v, the amplitude S in point x at the moment t and in the point $x + \Delta x$, $\Delta x = v\Delta t$ at the moment $t + \Delta t$, is the same and the operator τ is substituted by $\zeta - v\tau$.Thus, the combinatorial model can represent the *step-by-step* propagation of wave-trains associated to particles along certain directions, from one space-time interval to the subsequent one. The pulse is defined in any point on the same limited time interval, its shape being the same, and the absence of distortion is justified. It can be noticed that both the time interval on which the wave-train differs significantly to zero and the maximum amplitude *decrease* if n increases. This shows the importance of the total number of participants. Moreover, the decrease of this time interval will be greater when n changes from 50 to 100, than when n changes from 100 to 150, as it results from the considerations below.

Let's consider the normalized amplitude y is a function $y(t)$ of the normalized time $t = k/n$.

Using Stirling approximation:

$$\ln(\mathrm{n}!) \approx n \ln(\mathrm{n}) - \mathrm{n}$$
$$n! \approx \frac{n^n}{e^n} \tag{10 }$$

$$C_n^k \approx \frac{\frac{n^n}{e^n}}{\frac{(n-k)^{n-k}}{e^{n-k}} \frac{k^k}{e^k}} \approx \frac{n^n}{(n-k)^{n-k}\, k^k} \tag{11}$$

it results successively:

$$y(t) = \frac{C_n^k}{2^n} \approx \frac{n^n}{(n-k)^{n-k}\, k^k 2^n} \approx \frac{1}{\left(\frac{n-k}{n}\right)^{n-k} \left(\frac{k}{n}\right)^k 2^n}$$
$$y(t) = \frac{1}{(1-t)^{n-k} t^k 2^n} \tag{12}$$

Denoting:

$$g(t) = 2\,(1-t)^{(1-t)}\, t^t \tag{13}$$

the time function $f(t)$ can be written as:

$$y(t) = \left\{\frac{1}{g(t)}\right\}^n \tag{14}$$

It results:

$$\ln g(t) = \ln 2 + (1-t)\ln(1-t) + t \ln t \tag{15}$$

where $t \in [0,1]$. The derivative of $\ln g(t)$ with respect to time is

$$\frac{d \ln g(t)}{dt} = \ln t - \ln(1-t) \tag{16}$$

which is positive for $t > (1-t) \Rightarrow t > 1/2$, and negative for $t < (1-t) \Rightarrow t < 1/2$. It results that $ln\, g(t)$ and consequently $g(t)$ have a minimum value for $t = 1/2$. Substituting t by $1/2$ within $g(t)$, it follows that the minimum value of $g(t)$ equals unity, obtained at the middle of the interval $[0,1]$.

Therefore, on the time interval of definition of the alternating pulse, function $1/g(t)$ is less than or equal to unity and the normalized amplitude $y(t)$ decreases when n increases.

Since the "shrinking" of the active time interval t_{lim} is related to *n*- the power of the binomial- the difference between the corresponding t_{lim} when $n = n_1 = p$, or $n = n_2 = mp$ depends on the value $m = n_2/n_1$,. This shows that the difference in graph shapes between $n = 100$ and $n = 150$ is less obvious than the difference between graphs when $n = 50$ is substituted by $n = 100$.

Concerning the agreement between the combinatorial model and other formalisms, one can observe:

- Mathematical modeling using Stirling formula when n equals 50, 100, 150 and numerical computations of binomial coefficients lead to almost the same form of the graphs " normalized amplitude versus normalized time". Values along y axis can be simply transferred from the accurate graph into the approximate graph by multiplying with a certain scale constant;

- It was shown that, for alternating pulses, *n* corresponds to the number of oscillations within a certain time interval and can be associated to the frequency. The addition of a new individual is akin to *quantum aspects* in modern physics, where the energy of a particle or set of particles described by a certain state vector can vary by integral multiples of a quantum. Thus n can be considered as the number of interacting quanta, and total energy E as:

$$E = k\ \ln(Z) \tag{18}$$

where k is a proportionality constant, is proportional to this number.

This aspect is analogous to quantum representation of associated wave-function Ψ in quantum physics, for a free-propagating particle:

$$\Psi = const.e^{i\{\omega t - kx\}} = const.\exp\left\{i\left[\frac{E}{\hbar}t - \frac{p}{\hbar}x\right]\right\} \tag{19}$$

where E corresponds to energy, p to momentum along x axis, k to wave vector and ω to angular frequency. This means that the exponent of the associated wave-function is proportional to frequency, analogous to the case of the exponent n of quantity Z characterizing alternating pulses generated through cooperative behavior. The exponent of the associated wave-train is imaginary due to the use of the binomial $(1-\tau)$ instead of $(1+i^2\tau) = (1+\exp(i\pi))$ for generating alternating pulses. The imaginary number i does not appear in the expansion of $(1+i^2\tau)^n = (1+\exp(i\pi))^n$ if i^2 is substituted by (-1). Thus, the choice of $k\ \ln(Z)$ as a measure for energy is justified by considering n as number of quanta or as a quantity proportional to generated frequency.

3. Conclusions

This paper presents an original combinatorial model of the response of a finite set of *n* individual entities, acting together through a multiplicative interaction. As a basis of this model, the deterministic interaction launched at an initial time moment is described by $(1+\tau)$, τ corresponding to a delay operator, passing through all n interacting individuals. Multiplying n times the binomial expression, a time distribution of the response of all individuals is generated, similar to a pulse defined on a limited time interval.

By using this model, the paper analyses the importance of the number of participating individuals and the situation- often met in practical activities- when this number is changed during the already started process. Acting within the same limited time interval with the initial generated pulse, supplementary terms dependent on the time moment of this modification, occurred. We showed that the effect of this change is more significant when it is closer to the time origin. Therefore, the optimal time moment for such changes can be established in terms of the concrete situation. The number of efficient members was discussed in the paper.

In analogy with fast phenomena in physics, a relative high number of individuals (*n=100* and *n=150)* acting in a cooperative manner, was furţher analyzed. Number *n* was associated to frequency and the time-functions generated by the binomial correspond to the envelope of an alternating sinusoidal function, similar to a packet of waves with frequencies within a certain interval and with the same amplitude. The generated pulses present a maximum amplitude in the central zone, which decreases in normalized value with the number of *n* elements, similarly with the maximum generated by the overlap of waves in a wave-packet.

The wave-train differs significantly to zero on a time interval which decreases as n increases. Since the decrease is greater when n changes from 50 to 100, than when n changes from 100 to 150, it results that the final result of the process depends on the total number of participants.

These original considerations and conclusions model the cooperative behavior of a set of *n* elements involved in a creative activity, for building an efficient work team and accelerate the best decision process, as a part of the integrated QM. On the basis of our model, and according to the modern concept of OD, one can act within the first part (Plan) of the Deming Cycle (PDCA), and determine the adequate measures to be implemented during the process. We consider that our method can be taken into account in the frame of Six Sigma DMAIC (Define, Measure, Analyze, Improve, Control) instruments [20].

The change of the number of individuals during the started process was compared with some quantum aspects in modern physics, where the energy of a particle or set of particles described by a certain state vector can vary by integral multiples of a quantum. So we consider our model can open a way for explaining also other aspects, as for example the environmental influence [21], from economical domain, in connection with modern physics and strengthen the analogies between these domains, for obtaining a more accurate, quantitative analysis.

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