# REYNOLDS NUMBER EFECTS ON THE AERODYNAMIC PERFORMANCE OF SMALL VAWTs

Radu BOGATEANU[1], Alexandru DUMITRACHE[2], Horia DUMITRESCU[3], Cornel Ioan STOICA[4]

*Many small vertical axis wind tubines operate at Reynolds number around $10^5$, when the NACA series airfoils have some specific aerodynamic characteristics considered as "anomalies". These particularities influence on the turbine performance and their inability to self-start. Therefore, the present paper is focused on the symmetric NACA airfoils which for this Reynolds number range experience a dead band of negative torque at tip speed ratios (TSR) between 1 to 3. The effects of Reynolds number are discussed.*

**KeyWords:** small VAWT, self-starting behavior, Reynolds number effects, marching-vortex model

## 1. Introduction

The number of wind turbines that are being constructed is increasing substantially. This is mainly in response to the perceived need to reduce emissions of carbon and other pollutants. The current generation of wind turbines that are being deployed around the world feature, almost exclusively, a three-bladed rotor with a horizontal-axis configuration (HAWT) [1], [2]. In recent years, however, there has been a resurgence of interest in both large-scale and small-scale vertical-axis wind turbines (VAWT) [3], [4]. The development of large-scale VAWTs is mainly a response to a plateau in the improvement of the aerodynamic performance of HAWTs. The research in small-scale VAWTs, with rotor diameters of only several meters, is motivated by the future demand for a decentralised and sustainable energy supply in cities and rural communities [5].

There are three reasons to be particularly interested in performance at low wind speed: firstly, many small VAWTs are located close to the load they supply and this may not be a good wind site, secondly they are by design insensitive to the direction and the quality of the wind which seldom is not so good, and thirdly these turbines have a drive train that is situated on the ground, thereby reducing the loads on the tower, and facilitating the maintenance of its systems. However,

[1] PhD Student, University POLITEHNICA of Bucharest, Romania, e-mail: unquradu@gmail.com
[2] CS II, Institute of Mathematical Statistics and Applied Mathematics, Romania
[3] CS I, Institute of Mathematical Statistics and Applied Mathematics, Romania
[4] PhD Student, University POLITEHNICA of Bucharest, Romania

in general a VAWT with fixed pitch blades has the disadvantage that it is unable to start on itself. The main problem for the small wind turbines is the negative power coefficient at low tip speed ratios so that the self-starting is the major obstacle to be overcome for successful design of a VAWT [6], [7].

## 2. Basic aerodynamics

As the VAWT have a rotational axis perpendicular to the oncoming airflow, the aerodynamics involved is more complicated than of the more conventional HAWT [3]. The main benefit of this layout is the independence of wind direction. The main disadvantages are the high local angles of attack involved at start and the wake coming from the blades in the upwind part and front the axis. If the turbine is represented in a two-dimensional way these characteristics are more obvious (Fig. 1).

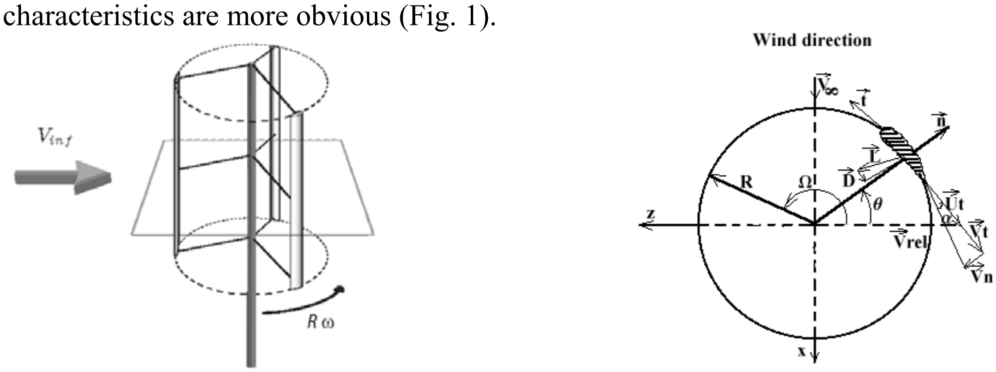



Fig. 1. Straight-wing (H-Darrieus) turbine.

The rotational speed can be varied by the turbines controller for a certain wind speed and this is therefore represented by the tip speed ratio TSR. This parameter gives the tip speed ratio $R\Omega$ as factor of the free stream velocity $V_{\text{inf}}$:

$$TSR = \frac{R\Omega}{V_{\text{inf}}} \tag{1}$$

The Reynolds number is a measure of the viscous behavior of air:

$$R_e = \frac{V_{\text{inf}}\,c}{\nu}\sqrt{1+(TSR)^2} \tag{2}$$

where the length and velocity scales are blade chord $c$ and tip speed $R\Omega$; $\nu$ is the kinematic viscosity of air.

The performance of the turbine is given by the power coefficient $C_P$. This coefficient represents the produced energy of the turbine as part of the total wind energy passing through the swept area of the turbine. This area equales the frontal area of the turbine given by the height times the diameter. This coefficient is normally plotted against the tip speed ratio TSR at a certain Reynolds number, Fig. 2.

The power coefficient $C_P$ is in this case dependent of TSR and Re number. The main problem for VAWTs (Darrieus) is the negative power coefficient at low tip speed ratios. If the coefficient is negative, the turbine needs extra power to be able to rotate. The region of negative $C_P$ is influenced by the airfoil (camber and thickness) and Reynolds number of blades.

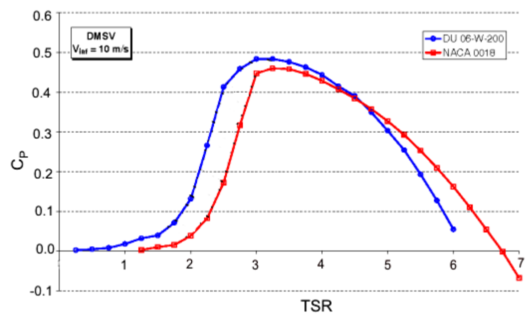



Fig. 2 Typical power efficiency for a VAWT.

Most vertical axis wind turbines built so far use airfoils from the NACA 4-series and most of them use symmetric airfoils. On the other hand these airfoils for the small wind turbines operate at Reynolds number around $10^5$ having specific aerodynamic characteristics which prevent the rotor to enter full-lift driven state (flapping-wing analogy) at the unity TSR.

The paper aimed at giving an insight into the small wind turbine starting behavior and its influence parameters.

### 3. Description of vortex model

A vortex model [8], [9], is introduced to predict aerodynamic performance of VAWT using NACA 4 series airfoil for the $10^5$ Reynolds number range. The general approach requires that the rotor blades are simply lifting surface of large span.

The production, convection, and interaction of vortex system springing from the each blade are modeled and used to predict the induced velocity (or perturbation velocity) at various points in the flow field. The induced velocity at a point is simply the velocity which is superimposed on the undisturbed wind stream by the wind turbine. Having obtained the induced velocities the lift and drag of the blade can be obtained using airfoil section data. Here, the aerodynamic characteristics for the symmetric NACA 4 series airfoil are available from wind tunnel results at Reynolds number varying from $10^4$ to $10^6$ [10].

A simple representation of the vortex system associated with a blade and its wakes is shown in Fig. 3. The airfoil blade is replaced by a bound vortex filament or a lifting line located along the rotor blade quarter chord line with the incident-flow boundary condition met at the three quarter chord location. The wake consists of shedding spanwise vortex filaments results from the temporal variation in loading distributions on blades according to Kelvin's theorem. The contour encloses both the airfoil and its wake and any change in the bound circulation must be accompanied by an equal and opposite change in circulation in the wake.

The model is based on the marching - vortex concept where motion begins from an impulsive start with the subsequent generation of a vortex wake modelled by a sequence of discrete vortices shed at equal time intervals. For steady - state motion, the force and moment responses are achieved asymptotically. The vortices which are shed during any given time period can be related to the changes in bound vortex with respect to time and position along the blade. Thus, all variables associated with a particular point - vortex as point coordinates, and velocities as well as vortex strengths are identified by a double subscript (*i*,*j*) . The first subscript denotes the blade element from which the point-vortex originated and second subscript denotes the time step at which the vortex originated.

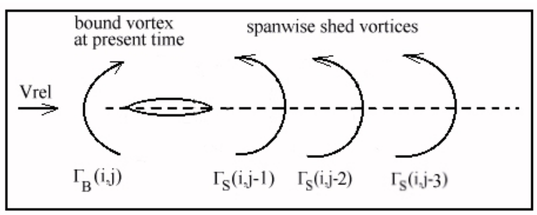



Fig. 3 Marching - vortex model.

Referring to Fig. 3, the spanwise shed vortex strengths can be written as

$$\Gamma_S(i, j-1) = \Gamma_B(i, j-1) - \Gamma_B(i, j) \ . \tag{3}$$

The discrete vortices $\Gamma_S(i, j)$ are assumed to move downstream with the local fluid velocities given by

$$\vec{V}(i, j) = \vec{V}_\infty + \vec{v}_I(i, j) \ , \tag{4}$$

where $\vec{V}_\infty$ is the undisturbed freestream velocity and $\vec{v}_I(i, j)$ is velocity induced by all discrete vortices in the flowfield, forming the vortex wake lattice. To convect all the lattice points in the wake, we use an explicit integration formula

$$\vec{r}(i, j) = \vec{r}(i, j-1) + \left[\frac{3}{2}\vec{V}(i, j) - \frac{1}{2}\vec{V}(i, j-1)\right]\Delta t \ , \tag{5}$$

The induced velocity at the wake lattice points is computed by application of Biot-Savart law. The closure of the vortex model is the relationship for the bound vortex strength ($\Gamma_B$) which can be related to the local relative air velocity ($V_{\text{rel}}$)

section chord ($c$) and section lift coefficient $\left((C_L(\alpha, \mathrm{Re})\right)$ through the equation [8], [11].

$$\Gamma_B = \frac{1}{2} C_L c V_{rel} \ , \tag{6}$$

where $C_L$ is given by measured lift coefficients tables [10]. It should be noted that the vortex model is valid only if the flow around airfoil is attached, excepting a small region at the trailing edge. According to [3] the minimal tip speed ratio necessary to prevent blade stall is equal to

$$\lambda_{\min} = 1/\left(\tan\alpha_s + \sigma\right) \ ,$$

where $\alpha_s$ is the maximum angle of attack of the airfoil at which attached flow is found, and σ is the rotor solidity. For the lower TSR when the flow is massively separated, the vortex-shedding effects are neglected and the blade element method is further used.

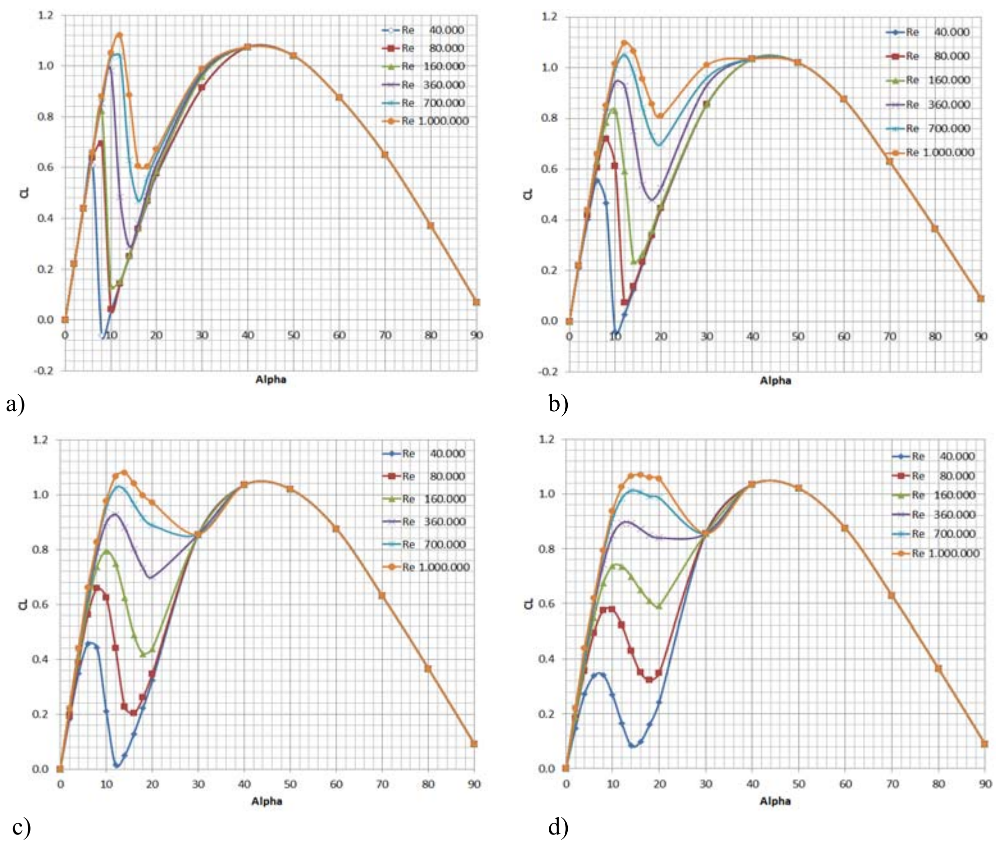



Fig. 4 Lift coefficients for symmetric NACA airfoils at various Reynolds numbers $\left(\mathrm{Re}\cdot 10^6\right)$:
a) NACA 0012; b) NACA 0015; c) NACA 0018; d) NACA 0021.

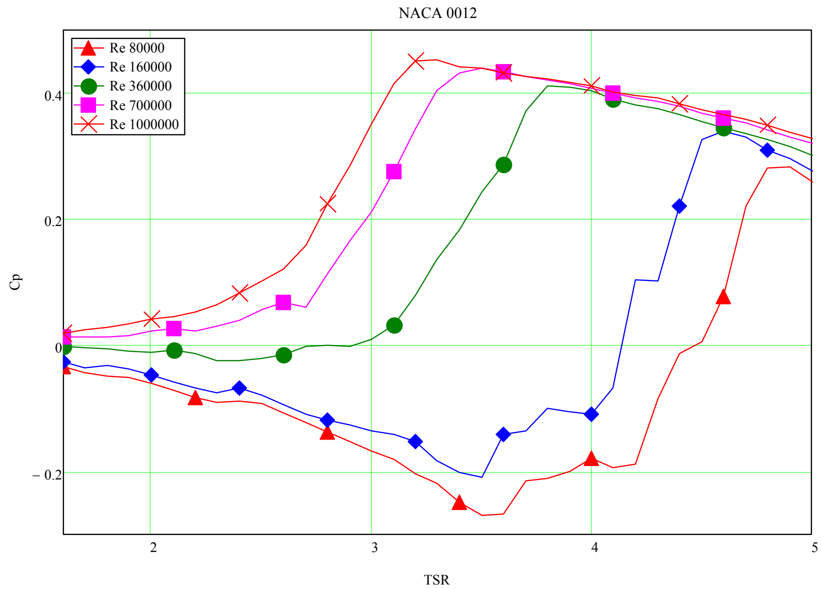
For the small VAWTs the Reynolds number is a critical parameter since if the Reynolds number is too low, the airfoil will suffer performance degradation (lift curve slope will be significantly reduced in comparison to a high Reynolds number), leading to a very small amount of torque. The wind tunnel results for the symmetric NACA 4 series at $\mathrm{Re} = 4 \cdot 10^4 - 10^6$ which are shown in Fig. 4 (a-d) will be used for this study to predict aerodynamic performance of three-straight-blade vertical axis wind turbine having a reduced solidity value of 0.125.

### 4. Results

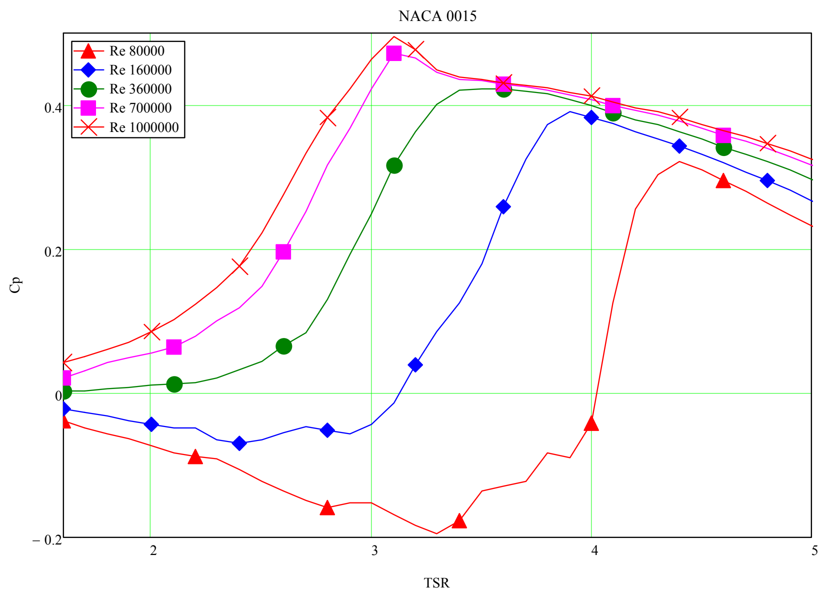
The measured data for symmetric NACA airfoil with various thicknesses at different Reynolds numbers was implemented into the computing program. The authors of Ref. [10] investigated seven symmetrical wing sections experimentally at Reynolds numbers from $0.35 \cdot 10^6$ to $1.76 \cdot 10^6$ through angles of attack up to 180 degrees. They point out that data at much lower Reynolds numbers are needed for VAWTs and present data down to $\mathrm{Re} = 10^4$. These data were computationally generated for the linear and early non-linear $C_L(\alpha)$ curve and the Re – independent data for larger angles of attack were obtained experimentally.
These data at low Reynolds number show anomalous feature, i.e. the linear dependency of $C_L$ on angle of attack extends to a few degrees followed by a drop to negative values of $C_L$. Though, the Figs. 4 show that their anomalous and the accompanying increase in drag coefficient numbers, seem to be a continuous function of Reynolds number, undistinguishable from the common stall behavior at larger Reynolds number. Understanding of flow phenomenon on airfoils at low Reynolds number requires further investigation. For the moment, the measured data for different symmetric NACA airfoils were only used into the simulation program for VAWT performance.

The main problem for Darrieus turbines is the negative power coefficient at low tip speed ratios. If the power coefficient is positive at TSR $\geq 1$, the turbine is able to rotate independently and produce power. If the coefficient is negative, the turbine is unable to start on itself and need extra power to enter continuous rotation.

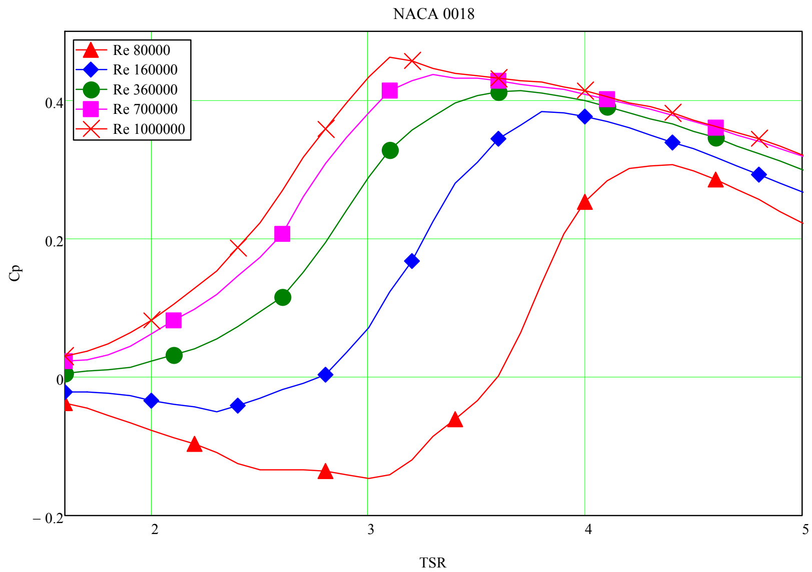
The region of negative $C_P$ is influenced strongly by the thickness and Reynolds number of blades. Figure 5 (a-d) show the influence of airfoil thickness on the turbine performance at five values of Reynolds number: $8 \cdot 10^4$, $1.6 \cdot 10^5$, $3.6 \cdot 10^5$, $7 \cdot 10^5$ and $10^6$.


NACA 0012
Re 80000
Re 160000
Re 360000
Re 700000
Re 1000000
Cp
0.4
0.2
0
− 0.2
2
3
4
5
TSR

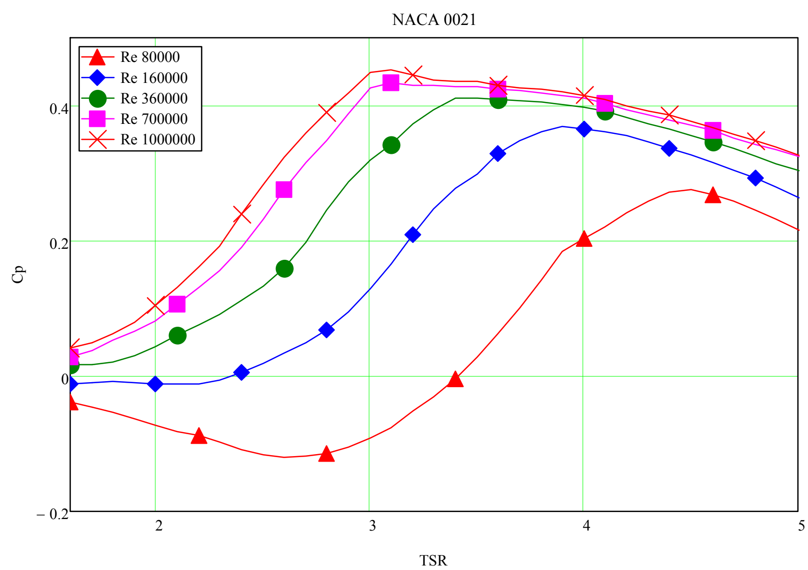
a)


NACA 0015
Re 80000
Re 160000
Re 360000
Re 700000
Re 1000000
Cp
0.4
0.2
0
− 0.2
2
3
4
5
TSR

b)




c)

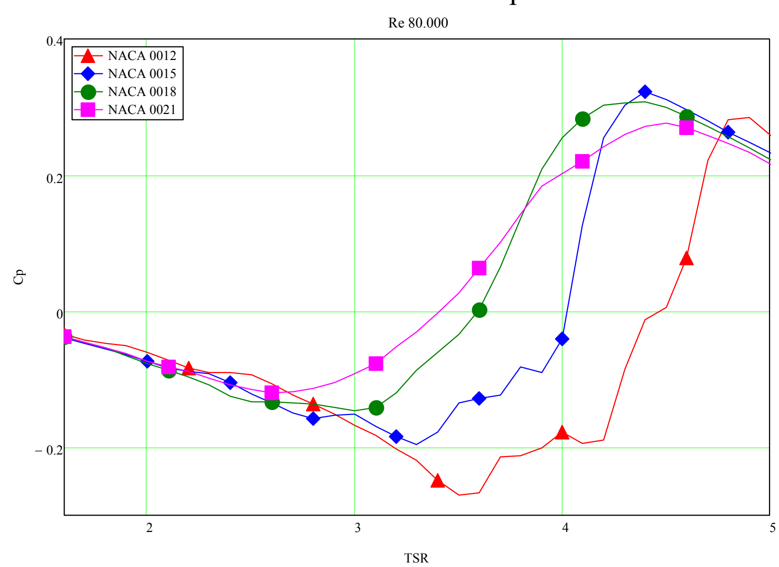



d)

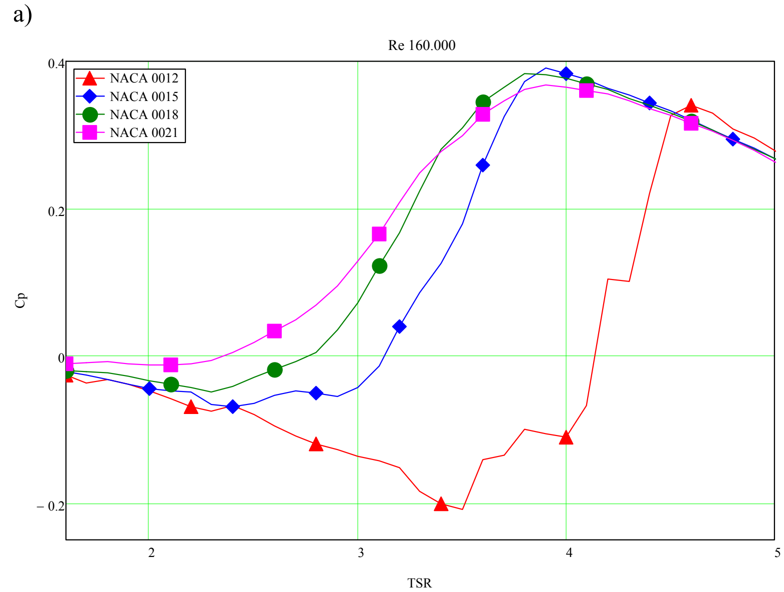
Fig. 5 The influence of airfoil thickness on the turbine performance at various Reynolds numbers (B = 3; σ = 0.125; c/R = 0.083) : a) NACA 0012; b) NACA 0015; c) NACA 0018; d) NACA 0021.

At all Reynolds numbers, by increasing the thickness the region with negative $C_P$ coefficients is drastically decreases. However, the blade thickness reduces the severity of the dead band of negative torque between TSR = 1 to 3, but does not eliminate it than for NACA 0021, NACA 0018 and NACA 0015 airfoils at $Re \geq 3.6 \cdot 10^5$. The NACA 0012 airfoil for $Re \leq 5 \cdot 10^5$ the dead band of negative torque is between TSR = 1 to 3 which shows that the small, NACA 0012 three-bladed turbine is unable to start on itself.

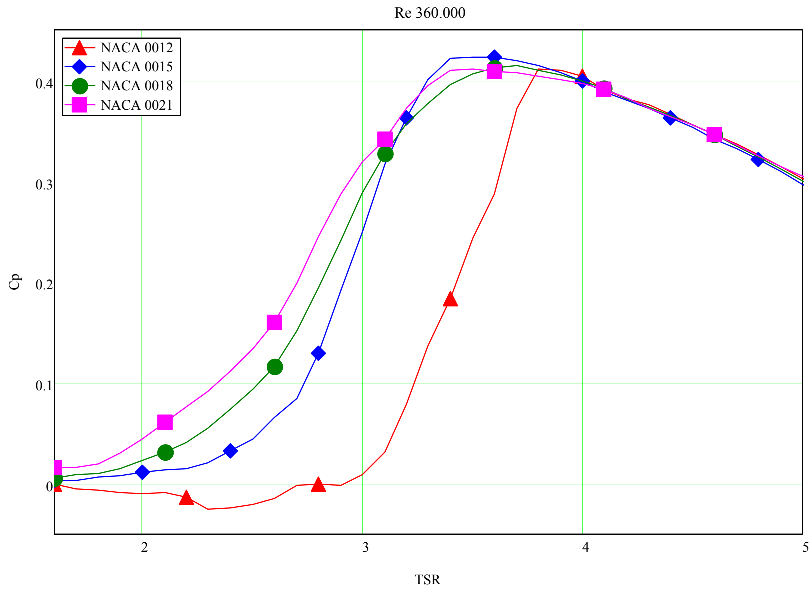
The increase of Reynolds number also diminishes the region of negative $C_P$ and even makes the $C_P$ values completely positive for NACA0012 if $Re \geq 7 \cdot 10^5$ (see Fig. 6). Figure 6 also shows that for a smallest turbine (R $\approx$1m) the NACA 0021 blade is able to self start at wind speed of order 10 m/s.



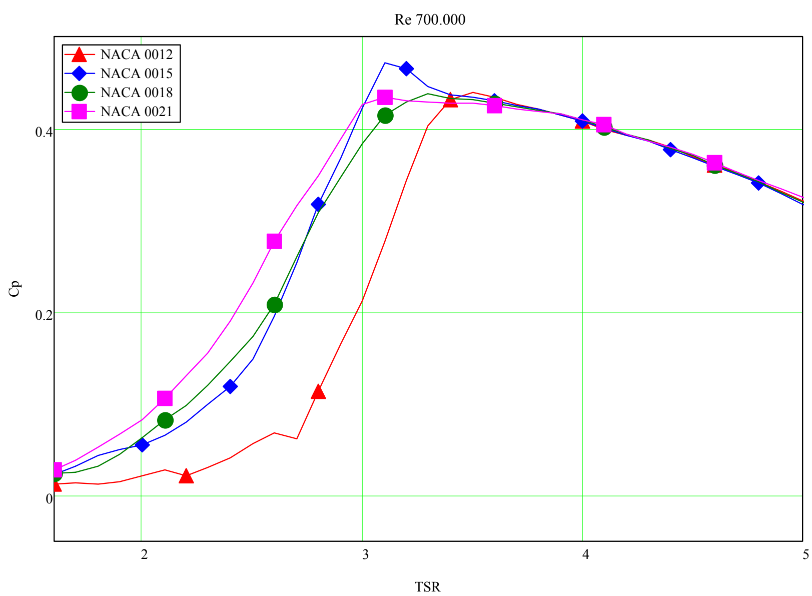

a)



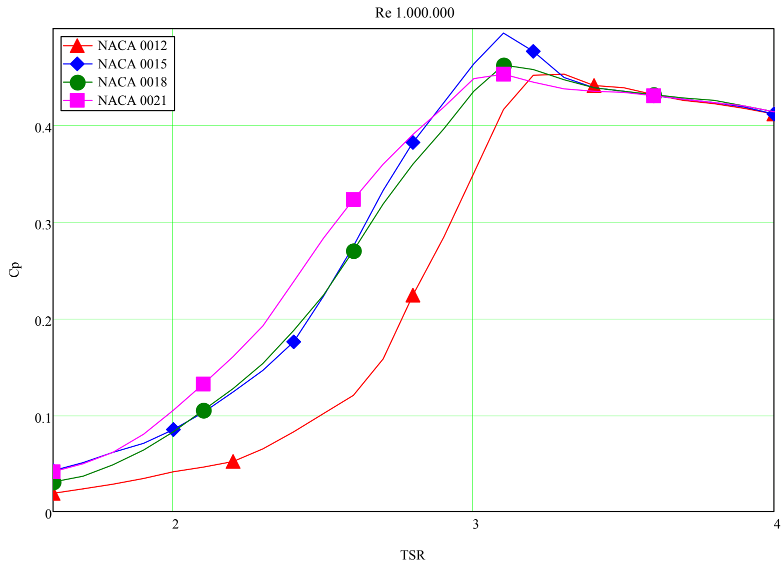

b)


Re 360.000
NACA 0012
NACA 0015
NACA 0018
NACA 0021
Cp
0.4
0.3
0.2
0.1
0
2
3
4
5
TSR

c)


Re 700.000
NACA 0012
NACA 0015
NACA 0018
NACA 0021
Cp
0.4
0.2
0
2
3
4
5
TSR

d)




e)

Fig. 6 The influence of Reynolds number on the turbine performance for various NACA airfoils (B = 3; σ = 0.125; c/R = 0.083): a) Re = 80000; b) Re = 160000; c) Re = 360000; d) Re = 700000; e) Re = 1000000.

## 5. Conclusions

The Darrieus turbine starting behavior consists of two-step process: first acceleration step between TSR = 0 to 1 (drag mode) and second acceleration between TSR = 1 to 3 (lift mode). In the first acceleration step the rotor is alternately driven by drag (angle of attack between 45 - 180 degrees) and lift (angle of attack between 0 - 45 degrees). In the second acceleration step, the rotor is solely driven by lift. The Reynolds number and thickness of symmetrical NACA blade are the main parameters which influences the second acceleration step. In this paper a parameter study on the self-starting behavior revealed the airfoils to promote continuous generation of thrust for small VAWTs.

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