DESIGN OF MICROSTRIP BANDPASS FILTERS WITH PRESCRIBED TRANSMISSION ZEROS AT FINITE FREQUENCIES

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Articolul prezintă o metodă analitică de sinteză pentru filtrele trece-bandă (FTB) cuasi-eliptice cu poli de atenuare impuşi funcţiei de transfer. În general, matricea filtrelor obţinută prin această procedură presupune existenţa unor cuplaje directe între toate rezonatoarele filtrului. Prin transformări de similitudine pot fi însă obţinute alte matrice care păstrează toate caracteristicile electrice ale filtrului corespunzător matricei iniţiale, dar care sunt mai convenabile din punct de vedere al realizării practice. Ca exemplu, în articol este prezentată proiectarea şi simularea unui FTB de ordin 4 cu rezonatoare cuplate încrucişat, care are o pereche de poli de atenuare la frecvenţe finite. Performanţele FTB obţinute prin simulare de circuit şi prin simulare electromagnetică concordă bine cu specificaţiile de proiectare, ceea ce validează metoda de calcul.

This paper describes a general method for the synthesis of quasi-elliptic bandpass filters (BPF), with prescribed attenuation poles in the stopband. This method generally leads to an extended coupling matrix that assumes multiple couplings between all the filter resonators. The extended matrix can be then reconfigured into a form suitable for practical filter realization, using similitude transformations. The design and the simulation of a 4-pole cross-coupled planar microwave BPF with a pair of attenuation poles at finite frequencies are presented, as an example. The circuit- and EM-simulated performances of this BPF show a good agreement to the specifications, validating the design method.

Keywords: filters, transmission zeros, cross-couplings, extended coupling matrix, similitude transformations, square resonator.

Introduction

Conventional bandpass filters (BPF) are usually composed from a number of in-line synchronously tuned resonators, each of its elements being coupled only with two other elements, the previous and the next one. Such structures are submitted to some well-known theoretical limitations, but a part of them can be avoided by using cross-couplings between the filter elements (Fig. 1).

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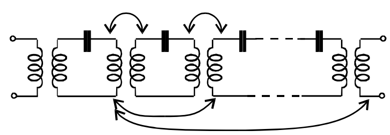
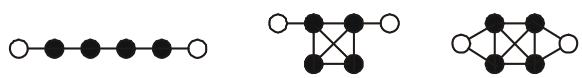


Fig. 1. BPF with multiple coupled resonators

The presence of multiple couplings can lead to a filter transfer function with zeros in the right half-plane of the complex frequency variable $s = \alpha + \mathrm{j}\omega$, and/or on the imaginary axis $s = \mathrm{j}\omega$. Consequently, it is possible to obtain transfer functions with transmission zeros at certain finite frequencies (attenuation poles in its stopband), hence a higher selectivity, (sharper cutoffs at the edges of the passband), and/or with an improved, flatter in-band group delay response.

The presence of cross-couplings between the bandpass filter resonators can be described by a coupling matrix, an extension of the coupling coefficients concept. Some elements of this coupling matrix, corresponding to pairs of filter elements that does not interact directly, can be zero.



a) b) c)

Fig. 2. Some BPF topologies (degree 4 examples):
a) in-line topology; b) with cross-coupled resonators;
c) with cross-coupled resonators and multiple
couplings with the input and output lines.

The design information for a bandpass filter structure can be extracted from its coupling matrix in a similar way to that usually derived from g (or $k - q$) values, for in-line filters.

The design of microwave bandpass filters composed from cross-coupled resonators and multiple coupled input/output lines is based on the concept of normalized bandpass filters. Such a normalized filter has a standard fractionary bandwidth $w = \Delta\omega / \omega_0 = 1$. The normalized couplings coefficients $k_{i,j}$ of a filter with n resonators are the elements of a normalized coupling matrix **k**, with n rows and n columns. These $k_{i,j}$ values, together with some extra information regarding the couplings between resonators and the I/O lines (the normalized coefficients q_i) determinate all the properties of the normalized bandpass filter. The microwave bandpass filters design stays in some de-normalizing operations on these normalized elements. The couplings coefficients between resonators pairs are obtained by de-normalizing the elements of the **k** matrix, while the external quality factors, representing the input and output couplings Q_{ei}, are derived from the q_i coefficients. Consequently, using this procedure, one gets the characteristic immittances of all inverters, from the general bandpass filter structure (Fig. 3).

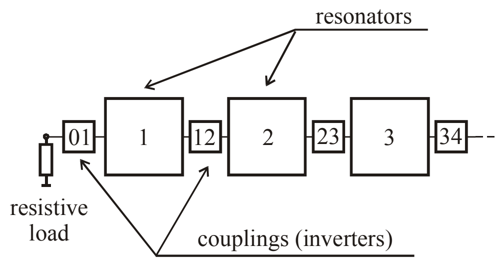


Fig. 3. Microwave BPF composed from identical resonators and inverters

The information contained in the normalized coupling matrix **k** together with that contained in the coefficients q_i, rewritten as

$$M_{0,i} = M_{i,0} = \frac{1}{\sqrt{q_i}}, \qquad M_{i,n+1} = M_{n+1,i} = \frac{1}{\sqrt{q_i}}, \tag{1}$$

can be included into a sole matrix, named the extended coupling matrix of the normalized filter, **M**. This matrix has n+2 rows and n+2 columns, the input and output lines being considered as its 0 and n+1 elements, respectively. The elements of the outer rows and columns of this matrix correspond to the normalized couplings between resonators and the I/O lines. Using a convenient de-normalizing procedure, from the elements on the outer rows of **M** one gets the characteristic parameters of the inverters connecting the filter resonators with its resistive loads, arbitrarily chosen. In this way, the extended couplings matrix **M** describes completely the structure of a normalized bandpass filter. The matrix **M** is useful especially when the I/O lines are multiple coupled to the resonators (Fig. 2.c).

A bandpass filters composed from synchronously tuned resonators has a **M** matrix with all diagonal coefficients $M_{i,i}$ equal to zero. In the more general case, of a bandpass filter with asynchronously tuned resonators, the $M_{i,i}$ terms of the extended coupling matrix represent the normalized frequency offsets of the resonators, with respect to the center frequency ω_0 of the filter.

The multiple couplings allow the design of bandpass filters with some special properties, needed in various telecommunications applications: filters with attenuation poles at finite frequencies in the stopband, filters with an improved group delay response in passband, etc.. The **M** matrix for such filters can be derived using exact synthesis procedures presented in [1], [2].

The de-normalizing procedure of **M** is simple, but this operation must be done carefully, due to the different significances of the matrix elements.

Thus, the elements of **M** matrix that correspond to couplings between pairs of resonators can be de-normalized with the relation:

$$c_{i,j} = wM_{i,j}, \tag{2.a}$$

where $c_{i,j}$ is the needed electromagnetic coupling coefficient between the resonators "*i*" and "*j*". For a filter composed only from parallel resonators and lumped elements and admittance inverters [7], the relation (2.*a*) becomes:

$$J_{i,j} = \frac{(\omega_2 - \omega_1)M_{i,j}}{\sqrt{C_i C_j}}, \tag{2.b}$$

where C_i and C_j are the capacitances of the two resonators and $J_{i,j}$ is the characteristic admittance of the inverter located between them.

The de-normalization of the elements that represent couplings of the resonators with the input and output lines can be done using the relations:

$$Q_{ei} = \frac{M_{0,i}^2}{w}, \qquad Q_{ei} = \frac{M_{i,n+1}^2}{w}, \tag{3.a}$$

for any resonator "*i*" coupled with the input line "0" and/or with the output line "*n*+1" (Fig. 2.c). In a filter composed from parallel resonators and inverters, the above relations become:

$$J_{0i} = M_{0,1}\sqrt{(\omega_2 - \omega_1)G_0 C_i}, \qquad J_{i,n+1} = M_{i,n+1}\sqrt{(\omega_2 - \omega_1)C_i G_{n+1}}, \tag{3.b}$$

where J_{0i}, $J_{i,n+1}$ are the characteristic immittances of the inverters between the "*i*" resonator and the load conductances of the filter, G_0 and G_{n+1}.

For a filter composed from inverters and lumped parallel resonators, the de-normalizing procedure of the $M_{i,i}$ elements of the **M** matrix leads to:

$$C_i' = wM_{i,i}C_i, \tag{4.a}$$

where C_i' is an extra parallel capacitance (positive or negative) that has to be added to the synchronously tuned L_iC_i resonator, to obtain the necessary frequency offset. In a general, regardless the type of the microwave resonator formulation, the resonant frequency ω_0' of the resonator must be slightly different from the frequency ω_0:

$$\omega_{0i}' = \frac{\omega_0}{\sqrt{1 + wM_{i,i}}} \tag{4.b}$$

2. Similitude transformations of the matrix M

The normalized extended coupling matrix, **M**, describes the properties of a normalized bandpass filter. A similitude transformation [1] of this matrix:

$$\mathbf{M}' = \mathbf{R} \cdot \mathbf{M} \cdot \mathbf{R}^T , \tag{5}$$

where **R** is a rotation matrix and $\mathbf{R}^T$ is its transpose, does not affect the matrix eigenvalues. Rotation matrices **R** are simple, being defined by a pivot (a pair of indexes *i*, *j*) and by a rotation angle θ. For instance, the rotation matrix of degree 6, with the pivot (3,5) and the rotation angle θ is:

$$R_6[(3,5),\theta] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos\theta & 0 & -\sin\theta & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & \sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

The transformation (5) affects only the elements on the "*i*" and "*j*" rows and columns of **M**.

The extended coupling matrix $\mathbf{M}'$ resulted from a transformation like (5) corresponds to a new filter, with a different structure, but with the same electrical features as the original filter described by **M**. Hence similitude transformations allow changes of the couplings between filter elements, while maintaining its original response. In this way, some elements of the coupling matrix can even be annihilated by a proper choice of similitude transformations. Usually, the annihilation of certain couplings simplifies the realization and/or the tuning of the filter.

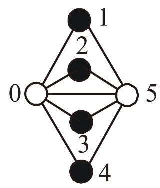


Fig. 4. Transversal canonical bandpass filter (of degree 4)

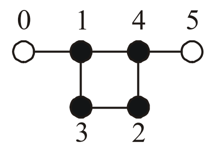


Fig. 5. The topology of a BPF corresponding to the matrix (9)

A recent paper [1] presents an exact analytical synthesis procedure, for bandpass filters with quasi-elliptic transfer functions and with prescribed transmission zeros in the passband. An improved version [2] of the above method stays in the synthesis of a filter with a special topology, where all the *n* resonators are coupled directly with both the input and output lines, without direct couplings between them (Fig. 4). For such topologies, the extended coupling matrix which assures the prescribed transmission zeros can be derived. Certainly, a practical realization of a filter with this topology is not an easy task, therefore a post-processing of this matrix by similitude transformations is required.

3. Design example

For testing the design method, a microwave planar bandpass filter was designed. This bandpass filter meets the following specifications: a center frequency $f_0 = 2400\,\mathrm{MHz}$, a frequency bandwidth $\Delta f = 120\,\mathrm{MHz}$, (a fractionary bandwidth $w = 0.05$), degree 4, Chebyshev response in the passband with a return loss $R_L = 20\,\mathrm{dB}$. The filter should exhibit two attenuation poles, at the frequencies $f_1 = 2280\,\mathrm{MHz}$ and $f_2 = 2520\,\mathrm{MHz}$.

Based on the procedure developed in [2] and using an original computational program, the extended coupling matrix **M** was computed for a normalized bandpass filter of degree 4, with a Chebyshev filtering function with a return loss $R_L = 20\,\mathrm{dB}$ in the passband and with two attenuation poles at the normalized frequencies:

$$f_{z1} = \frac{1}{FBW}\left(\frac{f_1}{f_0} - \frac{f_0}{f_1}\right) \cong \frac{f_1 - f_0}{\Delta f} = -2\,, \tag{7.a}$$

$$f_{z2} = \frac{1}{FBW}\left(\frac{f_2}{f_0} - \frac{f_0}{f_2}\right) \cong \frac{f_2 - f_0}{\Delta f} = 2\,. \tag{7.b}$$

The obtained matrix,

$$\mathbf{M}=\begin{bmatrix} 0 & 0.371111 & 0.62138 & -0.62138 & -0.37111 & 0 \\ 0.371111 & -1.2872 & 0 & 0 & 0 & 0.37111 \\ 0.62138 & 0 & 0.6904 & 0 & 0 & 0.62138 \\ -0.62138 & 0 & 0 & -0.6904 & 0 & 0.62138 \\ -0.37111 & 0 & 0 & 0 & 1.2872 & 0.37111 \\ 0 & 0.37111 & 0.62138 & 0.62138 & 0.37111 & 0 \end{bmatrix}, \qquad (8)$$

corresponds to a transversal canonical filter of order 4, satisfying the specified requirements.

As mentioned above, this filter is almost impossible to be fabricated. However, starting from this **M** matrix, other $\mathbf{M}'$ matrices corresponding to some forms suitable for filter realization can be derived using similitude transformations. Applying five times some properly chosen similitude transformations on the matrix (8), one gets:

$$\mathbf{M}'=\begin{bmatrix} 0 & -1.02356 & 0 & 0 & 0 & 0 \\ -1.02356 & 0 & 0 & -0.87057 & -0.17046 & 0 \\ 0 & 0 & 0 & -0.76726 & 0.87057 & 0 \\ 0 & -0.87057 & -0.76726 & 0 & 0 & 0 \\ 0 & -0.17046 & 0.87057 & 0 & 0 & 1.02356 \\ 0 & 0 & 0 & 0 & 1.02356 & 0 \end{bmatrix}. \qquad (9)$$

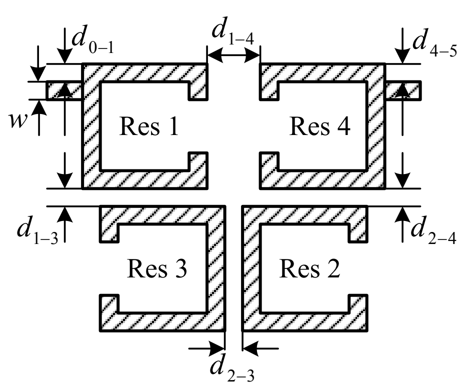


Fig. 6. Cross-coupled planar microwave bandpass filter

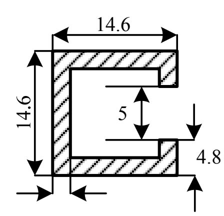


Fig. 7. The dimensions (in mm) of a square open-loop resonator from Fig. 6

The matrix (9) corresponds to a filter having the topology shown in Fig. 5.

In practice, this topology can be easily realized in the form of a planar bandpass filter, composed from four identical microstrip resonators [4]. The layout of such a filter, composed from four square open-loop resonators is given in Fig. 6. The substrate used in the design was Rogers RO3003, having a relative dielectric constant of 3, a thickness of 0.508 mm and a cooper metallization thickness of 0.035 mm. The input and output lines, directly coupled with resonators 1 and 4 (see Fig. 6) have widths of 1.3 mm and assure standard 50Ω loads for the filter. The square resonator details are shown in Fig. 7.

Table 1

The values of the external quality factors and of the coupling coefficients between filter resonators

$Q_{e0,1}$	$Q_{e4,5}$	k_{1-3}	k_{2-4}	k_{1-4}	k_{2-3}
19.08	19.08	0.04352	0.04352	0.00852	0.03836

The design of the filter from Fig. 6 stays in the finding all the distances *d*, in order to assure the external quality factors and the coupling coefficients derived by de-normalizing the extended coupling matrix $\mathbf{M}'$ [3], [4], [8]. The de-normalized values are shown in Table 1 and the distances, derived by full-wave EM simulations [9], are presented in Table 2.

Table 2

The distances between filter elements from Fig. 6, corresponding to the values from Table 1

d_{0-1} (mm)	d_{4-5} (mm)	d_{1-3} (mm)	d_{2-4} (mm)	d_{1-4} (mm)	d_{2-3} (mm)
0.6	0.6	0.4	0.4	1.0	0.6

A lumped elements model of this bandpass filter corresponding to the matrix (9) is also plotted in Fig. 8, and its elements values are given in Table 3.

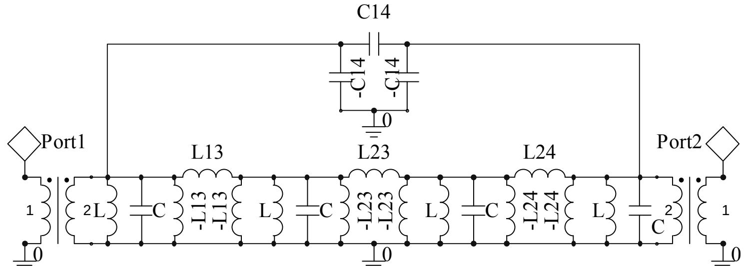


Fig. 8. Lumped elements model of the BPF

Table 3

The values of the lumped elements model of the BPF from Fig 8

n	L (nH)	C (pF)	L13 (nH)	L23 (nH)	L24 (nH)	C14 (pF)
0.198738	4.397621	1	101.0285	114.6318	101.0285	0.008596

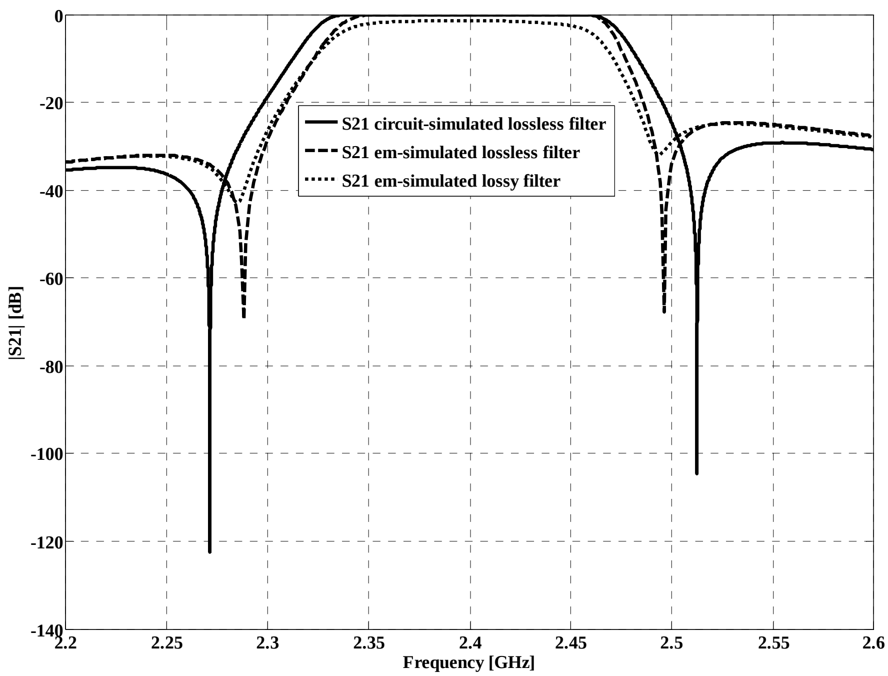


Fig. 9. The response $|S_{21}|$ of the BPFs from Fig. 6 and Fig. 8

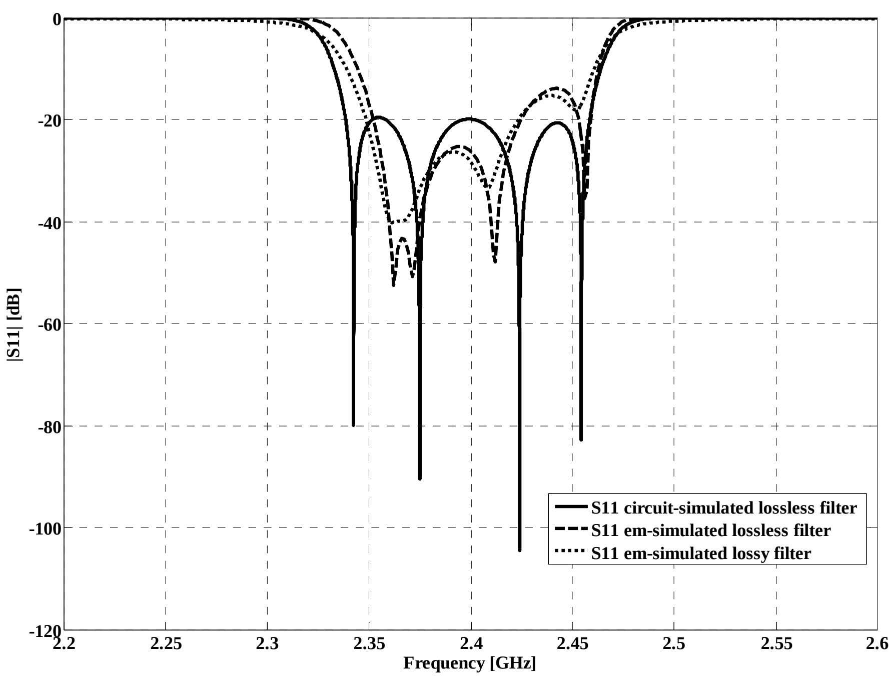


Fig. 10. The response $|S_{11}|$ of the BPFs shown in Fig. 6 and in Fig. 8

The circuit-simulated and the EM-simulated performances of the designed bandpass filter are plotted in Fig. 9 and in Fig. 10.

It can be noticed that simulated responses are very close to the filter requirements. However, some differences are present. The first transmission zero occurs practically exactly at the frequency f_1, of 2280 MHz, but the second transmission zero is located at a frequency f_2 of $2496\,\mathrm{MHz}$, slightly different to the requirement. Also, the resulted bandwidth is of only $110\,\mathrm{MHz}$, slightly inferior to the specifications. If the losses of the substrate and of the metallization are taken into account, then the EM-simulated performances anticipate an insertion loss of approximately $1.5\,\mathrm{dB}$ and a slightly narrower bandwidth, compared to the lossless model.

Conclusions

The above design example highlights the possibility of using the extended coupling matrix synthesis procedure, for the design of microwave bandpass filters with special features.

The circuit-simulated and EM-simulated performances of the planar 4-poles bandpass filter, with two prescribed transmission zeros, are in good agreement with the specifications, validating the design method.

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