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DYNAMIC IDENTIFICATION OF A HYDRODYNAMIC TORQUE CONVERTER

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Lucrarea prezintă un model teoretic pentru identificarea dinamică a convertoarelor de moment hidrodinamic. Convertorul de moment este considerat ca o maşină cu parametrii concentraţi, numită cuadripol.

Plecând de la legile de bază ale mecanicii şi considerând o curgere unidimensională în interiorul convertorului într-o suprafaţă medie toroidală, se realizează funcţiile de transfer şi matricea de transfer a maşinii. Aceste rezultate sunt folosite pentru a descrie comportamentul convertorului în regim de funcţionare nepermanent şi tranzitoriu.

The paper presents a theoretical model for the dynamic identification of hydrodynamic torque converters. The torque converter is considered like a machine with lumped parameters, namely a quadripole.

Starting from the basic laws of mechanics and considering an one-dimensional flow inside the torque converter in a mean toroidal surface there are developed the transfer functions and the transfer matrix of the machine. These results are useful to describe the behavior of the converter in no stationary and transient operation.

Keywords: Hydrodynamic transmissions, torque converters, dynamic identification, transfer matrix, transfer functions.

1. Introduction

The hydrodynamic torque converter consists from a pump impeller (P) one or more turbine runners (T) and stator stages (S). For the following mathematical developments it is considered the simple Főttinger transmission, fig. 1, but analogous relations may be obtained for Riesler, Lysholm-Smith or any other type of hydrodynamic torque converter [1], [3], [7], [10].

The dynamic response of the machine is determined through the harmonic sinusoidal perturbations on the global parameters of the hydrodynamic torque converter [4], [5], [6] and [8]. Using a parametric synthesis it is possible to model the linear dynamic behavior of the machine.

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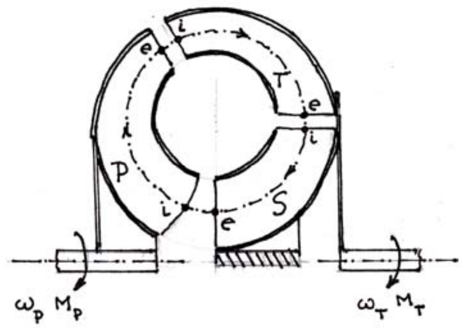

Fig. 1 Explanatory sketch for a hydrodynamic torque converter circuit. Axial section through the torus

2. Theoretical model

The mathematical approach refers to a transmission with a pump impeller, a turbine runner and a stator, fig. 1. The basic laws of fluid motion are applied to pump, turbine and stator which, from hydrodynamic point of view, form radial, radial-axial and axial cascades of blades [2], [9]. Fluid mechanics conservation laws are applied on rotors and stator generically sketched only a part, namely a pump channel, between two blades, in fig.2.

The momentum equation is applied to the control volume of which a channel is represented in fig.2. So :

$$\frac{d\vec{I}}{dt} = \sum \vec{F} \tag{1}$$

Developing:

$$\int_V \frac{\partial}{\partial t}\left(\rho \vec{v}\right) dV + \int_A \vec{v}\,\rho\vec{v}\cdot d\vec{A} = -\int_A p\, d\vec{A} + \int_V \rho \vec{f}\, dV + \overrightarrow{F_{ext}} \tag{2}$$

where $\widetilde{f}$ - mass specific force , ρ - density, V - volume, A - area, v - velocity, t - time and $\overrightarrow{F_{ext}}$ - external forces.

The moment of momentum in respect of the rotation axis gives:

$$\frac{d\vec{L}}{dt} = \sum \vec{r}\times\vec{F} \tag{3}$$

Again, for a liquid, in developed shape:

$$\rho \int_V \vec{r}\times\frac{\partial\vec{v}}{\partial t}\, dV + \rho\int_A (\vec{r}\times\vec{v})\,\vec{v}\cdot d\vec{A} = -\int_A p\,\vec{r}\times d\vec{A} + \rho\int_V \vec{r}\times\vec{f}\, dV + \vec{r}\times\overrightarrow{F_{ext}} \tag{4}$$

Projecting along the X axis, namely the rotation axis, it is obtained:

$$\rho \int_V r \frac{\partial v_u}{\partial t} dV + \rho Q \left(r\, v_u \right)\Big|_i^e = r\, F_{u\,ext} = M_x \tag{5}$$

Because $A = A_i \bigcup A_e$ and $A_i = \sum \Delta A_i$ respective $A_e = \sum \Delta A_e$. The surfaces A_i and A_e are symmetrical in respect of the X axis and the resultant of the superficial forces $\overrightarrow{F_s} = -p\ d\overrightarrow{A}$ intersects the rotation axis and $\vec{r} = 0$.

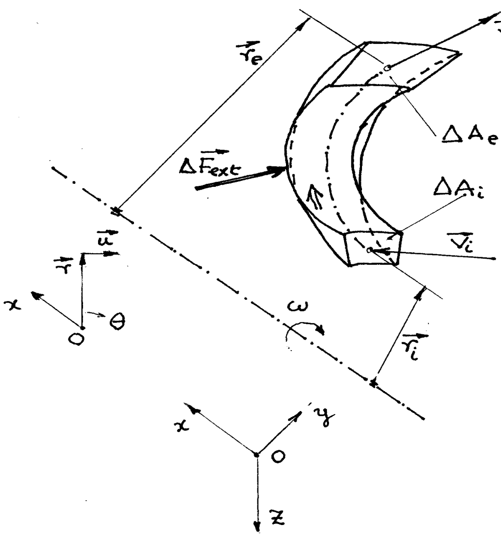

Fig. 2. A channel from a pump impeller cascade. Legend : index i - entrance, e – exit, of the liquid from the cascade ; ext – external.

Knowing that $\vec{f} = \vec{g}$ similar the weight of the fluid from the control volume intersects the rotation axis and $\vec{r} = 0$. So the integrals of the right side of the relation (4) vanish.

The external forces are transmit from the shafts to the rotors $\overrightarrow{F_{ext}} = \sum \Delta \overrightarrow{F_{ext}}$.

Taking into account the relation in the velocity triangle of the liquid, fig. 3:

$$v_u = u - v_m\, ctg\,\beta = r\,\omega - \frac{Q}{A} ctg\,\beta \tag{6}$$

Introducing rel. (6) in rel. (5) :

$$M_x = M_{hP} = \rho \int_V r \frac{\partial}{\partial t} \left(r\,\omega - \frac{Q}{A} ctg\,\beta \right) dV + \rho Q \left(r_e v_{ue} - r_i\, v_{ui} \right) \tag{7}$$

$$M_{hP} = \rho \int_V r^2 \frac{\partial \omega}{\partial t} dV - \frac{\rho}{A} \int_V r \frac{\partial Q}{\partial t} ctg\,\beta\, dV + \rho Q \left(r_e v_{ue} - r_i\, v_{ui} \right) \tag{8}$$

But

$$dV = A\, dl \tag{9}$$

and

$$dl = ds \sin\beta \tag{10}$$

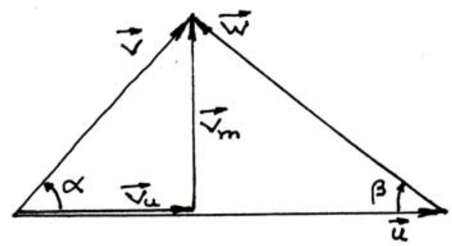

Fig. 3. The velocity triangle, of the liquid, in the rotors. v - absolute velocity, w – relative velocity and u – tangential rotation velocity.

Here dl - is the displacement element along the meridian velocity $\overrightarrow{v_m}$ and ds - is the displacement element along the channel (tangent to the relative velocity $\vec{w}$ in the rotors and tangent to the velocity $\vec{v}$ in the stator).

The moment of inertia of the liquid (oil) inside of the rotor is:

$$J_u = \int_m r^2\, dm = \rho \int_V r^2\, dV \tag{11}$$

With this rel. (8) for a pump impeller becomes:

$$M_{hP} = J_u \frac{\partial\omega}{\partial t} - \rho \int_i^e r\cos\beta \frac{\partial Q}{\partial t}\, ds + \rho Q\left(r_e v_{ue} - r_i v_{ui}\right) \tag{12}$$

For a turbine runner:

$$M_{hT} = -M_{hP} = -J_u \frac{\partial\omega}{\partial t} + \rho \int_i^e r\cos\beta \frac{\partial Q}{\partial t}\, ds - \rho Q\left(r_e v_{ue} - r_i v_{ui}\right) \tag{13}$$

Introducing, in accord with fig. 1, the equation of motion, namely the rotation of a body in respect of an axis in the form:

$$J_P \frac{d\omega}{dt} = M_P - M_{hP} \tag{14}$$

$$J_T \frac{d\omega}{dt} = M_{hT} - M_T \tag{15}$$

with J_P and J_T the moment of inertia of pump and turbine solid parts of rotors and shafts.

Combining rel. (12) with (14) respective rel. (13) with (15) results the following torques:

$$M_P = (J_P + J_u)\frac{\partial \omega_P}{\partial t} - \rho \int_i^e r\cos\beta \frac{\partial Q}{\partial t}\, ds + \rho Q (r_e v_{ue} - r_i v_{ui}) \tag{16}$$

$$M_T = -(J_u + J_T)\frac{\partial \omega}{\partial t} + \rho \int_i^e r\cos\beta \frac{\partial Q}{\partial t}\, ds + \rho Q (r_i v_{ui} - r_e v_{ue}) \tag{17}$$

for the pump and turbine shaft. For the stator carcass:

$$M_S = M_{hS} = \rho \int_i^e r\cos\alpha \frac{\partial Q}{\partial t}\, ds' + \rho Q (r_i v_{ui} - r_e v_{ue}) \tag{18}$$

where $dl = ds' \sin\alpha$.

It is obvious that in stationary regimes the transmission satisfy the equilibrium balance of the toques

$$M_P - M_T - M_S = 0 \tag{19}$$

Using the Bernoulli equation in relative motion in rotors:

$$z_i + \frac{p_i}{\rho g} + \frac{w_i^2 - u_i^2}{2g} = z_e + \frac{p_e}{\rho g} + \frac{w_e^2 - u_e^2}{2g} + \frac{1}{g}\int_i^e \frac{\partial w}{\partial t}\, ds + h_{p\,i-e} \tag{20}$$

With the hydraulic losses expressed through:

$$h_{p\,i-e} = \int_i^e \lambda \frac{ds}{4R_h}\frac{w^2}{2g}$$

and the analogous relations in stator it is possible to introduce the specific energies balance :

$$H_P = H_T + \sum h_{pc} + \sum h_{ps} \tag{21}$$

In which the pump head is equal with the turbine head and all the hydraulic losses and shock losses from the circuit added. For the expression of the shock losses in a hydrodynamic torque converter there was used the relations from pages 110 to 120 of [1].

Relations (16), (17) and (18) got the shape:

$$M_P = k_{11}\frac{d\omega_P}{dt} + k_{13}\frac{dQ}{dt} + k_{14} Q \omega_P + k_{16} Q^2 \tag{22}$$

$$M_T = k_{22}\frac{d\omega_T}{dt} + k_{23}\frac{dQ}{dt} + k_{24} Q \omega_P + k_{25} Q \omega_T + k_{26} Q^2 \tag{23}$$

$$M_S = k_{33}\frac{dQ}{dt} + k_{35} Q \omega_T + k_{36} Q^2 \tag{24}$$

Relation (21) with the above mentioned specifications is:

$$0 = k_{41}\frac{d\omega_P}{dt} + k_{42}\frac{d\omega_T}{dt} + k_{43}\frac{dQ}{dt} + k_{44} Q \omega_P + k_{45} Q \omega_T + k_{46} Q^2 + k_{47}\omega_P^2 + k_{48}\omega_T^2 \tag{25}$$

in which the introduced parameters are :

$$k_{11} = J_u + J_P$$

$$k_{13} = -\rho \int_{Pi}^{Pe} r \cos\beta \, ds$$

$$k_{14} = \rho \left(r_{Pe}^2 - r_{Pi}^2 \right)$$

$$k_{16} = -\rho \left(r_{Pi} \, ctg\, \beta_{Pi} - r_{Pe} \, ctg\, \beta_{Pe} \right)$$

$$k_{22} = -\left(J_u + J_T \right)$$

$$k_{23} = \rho \int_{Ti}^{Te} r \cos\beta \, ds$$

$$k_{24} = \rho \, r_{Ti}^2$$

$$k_{25} = -\rho \, r_{Te}^2$$

$$k_{26} = \rho \left(\frac{r_{Te}}{A_{Te}} ctg\, \beta_{Te} - \frac{r_{Ti}}{A_{Ti}} ctg\, \beta_{Ti} \right)$$

$$k_{33} = \rho \int_{Si}^{Se} r \cos\alpha \, ds'$$

$$k_{35} = -\rho \, r_{Se}^2$$

$$k_{36} = \rho \left(\frac{r_{Si}}{A_{Si}} ctg\, \alpha_{Si} - \frac{r_{Se}}{A_{Se}} ctg\, \alpha_{Se} \right)$$

$$k_{41} = -\int_{Pi}^{Pe} r \cos\beta \, ds$$

$$k_{42} = -\int_{Ti}^{Te} r \cos\beta \, ds$$

$$k_{43} = \int_{Pi}^{Pe} \frac{ds}{A_P \sin\beta} + \int_{Ti}^{Te} \frac{ds}{A_T \sin\beta} + \int_{Si}^{Se} \frac{ds}{A_S \sin\alpha}$$

$$k_{44} = \frac{1}{A} \left\{ \frac{r_{Pe}^2}{r_{Ti}} ctg\, \beta_{Ti} - r_{Pi} \; ctg\beta_{Pi} + r_{Pe} ctg\beta_{Pe} \left[1 - \left(\frac{r_{Pe}}{r_{Ti}} \right)^2 \right] \right\}$$

$$k_{45} = \frac{1}{A} \left\{ \frac{r_{Te}^2}{r_{Ti}} ctg\, \alpha_{Si} - r_{Ti} \; ctg\beta_{Ti} + r_{Te} ctg\beta_{Te} \left[1 - \left(\frac{r_{Te}}{r_{Si}} \right)^2 \right] \right\}$$

$$k_{46}=\frac{1}{2A^2}\left[\left(ctg\,\beta_{Pi}-\frac{r_{Se}}{r_{Pi}}ctg\,\alpha_{Se}\right)^2+\left(ctg\,\beta_{Ti}-\frac{r_{Pe}}{r_{Ti}}ctg\,\beta_{Pe}\right)^2+\left(ctg\,\alpha_{Si}-\frac{r_{Te}}{r_{Si}}ctg\,\beta_{Te}\right)^2\right]+$$
$$+\frac{\lambda}{8}\int_{i=P,T,S}\frac{U\,ds}{A^3\sin^3\beta}$$

By the parameters k_{44} k_{45} k_{46} : A - is the mean area, U - relative velocity in P and T and absolute velocity is stator and between the cascades and λ - the mean friction factor.

$$k_{47}=\frac{r_{Pi}^2}{2}+r_{Pe}^2\left[\frac{1}{2}\left(\frac{r_{Pe}}{r_{Si}}\right)^2-1\right]$$

$$k_{48}=\frac{r_{Ti}^2}{2}+r_{Te}^2\left[\frac{1}{2}\left(\frac{r_{Te}}{r_{Si}}\right)^2-1\right]$$

The dynamic identification is applied to the linear differential relations. So to the rel. (21) – (24) there are considered small periodic harmonic perturbations around a stationary operating regime. The functions M_P M_T and the variables Q ω_P ω_T may be written:

$$M_P = M_{P0} + \widetilde{M}_P \tag{26}$$

$$M_T = M_{T0} + \widetilde{M}_T \tag{27}$$

$$Q = Q_0 + \widetilde{Q} \tag{28}$$

$$\omega_P = \omega_{P0} + \widetilde{\omega}_P \tag{29}$$

$$\omega_T = \omega_{T0} + \widetilde{\omega}_T \tag{30}$$

M_{P0} and M_{T0} are obtained from rel. (22) (23) without the terms which contain $\frac{d\omega}{dt}$ and $\frac{dQ}{dt}$.

The linear fluctuations of the functions are:

$$\widetilde{M}_P = k_{11}\frac{d\widetilde{\omega}_P}{dt}+k_{13}\frac{d\widetilde{Q}}{dt}+k_{14}Q_0\widetilde{\omega}_P+\left(k_{14}\omega_{P0}+2k_{16}Q_0\right)\widetilde{Q} \tag{31}$$

$$\widetilde{M}_P = k_{22}\frac{d\widetilde{\omega}_T}{dt}+k_{23}\frac{d\widetilde{Q}}{dt}+k_{24}Q_0\widetilde{\omega}_P+k_{25}Q_0\widetilde{\omega}_T+ \left(k_{24}\omega_{P0}+k_{25}\omega_{T0}+2k_{26}Q_0\right)\widetilde{Q} \tag{32}$$

$$0 = k_{41}\frac{d\widetilde{\omega}_P}{dt} + k_{42}\frac{d\widetilde{\omega}_T}{dt} + k_{43}\frac{d\widetilde{Q}}{dt} + \left(k_{44}Q_0 + 2k_{47}\omega_{P0}\right) + \left(k_{45}Q_0 + k_{48}\omega_{T0}\right)\widetilde{\omega}_T + \left(k_{44}\omega_{P0} + k_{44}\omega_{T0} + 2k_{46}Q_0\right)\widetilde{Q} \quad (33)$$

Noting with:

$c_{11} = k_{14}\,Q_0$

$c_{13} = k_{14}\,\omega_{P0} + 2\,k_{16}\,Q_0$

$c_{21} = k_{24}\,Q_0$

$c_{22} = k_{25}\,Q_0$

$c_{23} = k_{24}\,\omega_{P0} + k_{25}\,\omega_{T0} + 2\,k_{26}\,Q_0$

$c_{41} = k_{44}\,Q_0 + 2\,k_{47}\,\omega_{P0}$

$c_{42} = k_{45}\,Q_0 + 2\,k_{48}\,\omega_{T0}$

$c_{43} = k_{44}\,\omega_{P0} + k_{45}\,\omega_{T0} + 2\,k_{46}\,Q_0$

Relations (31) – (33) became:

$$\widetilde{M}_P = k_{11}\frac{d\widetilde{\omega}_P}{dt} + k_{13}\frac{d\widetilde{Q}}{dt} + c_{11}\widetilde{\omega}_P + c_{13}\widetilde{Q} \quad (34)$$

$$\widetilde{M}_T = k_{22}\frac{d\widetilde{\omega}_T}{dt} + k_{23}\frac{d\widetilde{Q}}{dt} + c_{21}\widetilde{\omega}_P + c_{22}\widetilde{\omega}_T + c_{23}\widetilde{Q} \quad (35)$$

$$0 = k_{41}\frac{d\widetilde{\omega}_P}{dt} + k_{42}\frac{d\widetilde{\omega}_T}{dt} + k_{43}\frac{d\widetilde{Q}}{dt} + c_{41}\widetilde{\omega}_P + c_{42}\widetilde{\omega}_T + c_{13}\widetilde{Q} \quad (36)$$

Through Laplace transform from the time domain in the frequency domain it is introduced the complex variable $s = A + B\,i$. The torques equations and the specific energy balance got the expressions:

$$\widetilde{M}_P(s) = \left(k_{11}s + c_{11}\right)\widetilde{\omega}_P(s) + \left(k_{13}s + c_{13}\right)\widetilde{Q}(s) \quad (37)$$

$$\widetilde{M}_T(s) = c_{21}\widetilde{\omega}_P(s) + \left(k_{22}s + c_{22}\right)\widetilde{\omega}_T(s) + \left(k_{23}s + c_{23}\right)\widetilde{Q}(s) \quad (38)$$

$$0 = \left(k_{41}s + c_{41}\right)\widetilde{\omega}_P(s) + \left(k_{42}s + c_{42}\right)\widetilde{\omega}_T(s) + \left(k_{43}s + c_{43}\right)\widetilde{Q}(s) \quad (39)$$

In matrix form which describes the variation around an equilibrium point:

$$\begin{pmatrix} \widetilde{M}_P(s) \\ \widetilde{M}_T(s) \\ 0 \end{pmatrix} = \begin{pmatrix} k_{11}s + c_{11} & 0 & k_{13}s + c_{13} \\ c_{21} & k_{22}s + c_{22} & k_{23}s + c_{23} \\ k_{41}s + c_{41} & k_{42}s + c_{42} & k_{43}s + c_{43} \end{pmatrix} \begin{pmatrix} \widetilde{\omega}_P \\ \widetilde{M}_P \end{pmatrix} \quad (40)$$

Considering the hydrodynamic torque converter as a informational quadripole, see fig. 4, eliminating the fluctuating rate of flow, $\tilde{Q}(s)$, between equations (37) – (39) results the transfer matrix:

$$\begin{pmatrix} \tilde{\omega}_T \\ \tilde{M}_T \end{pmatrix} = \begin{pmatrix} F\left(\dfrac{\omega_T}{\omega_P}\right) & F\left(\dfrac{\omega_T}{M_P}\right) \\ F\left(\dfrac{M_T}{\omega_P}\right) & F\left(\dfrac{M_T}{M_P}\right) \end{pmatrix} \begin{pmatrix} \tilde{\omega}_P \\ \tilde{M}_P \end{pmatrix} \tag{41}$$

The component transfer functions of the transfer matrix are:

$$F\left(\frac{\omega_T}{\omega_P}\right) = \frac{\left(k_{11}s + c_{11}\right)\left(k_{43}\,s + c_{43}\right) - c_{13}\,c_{41}}{c_{12}\,c_{42}} \tag{42}$$

$$F\left(\frac{\omega_T}{M_P}\right) = -\frac{k_{43}\,s + c_{43}}{c_{13}\,c_{42}} \tag{43}$$

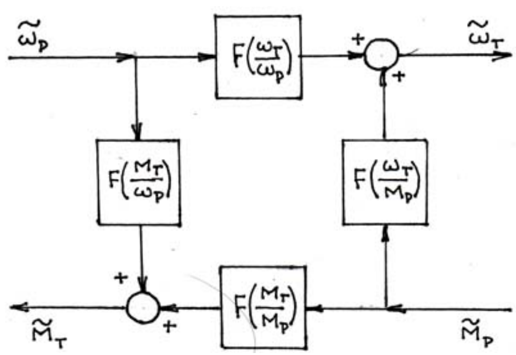

Fig. 4. Structure of the informational quadripole representing the dynamic behavior of the torque converter.

$$F\left(\frac{M_T}{\omega_P}\right) = \frac{c_{13}c_{21}c_{42} + \left(k_{11}s + c_{11}\right)\left(k_{22}s + c_{22}\right)\left(k_{43}s + c_{43}\right) - \left(k_{22}s + c_{22}\right)c_{13}c_{41} + \left(k_{11}s + c_{11}\right)c_{23}\,c_{42}}{c_{13}\,c_{42}} \tag{44}$$

$$F\left(\frac{M_T}{M_P}\right) = -\frac{\left(k_{22}\,s + c_{22}\right)\left(k_{43}\,s + c_{43}\right) - c_{23}\,c_{42}}{c_{13}\,c_{42}} \tag{45}$$

3. Conclusions

1. The paper establishes a theoretic model for the transfer matrix of a hydrodynamic torque converter in the shape of an informational quadripole.

2. There are given in linear, one-dimensional approximation and concentrated parameters the expressions for the transfer functions of the machine.
3. Based on the fundamental laws of mechanics and non-stationary fluid flows applied to a first class of hydrodynamic torque converter, the method used may be generalized to any kind of hydrodynamic transmission.
4. Experimental verification in a testing rig with harmonic perturbation will determine the precision of the model and the influence of the nonlinearities on the particular values of the expressions of the transfer functions.
5. Dynamic identification of the hydrodynamic torque converter permits the calculation of the non-stationary and transient behavior of the transmissions.

R E F E R E N C E S

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