U.P.B. Sci. Bull., Series A, Vol. 79, Iss. 1, 2017 ISSN 1223-7027

ON THE ACHIEVING CONTROL OF NOISE

Ruxandra ILIE[1], Veturia CHIROIU[2], Rodica IOAN[3]

A 2D acoustic cloak surrounding an object to make it acoustically "invisible" when the sound is incident from any direction and passes through and around the cloak, is discussed in this paper. The key for "invisibility" is the coordinate transformation which design the metamaterial required for a given application. The acoustic cloaking is based on the property of the Helmholtz equation to be invariant under the coordinate transformations, i.e. a specific compression is equivalent to a variation of the material parameters in the original space. The theory is simple and consists in transformation of an original domain with a given shape filled with a known material into a final domain, by applying a specific coordinate transformation. The final domain will have a desired shape and will be filled with a new desired material, strongly inhomogeneous and anisotropic. This new metamaterial must be engineered at the subwavelength scale in order to imitate the exotic properties provided by the wave equations. In this paper, we show that it is possible to design an acoustic cloak surrounding a noisy machine in order to control the noise. The cloak is characterized by a rank 4 material tensor that define the properties of new metamaterial. These properties are ideal and complex, being real challenges to experimentalists, but not impossible to be achieved.

Keywords: Acoustic cloaking, Helmholtz equation, coordinate transformation.

1. Introduction

An enhanced control of acoustic waves can be achieved through applying the coordinate transformations which bring the material parameters into their governing equations [1]. The coordinate transformation is a simple method for building acoustic cloaks with arbitrary sizes and shapes. The key is the property of the Helmholtz equation to be invariant to coordinate transformations [2]. The required material known as a metamaterial is engineered at the subwavelength scale [3-6] and its properties are designed by the coordinate transformation [7].

The purpose of the acoustic cloaking is to hidden for example a noisy machine. The functionality principle of the cloak to make its contents invisible to sound is discussed in [8-11]. Leonhardt [10] and Pendry [9] solve the problem of making the box invisible to light, valid for narrow bandwidths.

[1] Technical University of Civil Engineering, Bucharest, Romania
[2] Institute of Solid Mechanics, Romanian Academy, email: veturiachiroiuilie@gmail.com
[3] University Spiru Haret from Bucharest, Romania

As an alternative to a box made from metamaterial, sonic composites exhibit the full band-gaps, where the sound is not allowed to propagate due to complete reflections [12-14].

The acoustic field can be mimicked and cancelled out by using secondary sources. Such an approach is well known in acoustics as the *antinoise* [15-17]. An interesting review lecture in this direction has been addressed by Ffowcs [18].

Cummer [19] derived the mass density and bulk modulus of a spherical shell that can eliminate scattering from an arbitrary object in the interior of the acoustic shell. Results confirmed that the pressure and velocity fields were manipulated and excluded from the central region as for previously reported electromagnetic cloaking shells. It is also interesting to note that the ideal 3D acoustic cloaking parameters are similar in structure to the 2D electromagnetic and 2D acoustic parameters in that they contain singularities on the interior of the cloak. Cummer and Schurig [20] have demonstrated that the acoustic equations in a fluid are identical in form to the single polarization Maxwell equations via a variable exchange that also preserves boundary conditions. This is in contrast to the electromagnetic cloak [9], which does not contain singularities and for which scattering analysis does not require arguments to show the scattering is identically zero [22]. This is due to the electromagnetic and acoustic fields that are solutions of the Helmholtz equation and, therefore, the Bessel and spherical Bessel functions that do not all tend to zero as their argument approaches zero. By using this result, a class of rectangular cylindrical devices for noise shielding by using acoustic metamaterials was investigated by Liu and Huang [24]. Chen and Chan [21,22] designed a spherical acoustic cloaking shell by applying the isomorphism between the acoustic and electric equations. The correspondences among cloaks in electromagnetism and acoustics are studied in the context of exploiting the scalar nature of acoustic wave equations.

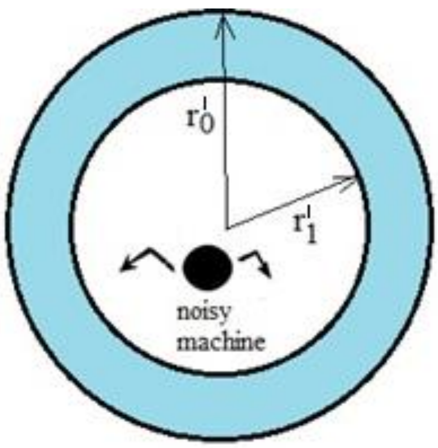

Fig.1. Sketch of the acoustic cloak surrounding a noisy machine.

These waves are all governed by a scalar partial differential equation invariant under geometric transformations [23]. These equations reduce to the Helmholtz equation which is the key for analyzing the band spectra of sound invisibility [26]. The design and experimental characterization of an almost perfect three-dimensional, broadband, and, most importantly, omnidirectional acoustic device that renders a region of space three wavelengths in diameter invisible to sound was performed in [27].

In this paper, a coordinate transformation is applied to obtain an annular acoustic cloak which surrounds a noisy machine (see Fig. 1). The original domain is a circular region D of radius r_0 filled with air. After transformation, new domain is an annular region D', of r_0' and r_1' the inner and outer radii, filled with new metamaterial. The expression of the acoustic tensor is the key for designing the heterogeneous and anisotropic properties of the new material. The results show that waves are smoothly bent around the central region inside the cloak. The waves inside the cloak are isolated from the region situated outside it. The sound is not detectable by an exterior observer because the wave amplitudes on the boundary vanish. The sound invisibility detected from an outside observer depends on the attenuation and frequency.

2. Problem formulation

Consider 2D problem of propagation of sound in a medium with constant speed of propagation c in the frequency domain at a fixed angular frequrncy ω, which is not a resonant frequency. The problem is to cloak a circular region D of r_0 and r_1 the inner and outer radii, from an incident known field of waves u_{inc}. We work in the frequency domain at a fixed angular frequency ω. In a medium with constant speed of propagation c, the wave field $u(x,y,\omega)$ satisfies the Helmholtz equation

$$\Delta u + k^2 u = 0, \tag{1}$$

where $k = \frac{2\pi f}{c}(1 + \mathrm{i}\delta_1)$ is the wavenumber, $\delta_1 = \delta/(40\pi \log_{10}(e))$ where $\delta(x,y)$ is the attenuation parameter, and $f = \omega/2\pi$ the frequency. For simplicity, we drop the dependency on the frequency and write $u(x,y) = u(x,y,\omega)$. In order to derive the acoustic pressure (the local deviation from the ambient pressure) is convenient to consider (1) for an abstract velocity potential $\Delta\phi + k^2\phi = 0$, by taking $u = \nabla\phi$ and $p = -\rho\phi_{,t}$. Therefore, the acoustic pressure is calculated as $\mathrm{Re}(u\exp(-2\mathrm{i}\pi ft))$.

Introducing the Green function

$$G(x,y,x_1,y_1) = \frac{\mathrm{i}}{4} H_0^{(1)}(k|d|), \tag{2}$$

where d is the distance between the point (x, y) and (x_1, y_1), and $H_n^{(1)}$ is the Hankel function of the first kind, the solution of (1) is given by [15, 29]

$$u_d(x, y) = \int_{\partial D} \left(-n\nabla u_{inc} G + u_{inc} n\nabla G\right) \mathrm{d}S\,, \tag{3}$$

where $n(x_1, y_1)$ is the unit outward normal to D at the point $(x_1, y_1) \in \partial D$.

We introduce the geometric transformation in cylindrical coordinates

$r = \sqrt{x^2 + y^2}$, $\theta = 2a\tan\left(\dfrac{y}{x + \sqrt{x^2 + y^2}}\right)$ [1]

$$r' = r_0 + \frac{r_1 - r_0}{r_1} r\,,\ \theta' = \theta \ \text{ for } r > r_1 \text{ and } r' = r\,,\ \theta' = \theta \ \text{ for } r \le r_1\,. \tag{4}$$

By application (4) to the domain $r' \in [r_0, r_1]$, the equation (1) is mapped into the equation

$$\Delta u + k'^2 u = 0\,, \tag{5}$$

where

$$k' = \frac{r - r_0}{r}\left(\frac{r_1}{r_1 - r_0}\right)^2 k\,. \tag{6}$$

The solution is mapped into [15, 29]

$$u'_d(x, y) = \int_{\partial D} \left(-n\nabla' u_{inc} G' + u_{inc} n\nabla' G'\right) \mathrm{d}S'\,, \tag{7}$$

$$G'(r') = \frac{\mathrm{i}}{4} H_0^{(1)}(k'|d'|)\,, \tag{8}$$

We see that the transformation (4) preserves the isotropy of the wave number, and (8) describes the property of the metamaterial which fills the annulus $r'_0 < r' < r'_1$.

By generalizing to 3D cases, the change of coordinates is characterized by the transformation of the differentials through the Jacobian $J_{xx'}$ of this transformation, i.e.

$$\begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \end{pmatrix} = J_{xx'} \begin{pmatrix} \mathrm{d}x' \\ \mathrm{d}y' \\ \mathrm{d}z' \end{pmatrix},\quad J_{xx'} = \frac{\partial(x, y, z)}{\partial(x', y', z')}\,. \tag{9}$$

From the geometrical point of view, the change of coordinates implies that, in the transformed region, one can work with an associated metric tensor [9, 15, 28, 29]

$$T = \frac{J_{xx'}^{\mathrm{T}} J_{xx'}}{\det(J_{xx'})}\,. \tag{10}$$

In terms of the acoustic parameters, one can replace the material from the original domain (homogeneous and isotropic) by an equivalent compressed one that is inhomogeneous (its characteristics depend on the spherical (r',θ',ϕ') coordinates) and anisotropic (described by a tensor), and whose properties, in terms of $J_{x'x}$ are given by

$$\kappa' = \kappa \det(J_{x'x}) , \tag{11}$$

or, equivalently, in terms of $J_{xx'}$

$$\kappa' = \frac{\kappa}{\det(J_{xx'})} . \tag{12}$$

3. Acoustic cloak

Our intention is to replace the initial material (air) which fills a circular region D by an equivalent compressed inhomogeneous anisotropic material described by (4) and (8). The new material obtained by compression of air, via coordinate transformation, is a material with subwavelength microstructures that are designed to have desired physical and acoustical properties. This material is not naturally occurring. Recent advances in metamaterials are encouraging for such an approach to constitutive parameters required for cloaking. To determine the material parameters, a unit cell is used to generate the cloak which has to be significantly subwavelength [19]. The unit cell is a circle of radius 50mm. Simulation of the acoustic wave propagation in the radial direction for $f = 200\text{Hz}$ and $f = 400\text{Hz}$ into the centre lane of the unit cell is presented in Fig. 2 for $\delta = 1$.

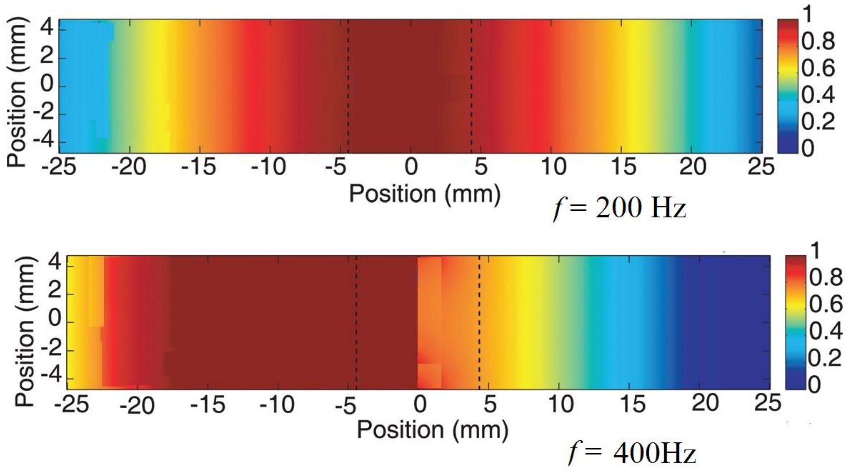

Fig. 2. Pressure field in the unit cell (arbitrary units) for $f = 200\text{Hz}$, $f = 400\text{Hz}$) and $\delta = 1$.

The metamaterial is composed of arrays of identical 2D unit cells. The recent progress on the experimental methods for fabrication of metamaterials based of unit cells make possible is based on direct laser writing, *i.e.*, a non-linear two-photon absorption process [30].

Fig. 3 displays the variation of the solution (7) u in the interior region $r \le r_1'$ of the cloak D' with respect to δ, for $f = 200\text{Hz}$. The attenuation parameter δ which appears in definition of k, is computed as $\delta_1 = \delta / (40\pi \log_{10}(e))$, where absorption $\delta[\text{dB}/\lambda]$ is defined, in the spirit of the Sabine equation, as the incident sound that is not reflected back.

In this paper, we consider the cases $\delta =$ 0.8, 0.9, 1, 1.2 and 1.4 [24].

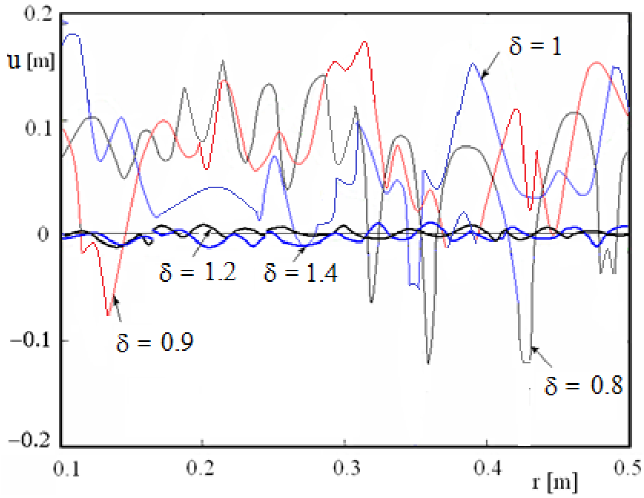

Fig. 3. Variation of the displacement amplitude with respect to δ in the region $r \le r_1'$.

Effective mass density in the metamaterial is presented in Fig.4 in the radial direction over the 100 Hz–2100Hz bandwidth. The effective mass densities vary few with frequency. The mass anisotropy ratio varies from 3.14 to 3.54 from 100–2100Hz.

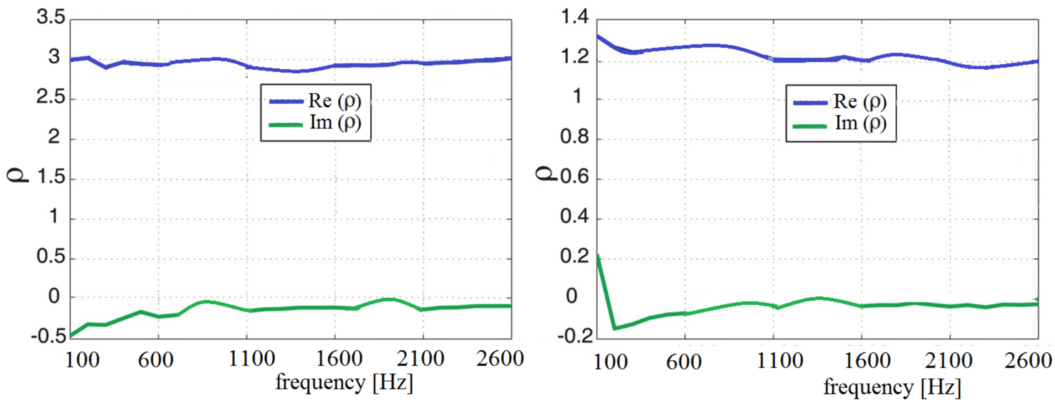

Fig. 4. Effective mass density in the radial direction over the 100 Hz–2100Hz bandwidth.

If we consider the case of an uncloaked region surrounding a noisy machine, the incident signals are scattered in all directions and creates standing waves as shown by the absolute pressure in Fig. 5 for $\delta = 1$ and $f = 200\text{Hz}$. The absolute pressure peaks at 1.77 Pa, which is almost double the amplitude of the noise machine incident wave. Fig. 6 shows the same plot with the cloak present for $\delta = 1$ and $f = 200\text{Hz}$. With the same range, the pressure variations with the cloak present would hardly be visible.

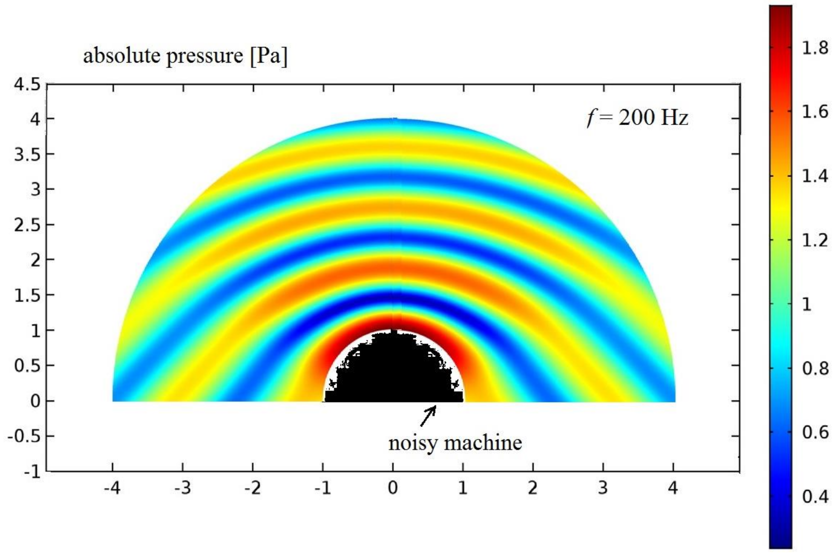

Fig. 5. Absolute value of the local pressure without the cloak for $\delta = 1$ and $f = 200\text{Hz}$.

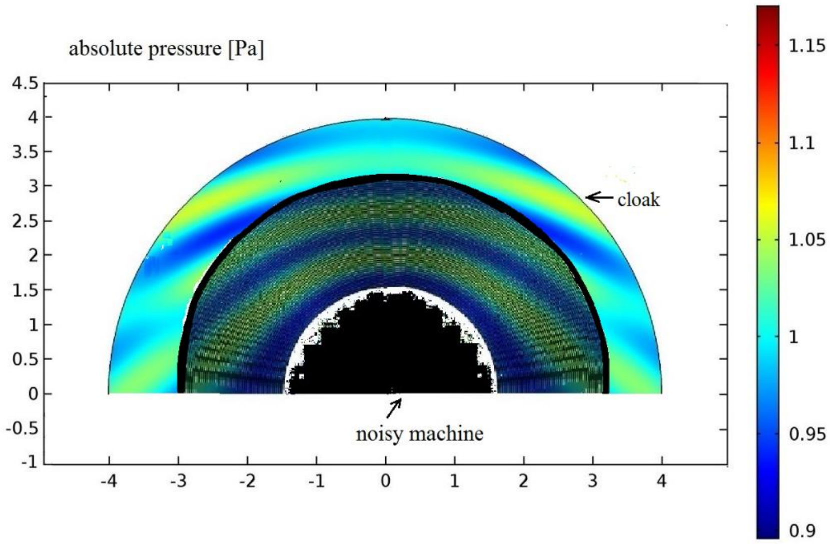

Fig. 6. Absolute pressure with the cloak for $\delta = 0.8$ and $f = 200\text{Hz}$.

To see the influence of the attenuation on the pressure field distribution, the absolute pressure with the cloak for $\delta = 0.8$ and $f = 200\text{Hz}$ is presented in Fig.6. We notice a more pronounced pressure inside the cloak.

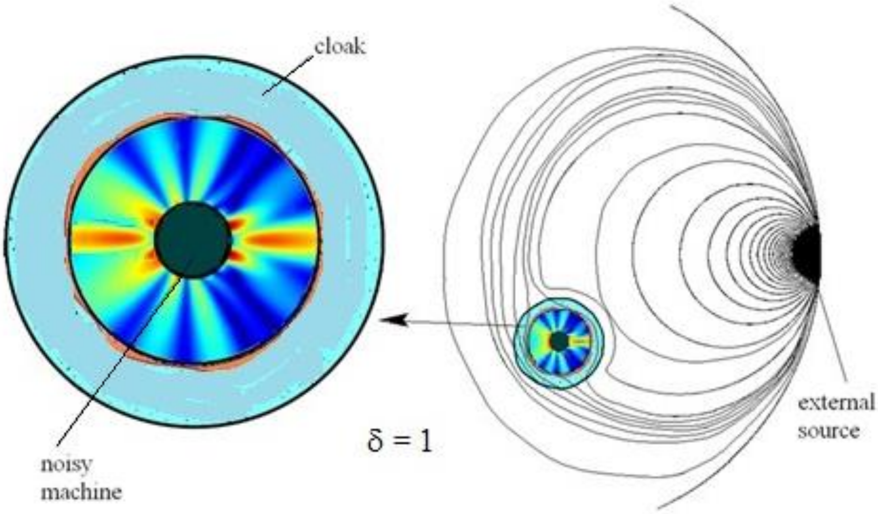

Fig. 7. Variation of the displacement u in the region $r_0' < r' < r_1'$ with respect to δ .

The absence of the scattering of waves generated by an external source outside the cloak is observed in Fig. 7 for $\delta = 1$ and $f = 200\text{Hz}$. The results show that the wave field inside the cloak, i.e. the inner region of radius r_1' which surrounds the noisy machine, is completely isolated from the region situated outside the cloak. The waves generated by the noisy source are smoothly confined inside the inner region of the cloak, and the sound invisibility detected from the observer depends on δ . The inner region is acoustically isolated and the sound is not detectable by an exterior observer because the amplitudes on the boundary vanish. The domain $r \le r_1'$ is an acoustic invisible domain for exterior observers. The waves generated by the exterior source outside the cloak do not interact with the interior field of waves. A possible interaction or coupling between the internal and external wave fields is cancelled out by the presence of the shell region $r_0' < r' < r_1'$ filled with metamaterial.

4. Conclusions

The control of sound propagation has been the goal of scientists involved in the design of acoustic cloaking. In this paper, we discuss a 2D acoustic cloak surrounding a noisy machine. The cloak make the noisy machine acoustically “invisible” when the sound is incident from any direction and passes through and around the cloak, is discussed in this paper. The original domain is a circular region D of radius r_0 filled with air. After transformation, the equivalent domain is an annular region D', of r_0' and r_1' the inner and outer radii, filled with new metamaterial. The key for "invisibility" is the coordinate transformation which design the metamaterial. The acoustic cloaking is based on the property of the Helmholtz equation to be invariant under the coordinate transformations, i.e. a specific compression is equivalent to a variation of the material parameters in the original space.

Acknowledgement

The authors gratefully acknowledge the financial support of the National Authority for Scientific Research ANCS/UEFISCDI through the project PN-II-ID-PCE-2012-4-0023, Contract nr.3/2013.

R E F E R E N C E S

[1] *Wolf , E., Habashy, T.,* Invisible bodies and uniqueness of the inverse scattering problem, J. Modern Opt., 40, 785–792, 1993.
[2] *Nicorovici, N.A., McPhedran, R.C., Milton, G.W*., Optical and dielectric properties of partially resonant composites, Phys. Rev. B, 490, 8479–8482, 1994.
[3] *Alu, A., Engheta, N*., Pairing an epsilon-negative slab with a mu-negative slab: resonance, tunneling and transparency, IEEE Trans. Antennas Propag., 51, 2558–2571, 2003.
[4] *Alu, A., Engheta, N*., Achieving transparency with plasmonic and metamaterial coatings, Phys. Rev. E, 72, 016623, 2005.
[5] *Milton, G.W., Briane, M., Willis, J.R*., On cloaking for elasticity and physical equations with a transformation invariant form, New Journal of Physics, 8, 248, 2006.
[6] *Milton, G.W., Nicorovici, N.A.*, On the cloaking effects associated with anomalous localized resonance, Proc. Roy. Soc. A 462, 3027–3059, 2006.
[7] *Milton, G.W.*, New metamaterials with macroscopic behavior outside that of continuum elastodynamics, New Journal of Physiscs, 9, 359– 372, 2007.
[8] *Guenneau, S., McPhedran, R.C., Enoch, S., Movchan, A.B., Farhat, M., Nicorovici, N.A.*, The colours of cloaks, Journal of Optics, 13, 2, 024014, 2011.
[9] *Pendry, J.B., Shurig, D., Smith, D.R*., Controlling electromagnetic fields, Science, 312, 1780–1782, 2006.
[10] *Leonhardt, U*., Optical conformal mapping, Science, 312, 1777–1780, 2006.
[11] *Miller, D.A.B.*, On perfect cloaking, Optical Society of America, 14,25, OPTICS EXPRESS 12465, 2006
[12] *Hirsekorn, M., Delsanto, P.P., Batra, N.K., Matic, P*., Modelling and simulation of acoustic wave propagation in locally resonant sonic materials, Ultrasonics, 42, 231–235, 2004.
[13] *Munteanu, L., Chiroiu, V.*, On the dynamics of locally resonant sonic composites, European Journal of Mechanics-A/Solids, 29(5), 871–878, 2010.
[14] *Munteanu, L., Chiroiu, V.*, On the three-dimensional spherical acoustic cloaking, New Journal of Physics, 13(8), 1-12, 2011.
[15] *Nelson, P.A., Elliott, S.J.*, Active Control of Sound, Academic Press, London, 290–293, 1992.
[16] *Friot, E., Bordier, C*., Real-time active suppression of scattered acoustic radiation, J. Sound Vib., 278, 563–580, 2004.
[17] *Friot, E., Guillermin, R., Winninger, M.*, Active control of scattered acoustic radiation: a real-time implementation for a three-dimensional object, Acta Acust. 92, 278–288, 2006.

[18] *Ffowcs Williams, J.E.*, Review Lecture: Anti-Sound, Proc. Roy. Soc. London A 395, 63–88, 1984
[19] *Cummer, S.A., Popa, B.I., Schurig, D., Smith, D.R., Pendry, J., Rahm, M., Starr,* A., Scattering theory derivation of a 3D acoustic cloaking shell, Physical Review Letters, 100, 024301, 2008.
[20] *Cummer, S.A., Schurig, D.*, One path to acoustic cloaking, New Journal of Physics, 9, 45, 2007.
[21] *Chen, H., Chan, C.T.*, Acoustic cloaking in three dimensions using acoustic metamaterials, Applied Physical Letters, 91, 183518, 2007.
[22] *Chen, H., Wu, B.I., Zhang, B., Kong, J.A.*, Electromagnetic wave interaction with a metamaterial cloak, Physical Review Letters, 99, 063903, 2007.
[23] *Chiroiu, V., Chiroiu, C.*, Inverse Problems in Mechanics, (in Romanian) Publishing House of the Romanian Academy, 2003.
[24] *Liu, B., Huang, J.P.*, Noise shielding using acoustic metamaterials, Communications in Theoretical Physics (Beijing, China), 53, 560–564, 2010.
[25] *Li, J., Chan, C.T.*, Anomalous Doppler effects in phononic band gaps, Phys. Rev. E, 70, 055602, 2004.
[26] *Torrent, D., Sanchez-Dehesa, J.*, Acoustic metamaterials for new two-dimensional sonic devices, New Journal of Physics, 9, 323, 2007.
[27] *Zigoneanu, L., Popa, B.I., Cummer, S.A.*, Three-dimensional broadband omnidirectional acoustic ground cloak, Nature Materials 13, 352–355, 2014.
[28] *Zolla, F., Guenneau, S., Nicolet, A., Pendry, J.B.*, Electromagnetic analysis of cylindrical invisibility cloaks and the mirage effect, Opt. Letters, 32, 1069–1071, 2007.
[29] *Larsson, E., Holmgren, S.,* A parallel domain decomposition method for the Helmholtz equation, Department of Scientific Computing, Information Technology, Uppsala University, Box 120, SE-751 04 Uppsala, Sweden, 1-20, 2000.
[30] *Liu, Y., Zhang, X.*, Metamaterials: a new frontier of science and technology, Chemical Society reviews, issue 5, 2011.