ALGORITHM DEVELOPMENT FOR THE OPTIMIZATION OF CUTTING TOOL TRAJECTORIES ON CNC MACHINE

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CNC machines are an essential tool in the manufacturing industry as they are used for producing complex parts. The precision and quality of these parts depend on the accuracy of the cutting trajectory and axis programming. To improve these performances, interpolation methods such as Cubic Spline and Lagrange interpolation exist. This method is capable of providing more precise trajectories and more uniform cutting speeds, resulting in superior cutting quality and precision. This paper presents a new algorithm for the optimization of cutting tool paths for machining on 5-axis CNC machines. This algorithm is based on the combination of Lagrange interpolation and salt cubic interpolation. The results of this study have been simulated and validated by a real case.

Keywords: Tool path; correction of trajectories; CAM; complex shaped; deflection; tolerances of shape

1. Introduction

The Computer Numerical Control (CNC) has revolutionized the manufacturing industry by enabling automated fabrication of complex parts with high precision. These machines use computer programs to control the movements of cutting tools and produce parts based on the provided drawings and specifications.

However, despite their numerous advantages, CNC machines also have their limitations. One of the main limitations of CNC lies in their ability to create complex shapes. While these machines are capable of producing parts with high precision, they may encounter difficulties when it comes to creating highly complex geometric shapes or very fine contours.

In order to improve CNC machines, several research initiatives have been undertaken. One notable study by Osama Abdulhameed and al highlighted the challenges of additive manufacturing (AM) like poor accuracy, surface quality, and low speed. They proposed overcoming these limitations by integrating AM with other processes, leading to the emergence of hybrid manufacturing (HM). In HM different AM methods are combined or supplemented with subtractive processes, resulting in improved tool life, faster material removal rates, enhanced

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dimensional accuracy, and reduced manufacturing times. HM combines the strengths of both additive and subtractive processes to address AM limitations, enabling the production of complex metallic parts with higher accuracy and surface finish [1].

Patel and al proposed a comparative study of zigzag and spiral tool path on sculpture surfaces, they showed that for zigzag tool path the surface finishing of sculpture is poor due to large variation in scallop height while the spiral tool path is good due to small variation in scallop height and the Tool utilization for spiral tool path is less comparatively zigzag tool path [2].

Guo and al introduced a novel smoothing method for continuous line-segment paths in CNC machining, known as the SSTI method. Unlike traditional approaches, the SSTI method does not directly modify the G-code tool path; instead, it employs real-time transformation of interpolation points to achieve path smoothing. The experimental results confirm that the SSTI method significantly enhances the quality of surface machining. In comparison to conventional smoothing algorithms for line-segment paths, the proposed method exhibits superior stability and versatility, contributing to improved CNC machining quality for continuous line-segment tool paths [3].

Li and Zhu's method for compensating deformation errors in five-axis flank milling involves predicting tool/workpiece deformations and incorporating them into the cutter envelope surface to construct the machined surface. The study evaluates machining errors by calculating signed distances between design surface points and the machined surface. The approach focuses on deriving differential increments to understand how surface errors change with tool path adjustments. They develop a mathematical model and algorithm to minimize deformation-induced surface errors by slightly optimizing tool path surface parameters. Experimental results in five-axis blade milling demonstrate that optimizing the tool path effectively reduces the impact of deformations on machining accuracy, while maintaining tool path smoothness through light adjustments to guiding curves' control points[4].

Pezer illustrates the efficiency of Tool Path Optimization Using Genetic Algorithm in Relation to the Optimization Achieved with the CAM Software.

The author was tried to find (by using of genetic algorithm) a sequence of drilling path that provides the shortest route, respectively, reduction of the total work time and increase efficiency, in relation to the route obtained by CAM software (WinCAM, CAMConcept and CATIA V5). Although, this software contains, or have built-in, modules for optimization of the tool path, the genetic algorithm provides a more favorable solutions or solutions closer to the optimum [5].

Beudaert and al proposed a feedrate interpolation method for 5-axis NURBS and G1 tool paths, considering axis jerk constraints. They highlighted the challenges in High Speed Machining, where high velocities and accelerations can adversely affect machine and workpiece surface quality. The authors emphasized the need to control kinematic parameters for each axis, including velocity, acceleration, and jerk, as well as those for tool-workpiece movement. Traditional control methods often sacrifice productivity without fully leveraging machining center capabilities. To address this, the authors presented a unified and efficient solution to minimize machining time by optimizing the machine tool's kinematic performance along the tool path for each axis [6].

Hsieh and Chu introduced an improved optimization method for tool path planning in 5-axis flank milling, employing advanced Particle Swarm Optimization (PSO) algorithms. Their study focused on optimizing tool paths for ruled surfaces, considering machining error as the primary objective. They utilized Advanced Particle Swarm Optimization (APSO) and Fully Informed Particle Swarm Optimization (FIPS) algorithms to enhance the quality of optimal solutions. Test results revealed that FIPS was particularly effective in reducing errors in all trials, while PSO excelled when the number of cutter locations was low. Overall, their research contributes to enhancing tool path planning in 5-axis flank milling by achieving smaller machining errors [7].

Yuwen Sun's and al introduced innovative methods for generating tool paths on mesh surfaces. The process involves transforming the surface using conformal mapping to achieve a genus-0 surface and generating tool paths within a rectangular region.[8].

Shuoxue Sun and al proposed an innovative tool path planning method for 5-axis flank milling of ruled surfaces. The focus is on optimal cutter locations (CLs) under multiple geometric constraints. The approach involves a three-point contact tool positioning model, utilizing a meta-heuristic algorithm to construct a tool orientation pool [9].

Marc-André Dittrich and al introduced a self-optimizing process planning method for 5-axis milling that addresses tool deflection. The approach combines material removal simulations, shape error measurements, and machine learning to predict and adapt the tool path automatically, resulting in a 50% reduction in shape error [10].

Wei He and al introduced an innovative optimization scheme for iso-parametric CNC tool paths, leveraging adaptive grid generation to tackle issues like overlap and inefficiencies stemming from the smallest intervals in CAM systems. The crux of the optimization involves employing an adaptive grid to generate an optimal tool path, potentially yielding the same number of discrete tool paths as the iso-parametric approach but with reduced machining errors [11].

Jiangang Li and al proposed a novel tool path optimization algorithm for five-axis machine tools within the postprocessor. The algorithm employs cubic spline interpolation to smooth the tool path, reducing non-linear errors. Data densification is utilized to add more points along the tool path, subsequently converted into NC codes [12].

Adam Jacso and al introduced a novel tool path optimization method for trochoidal milling using B-spline curves. Despite trochoidal milling's known productivity and tool life benefits, previous studies mostly focused on circular and cycloid-shaped tool paths. Jacso's algorithm aims to maximize the average material removal rate (MRR) and control tool load by optimizing B-spline curve control points through a differential evolution algorithm [13].

N. A. Fountas and al presented an innovative methodology for optimizing tool paths in 5-axis sculptured surface CNC machining. The approach leverages a virus-evolutionary algorithm with viral operators and intelligent system control to optimize both machining surface error and machining time [14].

D. H. Kim and al focused on optimizing tool paths in Selective Laser Sintering (SLS) processes, a method valued for its efficiency in creating intricate structures through additive manufacturing. The primary goal is to achieve optimal tool paths that minimize temperature gradients and unintended residual stresses in 3D printed structures [15].

SHI ZiKang and al proposed a G3 continuous toolpath smoothing method for a five-axis hybrid machining robot. By strategically inserting B-splines in the machine coordinate system (MCS) and employing the golden section method to estimate transition errors. The approach involves adaptive modifications of B-splines, adding anchor points and optimizing control points, with a bisection search method ensuring adherence to user-defined error tolerance limits. [16].

Zhou Feng and al focuses on advancing the optimization of numerical control programs and machining simulations, with a specific emphasis on the VERICUT platform. By targeting the machining of a locomotive's bogie frame, Zhou Feng employs a dynamic programming algorithm, alongside Creo and VERICUT software, to generate optimized tool paths, compile and validate numerical control programs, and simulate machining processes. The method, aiming to conserve machining time, refine cutting parameters, and ensure machining quality, lays a foundation for the integration of similar platforms [17].

Than Lin and al introduced a precise and efficient method for the five-axis CNC machining of free-form surfaces, employing a flat end mill cutter. The proposed algorithm autonomously selects the optimal tool and plans the toolpath through a combination of curvature matching and integrated inverse kinematics of the machine tool. Notably, the algorithm utilizes the real cutter contact toolpath generated by inverse kinematics, deviating from the conventional linearized piecewise real cutter location toolpath. The determination of the minimum tool inclination angle is a key aspect, strategically preventing gouging on the YL-ZL plane [18].

The study by Mohamed A CHAMI and al focuses on addressing workpiece positioning errors (WPE) in CNC milling to enhance the accuracy of manufactured parts. The research emphasizes the challenges of achieving dimensional accuracy in mass production due to various sources of errors, particularly workpiece positioning errors. It proposes a mathematical model to optimize these positioning errors and enhance the overall accuracy of the manufacturing process. The research outlines a four-step approach to develop a mathematical model for WPE, utilizing experimental data to refine calculations for optimal positioning. Future work will expand the model to include additional factors, particularly focusing on clamping constraints to further enhance accuracy in CNC machining. This study provides valuable insights into minimizing workpiece positioning errors in CNC milling, offering a comprehensive approach to enhancing the accuracy of manufactured parts [19] .

In the following article, we look at a new methodology designed to revolutionize the technology sector. By introducing a new approach, this paper aims to transcend existing boundaries, propelling the capabilities of technology to unprecedented heights. By exploring innovative avenues and challenging traditional norms, we believe this approach will usher in a new era of possibilities, redefining the very essence of technological progress.

2. Interpolation

Interpolation is the process of calculating the movements of the machine's tool based on the input design data. Interpolation is used to generate tool paths based on the design points defined by position and direction data for the machine's tool. The interpolators used for CNC determine the tool movements between trajectory points using various interpolation algorithms such as Hermite, Bezier, Chebychev, or cubic spline. Although interpolators generate tool movements between trajectory points, it is the CNC controller that determines how the machine's axes should be positioned to allow these movements and maintain machining accuracy. The choice of interpolation algorithm depends on the specific trajectory to be machined, the required precision, and the application for which the computer numerical control machine is used.

3. Limitations of interpolation

Interpolators can present certain limitations when used to calculate tool movements on CNC machines. Among these limitations, one can cite low accuracy in areas of high curvature, which can lead to deviations from the desired trajectory. This inaccuracy is often exacerbated in applications requiring high precision of movement, such as in the machining of complex shapes.

Moreover, the use of interpolators can increase the complexity of processing control parameters, as constant adjustments are needed to compensate for accuracy deviations. In complex geometries, this complexity results in longer computation times and difficulties in achieving satisfactory results, particularly when manual adjustments or compromises are required to maintain toolpath consistency. These constraints can limit the overall efficiency of the machine, making it difficult to achieve the desired tolerances in some production contexts.

4. Tool path optimization algorithm

The tool path optimization algorithm (TPOA) figure.1 calculates the error between two different interpolators using the original tool path as input. The first interpolator calculates an approximation of the path using predefined points, while the second interpolator uses a different interpolation method to calculate a different approximation. Then the two approximations are compared to calculate the error between the two interpolators.

To create a more accurate trajectory, the program uses the results of both interpolators by combining the two sets of points. This combination is done by finding the closest set of points to the original trajectory.

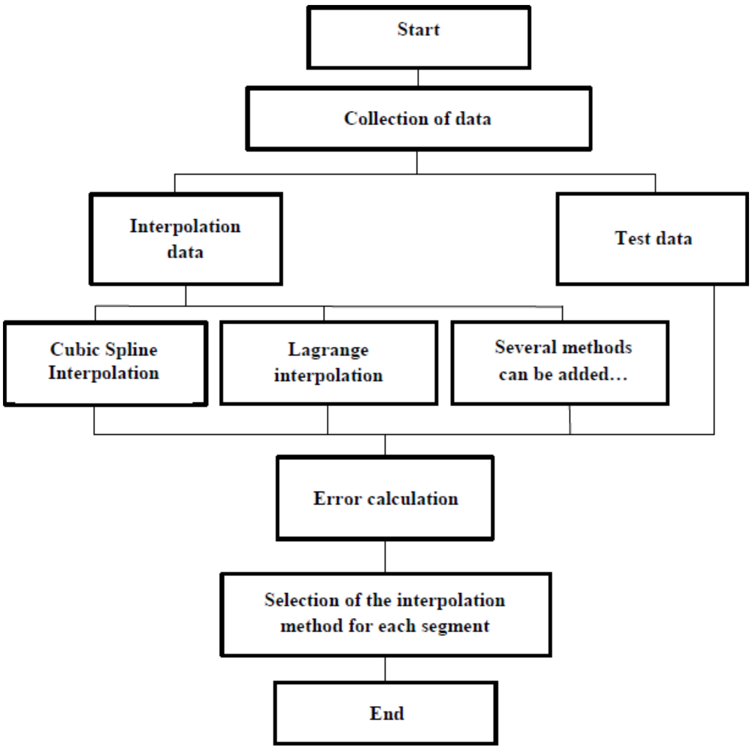

Fig. 1. Tool Path Optimization Algorithm (TPOA).

4.1. Combining interpolators

In this paper, two methods have been used, the first method is Cubic Spline Interpolation and the second method is Lagrange interpolation. It's important to note that the TPOA method can utilize more than two methods. The Cubic Spline Interpolation and Lagrange Interpolation methods were chosen as examples in this study. This implies that the TPOA method has the flexibility to incorporate various interpolation methods depending on the specific needs of the machining operation.

The concept of combining interpolators in CNC machining relates to using multiple interpolation algorithms in different sections of a tool path to achieve the desired out- come (Figure.2). For example, one algorithm may be better suited for curves, while another may be better for straight lines. By using a combination of algorithms, a CNC machine can achieve greater accuracy and efficiency in machining complex geometries. However, choosing the right combination of interpolators requires a thorough understanding of each algorithm and its limitations. Overall, this technique can be a powerful tool for achieving high-quality CNC machining results.

Figure 2 shows the transition between interpolators using the TPOA method.

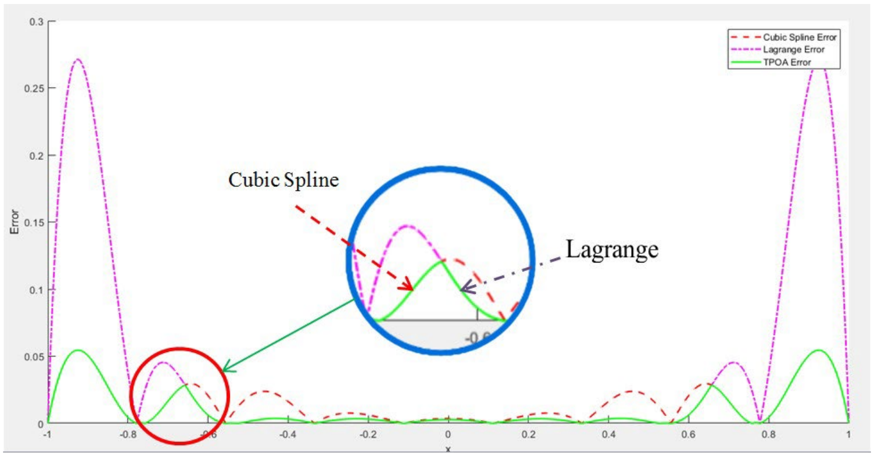

Fig. 2. Error graph of the TPOA compared to Cubic Spline and Lagrange.

4.2. Cubic Spline Interpolation

The goal of cubic spline interpolation [20] is to get an interpolation formula that is continuous in both the first and second derivatives, both within the intervals and at the interpolating nodes. This will give us a smoother interpolating function. The continuity of first derivative means that the graph y = S(x) will not have sharp corners. The continuity of second derivative means that the radius of curvature is defined at each point.

Given the n data points $(x_1,y_1),\ldots,(x_n,y_n)$, where x_i are distinct and in increasing order. A cubic spline S(x) through the data points $(x_1,y_1),\ldots,(x_n,y_n)$ is a set of cubic polynomials:

$$S_{n-1}(x) = y_{n-1} + b_{n-1}(x - x_{n-1}) + c_{n-1}(x - x_{n-1})^2 + d_{n-1}(x - x_{n-1})^3 \text{ on } [x_{n-1},x_n] \quad (1)$$

With the following conditions (known as properties):

a. $S_i(x_i) = y_i$ *and* $S_i(x_{i+1}) = y_{i+1}$ *for* $i=,\ldots,n-1$ (2)

This property guarantees that the spline S(x) interpolates the data points.

b. $S'_{i-1}(x_i) = S'_i(x_i)$ *for* $i=2,\ldots,n-1$ (3)

S'(x) is continuous on the interval $[x_1, x_n]$; this property forces the slopes of neighboring parts to agree when they meet.

a. $S'_{i-1}(x_i) = S'_i(x_i)$ *for* $i=2,\ldots,n-1$ (4)

S'(x) is continuous on the interval $[x_1, x_n]$, which also forces the neighboring spline to have the same curvature, to guarantee the smoothness.

4.3. Lagrange interpolation

Lagrange interpolation provides an alternative method for defining the polynomial P(x) without the need to solve complex systems of equations. This technique offers a more efficient approach, sidestepping the challenges associated with solving intricate equation systems.

Consider the function f : $[x_0, x_n] \rightarrow R$ given by the following table of values [21]:

x_k	x_0	x_1	. . .	x_n
$f(x_k)$	$f(x_0)$	$f(x_1)$	. . .	$f(x_n)$

x_k are called interpolation nodes, and they are not necessarily equally distanced from each other. Aiming to find a polynomial P(x) of degree (n) that approximates the function f(x) in the interpolation nodes, i.e.:

$$f(x_k) = P(x_k);\ k = 0, 1, 2, \ldots, n. \quad (5)$$

The Lagrange interpolation method finds such a polynomial without solving the system 5.

4.3.1. Theorem 1 Lagrange Interpolating Polynomial

The Lagrange interpolating polynomial [20] is the polynomial of degree (n) that passes through (n + 1) points $y_0 = f(x_0)$, $y_1 = f(x_1)$, . . . $y_n = f(x_n)$.

Then, the interpolating polynomial is simply:

$$P(x) = \sum_{j=0}^{n} P_j(x) \quad (6)$$

Where:

$$P_j(x) = y_j \prod_{k=0,k\neq j}^{n} \frac{x - xk}{xj - xk} \quad (7)$$

5. Simulation and experimental results

In order to verify the efficiency of the TPOA method proposed in this paper, the algorithm simulation and machining experiment are conducted. The algorithm simulation is conducted on a personal computer with an i3 Intel CPU, while the machining experiment is conducted on a tow-axis CNC machine tool The BoxFord 160 LTCI (figure.3).

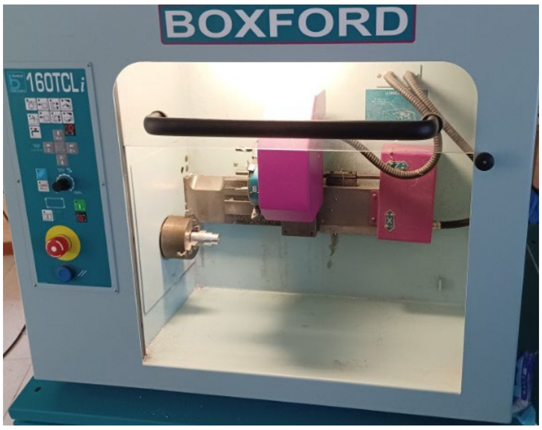

Fig. 3. Machine used (BoxFord 160 LTCI).

5.1. Matlab simulation

In this study, several tool paths were simulated in Matlab using three methods: cubic spline, Lagrange, and the TPOA method. The first method employed was cubic spline interpolation, which was compared to the original trajectory, the ideal path that the CNC machine should follow. The second method, Lagrange interpolation, was also compared to the same original trajectory used with cubic spline interpolation. This comparison allowed us to understand the advantages and disadvantages of each interpolation method. Additionally, we analyzed the TPOA method in comparison to the same original trajectory used with both cubic spline and Lagrange interpolations. Notably, the TPOA method dynamically switches between cubic spline and Lagrange on each segment by calculating the interpolation method that makes less error.

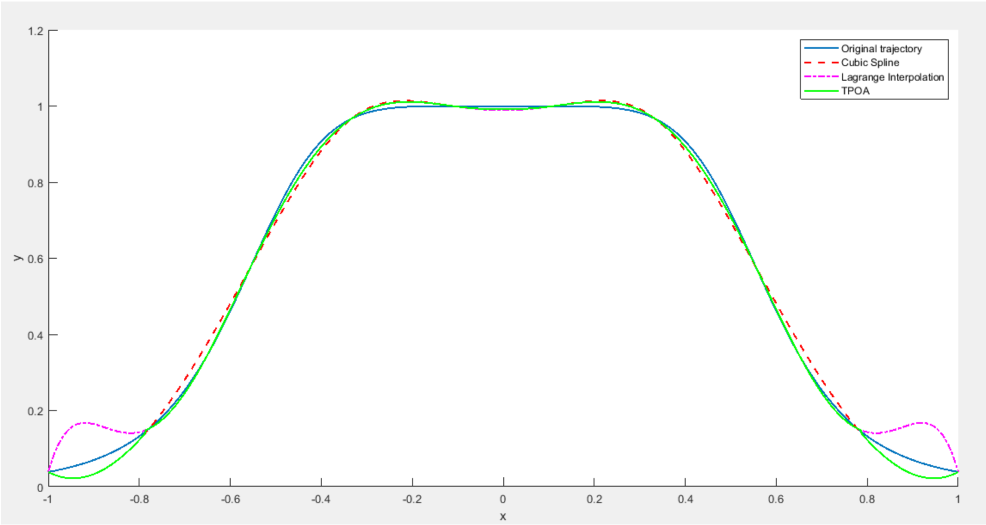

Fig. 4. Comparison between the three methods.

The cubic spline interpolation method can introduce errors depending on the characteristics of the data and the chosen spline parameters. In the following graph, we observe the errors generated during the interpolation of the original trajectory. When examining the error of the Lagrange interpolation, we notice the appearance of Runge's phenomenon and an increase in error, particularly near the boundaries of the data. However, when evaluating the error of the TPOA method, we observe that this method produces less error than the two interpolators used in this experimental setup. In other word, the TPOA method effectively reduces the error in the tool path.

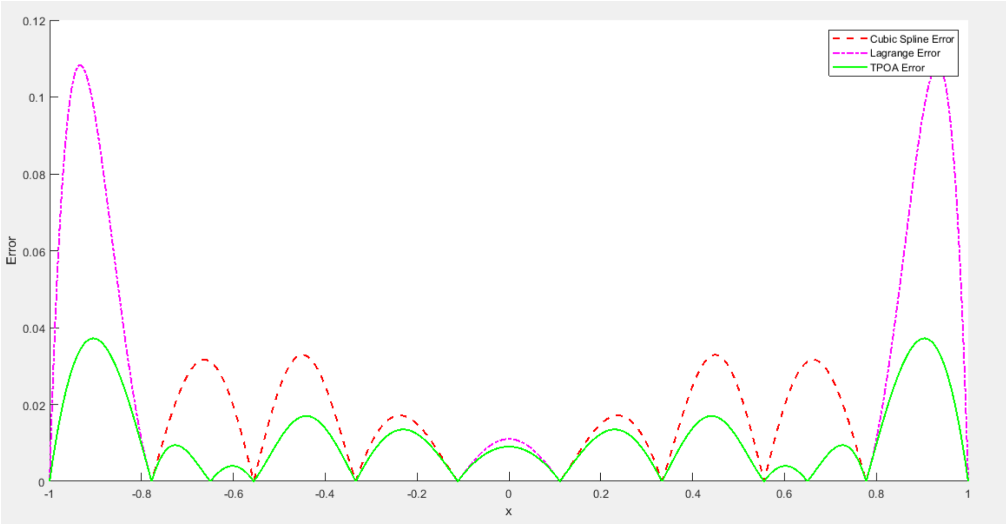

Fig. 5. Errors Comparison of Three Methods.

Table 1 presents the Root Mean Square Error (RMSE) calculations for three methods: (RMSE T) for the TPOA method, (RMSE LA) for the Lagrange method, and (RMSE C) for the Cubic Spline method. We observe that the TPOA method incurs fewer errors than the other two methods. This is because it combines the two tool paths by calculating the error for each point and selecting the method with the minimum error for each segment. In other words, the TPOA method utilizes both the Cubic Spline method and the Lagrange method to generate a single tool path with reduced error.

Table 1

The root mean square error (RMSE)

Example	**RMSE T**	**RMSE LA**	**RMSE C**
1	0.04019	0.10954	0.040932
2	0.018384	0.087975	0.020865
3	0.0049361	0.022587	0.010134
4	0.00040887	0.0061507	0.00052898
5	4.5591e-07	4.5595e-07	9.0081e-07
6	3.558e-15	4.422e-14	3.8772e-15
7	5.0573e-14	5.0601e-14	0.01457
8	7.4945e-16	5.3273e-15	8.666e-16
9	1.7621e-15	7.7915e-15	2.0831e-15
10	2.0555e-15	9.0894e-15	2.7209e-15
11	1.8342e-16	7.5478e-16	2.7304e-16

Several random tool paths were tested to see the advantage of the TPOA method and where each interpolator method could bring a better result.

The TPOA method uses two interpolation methods to generate one single path, as shown in Example 3 (Table 1) where RMSE LA is greater than RMSE C, noticing that that the majority of tool paths were generated using the Cubic Spline method, while in Example 7 (Figure 6), the majority of tool paths were generated using the Lagrange method. This means that the algorithm uses the method with the least error in each segment.

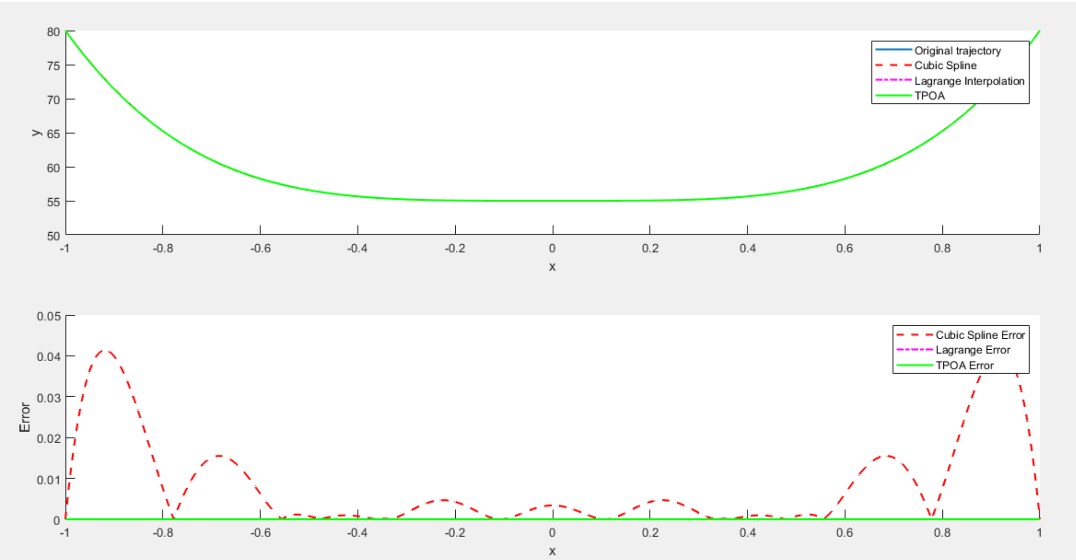

Fig. 6. Trajectory and error graph for example 7.

5.2. TPOA tool path generation for the CNC machine

The BoxFord 160 LTCI is a compact bench lathe that allows machining of mild steel. Its 8-station programmable turret enables a wide variety of internal and external machining. The Boxford V10 CAD/CAM software is used for designing and programming machining operations on this lathe.

During testing on the BoxFord 160 LTCI, the TPOA method was utilized to generate an optimized tool path for a simple cylindrical shape. The Boxford V10 CAD/CAM software was employed in this process, considering cutting parameters, tolerances, and part specifications.

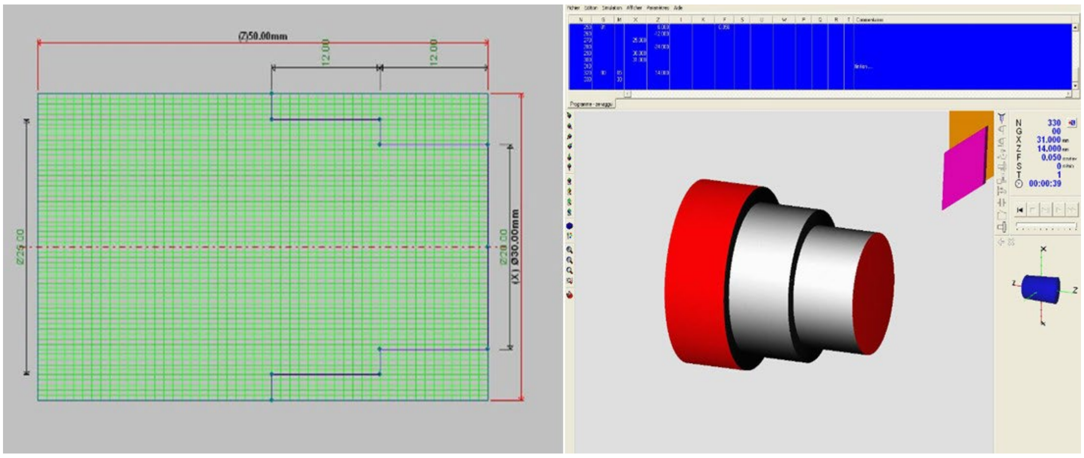

Fig. 7. Software Boxford V10 CAD/CAM.

5.2.1. Cutting parameters

As part of the evaluation of the TPOA method, we employed the specified cutting parameters from Table 2. It is essential to note that the feed rate varied, while the other parameters remained constant. The objective was twofold: first, to minimize surface roughness, and second, to analyze the effect of TPOA on this roughness, as well as the impact of feed rate on method precision.

Table 2

Cutting parameters used in The BoxFord 160 LTCI.

Spindle Speed, n (rpm)	Feed, f (mm/rev)	Cutting Speed, vc (m/mn)
2196,35	0,05	175
	0,08	
	0,11	
	0,19	
	0,25	

6. Results and discussion:

The TPOA method offers significant potential for enhancing machine efficiency and improving the quality of manufacturing operations. By improving cutting precision, it yields more reliable results and enhances the smoothness and precision of tool movement. This method significantly impacts machining due to its unique characteristics that utilize multiple interpolators, leading to a reduction in errors during the trajectory interpolation process and improving the roughness of the part. Even with an increased feed rate, the roughness remains lower compared to other methods. Figure 8 represents the machined part using this method.

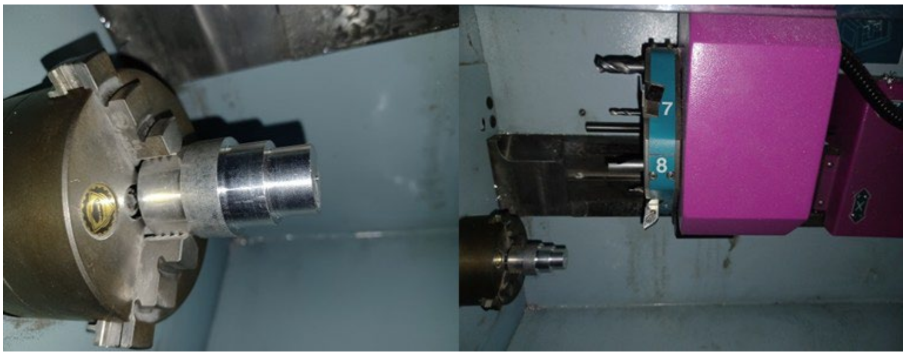

Fig. 8. Work piece after cutting using TPOA method.

Table 3 Surface Roughness, Ra (μm) for different Feed, f (mm/rev).

Feed, f (mm/rev)	0,05	0,08	0,11	0,19	0 ,25
Surface Roughness, Ra (µm)					
Lagrange	0,23	0,43	0,59	0,70	0,77
Cubic Spline	0,37	0,65	0,70	0,75	0,83
TPOA	0,06	0,1	0,14	0,22	0,35

As observed from Table 3, The TPOA method has had a significant impact on machining due to its unique characteristics that allow it to take advantage of multiple interpolators. This has reduced errors during the trajectory interpolation process, which has improved the roughness of the part. Even when increasing the feed rate, the roughness remained low compared to other methods.

Furthermore, optimizing the manufacturing process, taking into account the dynamic performance of the machine used, is another advantage of the TPOA method. For example, the choice of machining strategy and the type of interpolation can have a significant influence on the quality and machining time.

7. Conclusion

The TPOA (Tool Path Optimization Algorithm) combines the Cubic Spline and La- grange methods to generate tool trajectories with a lower average error. This method is particularly useful in applications where trajectory accuracy is crucial, such as precision manufacturing and additive manufacturing.

- Advantages of TPOA:
  - Reduced Error: By leveraging the strengths of both the Cubic

Spline and La- grange methods, the TPOA can effectively reduce errors in tool trajectories, leading to improved accuracy in manufacturing processes.

- Minimized Unnecessary Movement: The TPOA aims to minimize unnecessary movement of the CNC (Computer Numerical Control) machine, resulting in more efficient tool paths Improved Efficiency for Complex Tool Paths: The TPOA's efficiency increases as the complexity of the tool path increases. This means that for intricate tool paths, the TPOA can provide significant benefits in terms of trajectory accuracy and optimization.

• Drawbacks of TPOA:

- Increased Computational Time: One drawback of the TPOA is that it can take longer to generate trajectories compared to each individual method. This is because the TPOA combines the strengths of both methods, which requires additional computational time. However, the trade-off is improved trajectory accuracy.

Overall, the TPOA is a valuable tool for applications that prioritize trajectory accuracy, especially in scenarios involving complex tool paths. While it may require more computational time, the benefits of reduced error and minimized unnecessary movement make it a worthwhile approach in precision manufacturing and additive manufacturing.

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