SOFT LATTICES(IDEALS, FILTERS) RELATED TO FUZZY POINT

Şerife Yilmaz[1], Osman Kazanci[2]

Inspired by the study of algebraic structures of soft sets, our aim in this paper is to initiate research on the connection between soft sets and lattices. We apply the notion of soft sets to lattices. Some related notions are defined and several basic properties are discussed by using the soft set theory. Furthermore, the concept of $\in$-soft set, q-soft set and q_k-soft set is introduced and some interesting properties are investigated. Using the notion of "belongingness" and "quasicoincidence" of fuzzy points and fuzzy sets, characterizations for an $\in$-soft set, q-soft set and q_k-soft set to be a soft ideal(filter) are established.

Keywords: Sublattice; Fuzzy sublattice; Soft set; Soft lattice(ideal,filter); Soft function.
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1. Introduction

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces and the like. This provides sufficient motivation to researchers to review various concepts and results from the realm of abstract algebra in the broader framework of fuzzy setting. One of the structures that are most extensively used and discussed in mathematics and its applications is lattice theory. As it is well known, it is considered as a relational, ordered structure, on one hand, and as an algebra, on the other hand (see e.g. [4, 7]). Uncertainties, which could be caused by information incompleteness, randomness, limitations of measuring instruments, etc., are pervasive in many complicated problems in engineering, economics, environment science, medical science and social science [19]. Several theories like probability theory, fuzzy set theory [28], vague set theory [9], rough set theory [23] and interval mathematics [11], can be considered as mathematical tools for modeling uncertainties. However, as pointed out by Molodtsov in [21], all of these theories have their own difficulties, and one of the major reasons for these difficulties is the inadequacy of the parametrization

[1] Department of Mathematics, Karadeniz Technical University, 61080, Trabzon, Turkey, E-mail: serifeyilmaz@ktu.edu.tr

[2] Department of Mathematics, Karadeniz Technical University, 61080, Trabzon, Turkey, E-mail: kazancio@yahoo.com

tools for these theories. Therefore, Molodtsov proposed the soft set theory, as a new mathematical tool for dealing with uncertainties, which is free from the difficulties existing in those theories mentioned above. Furthermore, he demonstrated that soft set theory has potential applications in many directions, including function smoothness, Riemann integration, Perron integration, probability theory, measurement theory, game theory and operations research. In recent years, research on soft set theory, as well as its applications, especially the application in decision making, has received wide attention and achieved great progress [19, 20, 5, 18, 2, 27]. At the same time, work on theoretical aspects of soft sets is also very active. After Molodtsovs pioneer work [21], Maji et al. [19] gave further a detailed theoretical study on soft sets. On the basis of the analysis of several operations on soft sets introduced in [19], Ali et al. [2] proposed some new operations such as restricted intersection, restricted union, restricted difference and extended intersection of two soft sets. Xiao et al. [27] presented the concept of exclusive disjunctive soft sets, which is an extended concept for soft sets. Up to now, the algebraic structure of the soft sets has been investigated by some authors.(see [1, 12, 13, 14, 8, 15, 16]). The theory of fuzzy sets which was introduced by Zadeh [28] is applied to many mathematical branches. Goguen generalized them to the notion of L-fuzzy sets [10]. On the other hand, few years after the inception of the notion of fuzzy set, Rosenfeld started the pioneer work in the domain of fuzzification of the algebraic objects, with his work on fuzzy groups [24]. This work is a contribution to the theory founded on the ideas of those authors and their followers. Das [6] characterized fuzzy subgroups by their level subgroups. A new type of fuzzy subgroup (viz, $(\in, \in \vee q)$-fuzzy subgroup) was introduced in an earlier paper of Bhakat and Das [3] by using the combined notions of "belongingness" and "quasicoincidence" of fuzzy points and fuzzy sets. In fact, $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup. Since then many papers concerning various fuzzy algebraic structures have appeared in the literature. In [25], Yuan Bo and Wu Wangming introduced the concept of fuzzy ideal of a distributive lattice. Also, see [26, 17].

Inspired by the study of algebraic structures of soft sets, our aim in this paper is to initiate research on the connection between soft sets and lattice structures. In this paper, we apply the notion of soft sets to lattices. Some related notions, such as those of soft lattices(ideals, filters), are defined, and several basic properties are discussed by using the soft set theory. Furthermore, the notion soft homomorphism is introduced and its basic properties are studied. Finally, using the notions of a fuzzy ideal(filter), an $(\in, \in \vee q_k)$-fuzzy ideal(filter) and an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy ideal(filter) in lattices, we provide characterizations for $\in$-soft sets, q-soft sets and q_k soft sets to be soft ideals(filters).

2. **Preliminaries of lattice, fuzzy lattice and soft set**

In this section, we recall some definitions and results which will be used in what follows. Throughout the paper, we shall denote a lattice $(L, \vee, \wedge)$ by L, where the join and meet operations are denoted by $\vee$ and $\wedge$ in L, respectively.

Definition 2.1. Let L be a lattice and $L' \neq \emptyset$ is a subset of L such that for every pair of elements a, b in L' both $a \vee b$ and $a \wedge b$ are in L', then we say that L' with the same operations is a *sublattice* of L.

Definition 2.2. Let L be a lattice. An *ideal* I of L is a nonempty subset of L such that

(i) $a, b \in I \;\Rightarrow\; a \vee b \in I$,
(ii) $a \in I,\; b \in L$ and $b \leq a \;\Rightarrow\; b \in I$.

A *filter* of L is the dual concept of an ideal.

Throughout this paper $L = \langle L, \vee, \wedge \rangle$ denotes a lattice and $\langle [0,1], \sqcup, \sqcap \rangle$ is a complete lattice, where $[0,1]$ is the set of reals between 0 and 1 and $x \sqcup y = \max\{x, y\}$, $x \sqcap y = \min\{x, y\}$.

Let X be a non-empty set. A fuzzy subset μ of X is a function $\mu : X \longrightarrow [0,1]$. Let μ be any fuzzy subset of X. The set $\mu_t = \{x \in X |\; t \leq \mu(x)\}$, $t \in [0,1]$, is called a *level subset* of μ.

Definition 2.3. ([25]) Let L be a lattice and μ be a fuzzy subset of L. We say that μ is a *fuzzy sublattice* of L if for all $x, y \in L$, $\mu(x) \sqcap \mu(y) \leq \mu(x \vee y) \sqcap \mu(x \wedge y)$.

Definition 2.4. ([25])

(i) A fuzzy sublattice μ is called a *fuzzy ideal* if $\mu(x \vee y) = \mu(x) \sqcap \mu(y)$ for all $x, y \in L$.
(ii) A fuzzy sublattice μ is called a *fuzzy filter* if $\mu(x \wedge y) = \mu(x) \sqcap \mu(y)$ for all $x, y \in L$. It is easy to see that a fuzzy sublattice μ of L is a fuzzy ideal(filter) of L if and only if $x \leq y$ implies that $\mu(x) \geq \mu(y)(\mu(x) \leq \mu(y))$ for all $x, y \in L$.

A fuzzy set μ, on L which takes the value $t \in (0,1]$ at some $x \in L$ and takes the value 0 for all $y \in L$ expect x is called a *fuzzy point* and is denoted by x_t, where the point x is called its *support point* and t is called its *value*.

In what follows let k denote an arbitrary element of [0, 1) unless otherwise specified. For a fuzzy point x_t is said to be

(i) *belong* to (resp. be *k- quasi-coincident* with) a fuzzy set μ, written as $x_t \in \mu$ $(resp. x_t q_k \mu)$ if $\mu(x) \geq t (resp. \mu(x) + t + k > 1)$.
(ii) $x_t \in \mu$ or $x_t q_k \mu$, then we write $x_t \in \vee q_k \mu$.
(iii) The symbol $x_t \overline{\alpha} \mu$ if $x_t \overline{\alpha} \mu$ does not hold for $\alpha \in \{\in, q_k, \in \vee q_k\}$.

Definition 2.5. ([17]) A fuzzy subset μ of a lattice L is said to be an $(\in, \in \vee q_k)$-*fuzzy sublattice of* L if for all $t, r \in (0,1]$ and $x, y \in L$,

(i) $x_t,\ y_r \in \mu$ implies $(x \vee y)_{t \sqcap r} \in \vee q_k \mu$,
(ii) $x_t,\ y_r \in \mu$ implies $(x \wedge y)_{t \sqcap r} \in \vee q_k \mu$.

μ is called an $(\in, \in \vee q_k)$-*fuzzy ideal of* L if μ is an $(\in, \in \vee q_k)$-fuzzy sublattice of L and

(iii) $x_t \in \mu$ with $y \leq x$ implies $y_t \in \vee q_k \mu$,

μ is called an $(\in, \in \vee q_k)$-*fuzzy filter of* L if μ is an $(\in, \in \vee q_k)$-fuzzy sublattice of L and

(iv) $y_t \in \mu$ and $y \leq x$ implies $x_t \in \vee q_k \mu$.

Definition 2.6. ([17]) Let μ be a fuzzy subset of a lattice L. Then,
μ is an $(\in, \in \vee q_k)$-fuzzy sublattice of L if and only if,

(i) $\mu(x) \sqcap \mu(y) \sqcap \frac{1-k}{2} \leq \mu(x \vee y)$,
(ii) $\mu(x) \sqcap \mu(y) \sqcap \frac{1-k}{2} \leq \mu(x \wedge y)$,

μ is an $(\in, \in \vee q_k)$- fuzzy ideal of L if and only if,

(iii) μ is an $(\in, \in \vee q_k)$-fuzzy sublattice of L and $y \leq x$ implies $\mu(x) \sqcap \frac{1-k}{2} \leq \mu(y)$,

μ is an $(\in, \in \vee q_k)$- fuzzy filter of L if and only if,

(iv) μ is an $(\in, \in \vee q_k)$-fuzzy sublattice of L and $y \leq x$ implies $\mu(y) \sqcap \frac{1-k}{2} \leq \mu(x)$,

for all $x, y \in L$.

Definition 2.7. ([17]) A fuzzy subset μ of a lattice L is called an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-*fuzzy sublattice* of L if and only if

(i) $(x \vee y)_{t \sqcap r} \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q_k} \mu$ or $y_r \overline{\in} \vee \overline{q_k} \mu$,
(ii) $(x \wedge y)_{t \sqcap r} \overline{\in} \mu$ implies $x_t \overline{\in} \vee \overline{q_k} \mu$ or $y_r \overline{\in} \vee \overline{q_k} \mu$.

A fuzzy sublattice μ of a lattice L is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy ideal of L if and only if

(iii) $y_t \overline{\in} \mu$ and $y \leq x$ implies $x_t \overline{\in} \vee \overline{q_k} \mu$,

A fuzzy sublattice μ of a lattice L is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy filter of L if and only if

(iv) $x_t \overline{\in} \mu$ and $y \leq x$ implies $y_t \overline{\in} \vee \overline{q_k} \mu$,

for all $x, y \in L$.

Definition 2.8. ([17]) Let μ be a fuzzy subset of a lattice L. Then,
μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy sublattice of L if and only if,

(i) $\mu(x) \sqcap \mu(y) \leq \mu(x \vee y) \sqcup \frac{1-k}{2}$,
(ii) $\mu(x) \sqcap \mu(y) \leq \mu(x \wedge y) \sqcup \frac{1-k}{2}$,

μ is an $(\in, \in \vee q_k)$-*fuzzy ideal of* L if and only if,

(iii) μ is an $(\in, \in \vee q_k)$-fuzzy sublattice of L and $y \leq x$ implies $\mu(x) \leq \mu(y) \sqcup \frac{1-k}{2}$,

μ is an $(\in, \in \vee q_k)$-*fuzzy filter of* L if and only if,

(iv) μ is an $(\in, \in \vee q_k)$-fuzzy sublattice of L and $y \leq x$ implies $\mu(y) \leq \mu(x) \sqcup \frac{1-k}{2}$, for all $x, y \in L$.

Definition 2.9. ([17]) Let $\alpha, \beta \in [0,1]$ and $\alpha < \beta$. Let μ be a fuzzy subset of a lattice L. Then μ is called a *fuzzy sublattice with thresholds of* L, if for all $x, y \in L$,

(1) $\mu(x) \sqcap \mu(y) \sqcap \beta \leq \mu(x \vee y) \sqcup \alpha$,
(2) $\mu(x) \sqcap \mu(y) \sqcap \beta \leq \mu(x \wedge y) \sqcup \alpha$,

Moreover, μ is a *fuzzy ideal with thresholds of* L, if and only if μ satisfies the conditions (1) and (2) and satisfies the following condition:

(3) $y \leq x$ implies $\mu(x) \sqcap \beta \leq \mu(y) \sqcup \alpha$.

Finally, μ is a *fuzzy filter with thresholds of* L, if and only if μ satisfies the conditions (1) and (2) and satisfies the following condition:

(4) $y \leq x$ implies $\mu(y) \sqcap \beta \leq \mu(x) \sqcup \alpha$.

Now, we review some notions concerning soft sets. The definitions may be found in references[19, 21, 2, 27]. Let U be an initial universe set and E be a set of parameters. $P(U)$ denotes the power set of U and $A \subseteq E$.

Definition 2.10. ([21]) A pair (η, A) is called a soft set over U, where η is a set-valued function $\eta : A \to P(U)$ can be defined as $\eta(x) = \{y \in U \mid (x, y) \in R\}$ for all $x \in A$ and R will refer to an arbitrary binary relation between an element of A and an element of U, that is, R is a subset of $A \times U$. In fact, a soft set over U is a parameterized family of subsets of the universe U. For a soft set (η, A), the set $Supp(\eta, A) = \{x \in A \mid \eta(x) \neq \emptyset\}$ is called the *support* of the soft set (η, A) and the soft set (η, A) is called a *non-null* if $Supp(\eta, A) \neq \emptyset$ [18].

Definition 2.11. ([15]) Let $(\eta, A), (\gamma, B)$ be soft sets over a common universe U.

(i) (η, A) is said to be a soft subset of (γ, B), denoted $(\eta, A)\widetilde{\subseteq}(\gamma, B)$, if $A \subseteq B$ and $\eta(a) \subseteq \gamma(a)$ for all $a \in A$,
(ii) (η, A) and (γ, B) are said to be soft equal, denoted $(\eta, A) = (\gamma, B)$, if $(\eta, A)\widetilde{\subseteq}(\gamma, B)$ and $(\gamma, B)\widetilde{\subseteq}(\eta, A)$.

Definition 2.12. ([19, 21])

(i) The *restricted-intersection*(or bi-intersection) of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A) \cap (\gamma, B)$, where $C = A \cap B \neq \emptyset$, and $\delta(c) = \eta(c) \cap \gamma(c)$ for all $c \in C$.
(ii) The *extended intersection* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A)\widetilde{\cap}(\gamma, B)$, where $C = A \cup B$, and for all $c \in C$,

$$\delta(c) = \begin{cases} \eta(c) & \text{if} \quad c \in A \backslash B \\ \gamma(c) & \text{if} \quad c \in B \backslash A \\ \eta(c) \cap \gamma(c) & \text{if} \quad c \in A \cap B \end{cases}$$

Definition 2.13. [19]

(i) The *restricted-union* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A) \cup (\gamma, B)$, where $C = A \cap B \neq \emptyset$, and $\delta(c) = \eta(c) \cup \gamma(c)$ for all $c \in C$.

(ii) The *extended-union* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A)\widetilde{\cup}(\gamma, B)$, where $C = A \cup B$, and for all $c \in C$,

$$\delta(c) = \begin{cases} \eta(c) & \text{if } \quad c \in A \backslash B \\ \gamma(c) & \text{if } \quad c \in B \backslash A \\ \eta(c) \cup \gamma(c) & \text{if } \quad c \in A \cap B \end{cases}$$

Definition 2.14. ([21, 2])

(i) The $\wedge$-*intersection* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A)\widetilde{\wedge}(\gamma, B)$ where $C = A \times B$, and $\delta(a, b) = \eta(a) \cap \gamma(b)$ for all $(a, b) \in C$;

(ii) The $\vee$- *union* of two soft sets (η, A) and (γ, B) over a common universe U is defined as the soft set $(\delta, C) = (\eta, A)\widetilde{\vee}(\gamma, B)$, where $C = A \times B$, and $\delta(a, b) = \eta(a) \cup \gamma(b)$ for all $(a, b) \in C$.

Definition 2.15. ([2]) Let (η, A) and (γ, B) be two soft sets over U and V, respectively. The *cartesian product* of two soft sets (η, A) and (γ, B) is defined as the soft set $(\delta, A \times B) = (\eta, A)\widetilde{\times}(\gamma, B)$, where $\delta(x, y) = \eta(x) \times \gamma(y)$ for all $(x, y) \in A \times B$.

3. **Soft lattices(ideals, filters)**

Definition 3.1. Let (η, A) be a soft set over L. Then (η, A) is called a *soft lattice* (*ideal,filter*) over L if $\eta(x)$ is a *sublattice*(*ideal,filter*) of L for all $x \in A$; for our convenience, the empty set $\emptyset$ is regarded as a sublattice (ideal, filter) of L.

Example 3.1. Let $L = \{1, a, b, c, d, e, f, 0\}$ be a lattice with the following Hasse diagram: Let (η, A) be a soft set over L, where $A = \{1, 2, 3\}$ and $\eta : A \to P(L)$ be

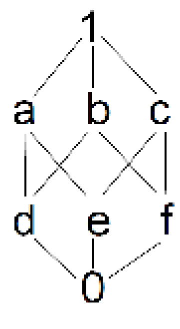

FIGURE 1. Hasse diagram of the lattice L

a set-valued function defined by

$$\eta(x) = \{y \in L \mid (x, y) \in R\}$$

for all $x \in A$, where $R = \{(1,1),(1,a),(1,c),(1,e),(2,0),(3,1),(3,d)\}$. Then $\eta(1) = \{a,c,e,1\}$, $\eta(2) = \{0\}$ and $\eta(3) = \{1,d\}$ are sublattices of L for all $x \in A$. Therefore (η, A) is a soft lattice over L.

Example 3.2. Consider the lattice $(\mathbb{N}, \leq)$, where the order $'' \leq ''$ defined by $''a \leq b \Leftrightarrow a \mid b''$. Let $A = \{0,1,2,3,4\}$ and define
$R = \{(0,1),(1,2),(1,15),(2,3),(2,6),(3,3),(3,30),(4,6),(4,8)\}$. Then $\eta(0) = \{1\}$, $\eta(1) = \{2,15\}$, $\eta(2) = \{3,6\}$, $\eta(3) = \{3,30\}$ and $\eta(4) = \{6,8\}$. Since $\eta(1)$ and $\eta(4)$ are not sublattices of L, (η, A) is not a soft lattice over L.

Theorem 3.1. Let (η, A) and (γ, B) be two soft lattices(ideals, filters) over L.

(i) The extended intersection $(\eta, A)\widetilde{\cap}(\gamma, B)$ is a soft lattice (ideal,filter) over L.
(ii) The restricted intersection $(\eta, A) \cap (\gamma, B)$ with $A \cap B \neq \emptyset$ is a soft lattice (ideal,filter) over L.
(iii) The $\wedge$-intersection $(\eta, A)\widetilde{\wedge}(\gamma, B)$ is a soft lattice (ideals, filters) over L.

Proof. It is obtained by Definition 2.12 (i-ii) and Definition 2.14(i). □

Corollary 3.1. Let (η, A) and (γ, A) be two soft lattices(ideals,filters) over L. Then extended intersection $(\eta, A)\widetilde{\cap}(\gamma, A)$ is a soft lattice (ideal,filter) over L.

Proof. Straightforward. □

Theorem 3.2. Let (η, A) and (γ, B) be two soft lattices(ideals, filters) over L.

(i) If $A \cap B = \emptyset$, then the extended union $(\eta, A)\widetilde{\cup}(\gamma, B)$ is a soft lattice (ideal,filter) over L.
(ii) If $\eta(x) \subseteq \gamma(x)$ or $\gamma(x) \subseteq \eta(x)$ for all $x \in A \cap B$, then restricted union $(\eta, A) \cup (\gamma, B)$ is a soft lattice (ideal,filter) over L.
(iii) If $\eta(a) \subseteq \gamma(b)$ or $\gamma(b) \subseteq \eta(a)$ for all $(a,b) \in A \times B$, then the $\vee$-union $(\eta, A)\widetilde{\vee}(\gamma, B)$ is a soft lattice(ideal, filter) over L.

Proof. (i)It follows from Definition 2.13 and 2.14. □

The following example shows that Theorem 3.2 (i) is not true in general if A and B are not disjoint.

Example 3.3. Let $L = \{1,a,b,c,0\}$ be a lattice with the following diagram: Let (η, A) be a soft set over L, where $A = \{1,2,3\}$ and define
$R_1 = \{(1,1),(1,a),(1,0),(2,a),(2,0),(3,a),(3,b)\}$. Then $\eta(1) = \{0,a,1\}$, $\eta(2) = \{0,a\}$ and $\eta(3) = \{a,b\}$ are sublattices of L for all $x \in A$. Therefore (η, A) is a soft lattice over L. Let (γ, B) be a soft set over L, where $B = \{1,3,4\}$ and define $R_2 = \{(1,a),(1,b),(1,c),(1,0),(3,c),(4,a),(4,c)\}$. Then $\gamma(1) = \{a,b,c,0\}$, $\gamma(3) = \{c\}$ and $\gamma(4) = \{a,c\}$ are sublattices of L for all $x \in B$. Hence (γ, B) is a soft lattice over L. However, $(\delta, C) = (\eta, A) \cup (\gamma, B)$ is not a soft lattice over L because $\delta(3) = \eta(3) \cup \gamma(3) = \{a,b,c\}$ is not a sublattice of L.

Theorem 3.3. Let (η, A) and (γ, B) be two soft lattices(ideals, filters) over L_1, L_2, respectively. Then the cartesian product $(\eta, A)\widetilde{\times}(\gamma, B)$ is a soft lattice(ideal, filter) over $L_1 \times L_2$.

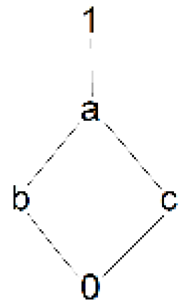

FIGURE 2. Hasse diagram of the lattice L

Proof. It is obvious from Definition 2.15 and Definition 3.1.

Definition 3.2. Let (η, A) be a soft lattice(ideal, filter) over L.

(i) (η, A) is called a *trivial* soft lattice(ideal, filter) over L if $\eta(x) = \{0\}$ for all $x \in A$.

(ii) (η, A) is called a *whole* soft lattice(ideal, filter) over L if $\eta(x) = L$ for all $x \in A$.

Example 3.4. (i) Consider the lattice $L = \{1, a, b, c, 0\}$ with the following diagram: Let (η, A) be a soft set over L, where $A = \{a, b\}$ and $\eta : A \to P(L)$ be a set-valued

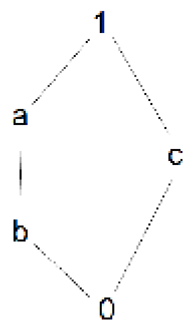

FIGURE 3. Hasse diagram the lattice L

function defined by

$$\eta(x) = \{y \in L \mid x \wedge y = 0\}$$

for all $x \in A$. Then $\eta(a) = \eta(b) = \{0\}$ and whence (η, A) is a trivial soft lattice.
(ii)Let $L = \{1, a, b, c, 0\}$ be a lattice with the following diagram: Let (η, A) be a soft set over L, where $A = \{a, 0\}$ and $\eta : A \to P(L)$ be a set-valued function defined by

$$\eta(x) = \{y \in L \mid x \wedge y \in \{0, a\}\}$$

for all $x \in A$. Then $\eta(a) = \eta(0) = L$ and whence (η, A) is a whole soft lattice.

Proposition 3.1. Let (η, A) and (γ, B) be two soft lattices(ideals, filters) over L.

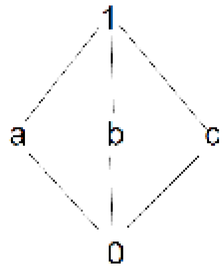

FIGURE 4. Hasse diagram the lattice L

(i) If (η, A) and (γ, B) are trivial soft lattices(ideals, filters) over L, then the restricted intersection $(\eta, A) \cap (\gamma, B)$ is a trivial soft lattice(ideal, filter) over L.
(ii) If (η, A) and (γ, B) are whole soft lattices(ideals, filters) over L, then the restricted intersection $(\eta, A) \cap (\gamma, B)$ is a whole soft lattices(ideals, filters) over L.
(iii) If (η, A) is a trivial trivial soft lattice(ideal, filter) over L and (γ, B) is a whole soft lattice(ideal, filter) over L, then the restricted intersection $(\eta, A) \cap (\gamma, B)$ is a trivial soft lattice(ideal, filter) over L.

*Proof.*It is straightforward. □

Proposition 3.2. Let (η, A) and (γ, B) be soft lattices(ideals, filters) over L_1 and L_2, respectively.

(i) If (η, A) and (γ, B) are trivial soft lattice(ideal,filter) over L_1 and L_2, respectively. Then $(\eta, A)\widetilde{\times}(\gamma, B)$ are trivial soft lattice(ideal, filter) over $L_1 \times L_2$.
(ii) If (η, A) and (γ, B) are whole soft lattice(ideal,filter) over L_1 and L_2, respectively. Then $(\eta, A)\widetilde{\times}(\gamma, B)$ are whole soft lattice(ideal, filter) over $L_1 \times L_2$.

*Proof.*It is straightforward. □

Definition 3.3. [16] Let (η, A) and (γ, B) be two soft sets over L_1 and L_2 respectively, $f : L_1 \to L_2$, $g : A \to B$ be two functions. Then the pair (f, g) is called a soft function from (η, A) to (γ, B), denoted by $(f, g) : (\eta, A) \to (\gamma, B)$ if $f(\eta(x)) = \gamma(g(x))$ for all $x \in A$. If f, g are injective(resp. surjective, bijective), then (f, g) are called injective(resp. surjective, bijective).

Definition 3.4. [16] Let (η, A) and (γ, B) two soft sets over L_1 and L_2 respectively, (f, g) a soft function from (η, A) to (γ, B).

(i) The *image* of (η, A) under the soft function (f, g), denoted by $(f, g)(\eta, A) = (f(\eta), B)$, is the soft set over L_2 defined by

$$f(\eta)(x) = \begin{cases} \cup_{g(x)=y} f(\eta(x)) & \text{if } \; y \in Img \\ \emptyset & \text{if } \; otherwise \end{cases}$$

for all $y \in B$.

(ii) The *pre-image* of (γ, B) under the soft function (f, g), denoted by $(f, g)^{-1}(\gamma, B) = (f^{-1}(\gamma), A)$, is the soft set over L_1 defined by $f^{-1}(\gamma)(x) = f^{-1}(\gamma(g(x)))$ for all $x \in A$.

It is clear that $(f, g)(\eta, A)$ is a soft subset of (γ, B) and (η, A) is a soft subset $(f, g)^{-1}(\gamma, B)$.

Definition 3.5. [16] Let (η, A) and (γ, B) be two soft lattice over L_1 and L_2 respectively and (f, g) a soft function from (η, A) to (γ, B). If f is a lattice homomorphism from L_1 to L_2, then (f, g) is called a *soft lattice homomorphism*, and we say that (η, A) is soft homomorphic to (γ, B) under the soft homomorphism (f, g). In this definition, if f is an isomorphism from L_1 to L_2 and g is a surjective mapping from A to B, then we say that (f, g) is a soft lattice isomorphism and that (η, A) is soft isomorphic to (γ, B) under the soft homomorphism (f, g), which is denoted by $(\eta, A) \simeq (\gamma, B)$.

Proposition 3.3. The relation $\simeq$ is an equivalence relation on soft lattices.

Proof. Straightforward. □

Theorem 3.4. Let (η, A) and (γ, B) be two soft lattice over L_1 and L_2 respectively and (f, g) a soft lattice homomorphism from (η, A) to (γ, B).

(i) If g is injective, f is surjective and (η, A) is a soft lattice(ideal, filter) over L_1, then $(f, g)(\eta, A)$ is a soft lattice(ideal, filter) over L_2.
(ii) If (γ, B) is a soft lattice(ideal, filter) over L_2, then $(f, g)^{-1}(\gamma, B)$ is a soft lattice(ideal, filter) over L_1.

*Proof.*Straightforward. □

Theorem 3.5. Let (η, A) and (γ, B) be two soft lattice(ideal, filter) over L_1 and L_2 respectively and (f, g) a soft lattice homomorphism from (η, A) to (γ, B).

(i) If $\eta(x) = ker(f)$ for all $x \in A$, then $(f, g)(\eta, A)$ is the trivial soft lattice(ideal, filter) over L_2.
(ii) If f is onto and (η, A) is whole, then $(f, g)(\eta, A)$ is the whole soft lattice(ideal, filter) over L_2.
(iii) If $\gamma(y) = f(L_1)$ for all $y \in B$, then $(f, g)^{-1}(\gamma, B)$ is the whole soft lattice(ideal, filter) over L_1.
(iv) If f is injective and (γ, B) is trivial, then $(f, g)^{-1}(\gamma, B)$ is the trivial soft lattice(ideal, filter) over L_1.

Proof. Straightforward. □

Proposition 3.4. Let L_1, L_2 and L_3 be lattices and (η, A), (γ, B) and (δ, C) soft lattices(ideals,filters) over L_1, L_2, and L_3 respectively. Let the soft function (f, g) from (η, A) to (γ, B) be a soft homomorphism from L_1 to L_2 and the soft function (f', g') from (γ, B) to (δ, C) be a soft homomorphism from L_2 to L_3. Then the the soft function $(f' \circ f, g' \circ g)$ from (η, A) to (δ, C) is a soft homomorphism from L_1 to L_3.

Proof. Straightforward. □

Theorem 3.6. Let L_1 and L_2 be lattices and (η, A), (γ, B) soft sets over L_1 and L_2 respectively. If (η, A) is a soft lattice(ideal, filter) over L_1 and $(\eta, A) \simeq (\gamma, B)$, then (γ, B) is a soft soft lattice(ideal,filter) over L_2.

Proof. The proof is obtained by Definition 3.4 and Proposition 3.3. □

Theorem 3.7. Let $f : L_1 \to L_2$ be an onto homomorphism of lattices and (η, A) and (γ, B) be two soft lattices(ideals,filters) over L_1 and L_2 respectively.

(i) The soft function (f, I_A) from (η, A) to (δ, A) is a soft homomorphism from L_1 to L_2 where $I_A : A \to A$ identity mapping and the set-valued function $\delta : A \to P(L_2)$ defined by $\delta(x) = f(\eta(x))$ for all $x \in A$.

(ii) If $f : L_1 \to L_2$ is an isomorphism, then the soft function (f^{-1}, I_B) from (γ, B) to (ν, B) is a soft homomorphism from L_2 to L_1 where $I_B : B \to B$ identity mapping and the set-valued function $\nu : B \to P(X)$ defined by $\nu(x) = f^{-1}(\gamma(x))$ for all $x \in B$.

Proof. The proof is obtained by Theorem 3.4. □

4. **Soft ideal(filter) related to fuzzy point**

In this section, we present some basic results which provide a connection between fuzzy ideals(filters) and soft ideals(filters) over a lattice. As we are interested in soft sets in fuzzy context so we confine ourself to those particular soft sets which originate from fuzzy subsets of lattices.

Given a fuzzy set μ in any lattice L and $A \subseteq [0, 1]$, consider three set-valued function over L. $\eta : A \to P(L)$, $\eta_q : A \to P(L)$ and $\eta_{q_k} : A \to P(L)$ defined by

$$\eta(t) = \{x \in S \mid x_t \in \mu\} \qquad \eta_q(t) = \{x \in S \mid x_t q \mu\} \quad \textit{and} \qquad \eta_{q_k}(t) = \{x \in S \mid x_t q_k \mu\}$$

for all $t \in A$, respectively. Then (η, A), (η_q, A) and (η_{q_k}, A) are soft sets over L, which are called an $\in$- soft set, a q-soft set and a q_k-soft set over L, respectively.

Theorem 4.1. Let μ be a fuzzy set of L and (η, A) be an $\in$-soft set over L with $A = (0, 1]$. Then (η, A) is a soft ideal (filter) over L if and only if μ is a fuzzy ideal(filter) of L.

Proof. Suppose that (η, A) be a soft ideal over L. If there exist $x, y \in L$ such that $\mu(x) \sqcap \mu(y) > \mu(x \vee y)$ and $\mu(x) \sqcap \mu(y) > \mu(x \wedge y)$, then we can choose, $t, s \in A$ such that $\mu(x) \sqcap \mu(y) > t \geq \mu(x \vee y)$ and $\mu(x) \sqcap \mu(y) > s \geq \mu(x \wedge y)$ Then $x_t, y_t \in \mu$ and $x_s, y_s \in \mu$, but $(x \vee y)_t, (x \wedge y)_s \notin \mu$, that is, $x_t, y_t \in \eta(t)$ and $x_s, y_s \in \eta(s)$, but $(x \vee y)_t \notin \eta(t)$ and $(x \wedge y)_s \notin \eta(s)$. This is a contradiction. Therefore $\mu(x) \sqcap \mu(y) \leq \mu(x \vee y)$ and $\mu(x) \sqcap \mu(y) \leq \mu(x \wedge y)$ for all $x, y \in L$, that is, μ is a fuzzy sublattice of L. If there exist $x, y \in L$ with $y \leq x$ such that $\mu(x) \geq \mu(y)$, then we can choose $t \in A$ such that $\mu(x) \geq t > \mu(y)$. Then $x_t \in \mu$, but $y_t \notin \mu$, that is, $x \in \eta(t)$ and $y \leq x$ but $y \notin \eta(t)$. This is a contradiction. Therefore $\mu(x) \leq \mu(y)$ with $y \leq x$ for all $x, y \in L$. Hence μ is a fuzzy ideal of L.

Conversely, suppose that μ is a fuzzy ideal of L. Let $t \in A$ and $x, y \in \eta(t)$. Then $x_t \in \mu$ and $y_t \in \mu$, that is, $\mu(x) \geq t$ and $\mu(y) \geq t$. Since μ is a fuzzy sublattice of L, it follows that $\mu(x \vee y) \geq \mu(x) \sqcap \mu(y) \geq t \sqcap t = t$ and $\mu(x \wedge y) \geq \mu(x) \sqcap \mu(y) \geq t \sqcap t = t$ so $(x \vee y)_t, (x \wedge y)_t \in \mu$. Hence $x \vee y, x \wedge y \in \eta(t)$. Thus $\eta(t)$ is a sublattice of L, i.e., (η, A) is a soft lattice over L. Now, let $x \in \eta(t)$, $t \in A$ and $y \in L$ with $y \leq x$. Then $x_t \in \mu$, that is, $\mu(x) \geq t$. Since μ is a fuzzy ideal of L, we obtain $\mu(y) \geq \mu(x) \geq t$ and so $y_t \in \mu$. Hence $y \in \eta(t)$. Thus $\eta(t)$ is an ideal of L for all $t \in A$,i.e., (η, A) is a soft ideal over L.

The proof of the filter case is similar. □

Theorem 4.2. Let μ be a fuzzy set of L and (η_q, A) an q-soft set over L with $A = (0, 1]$. Then μ is a fuzzy ideal(filter) of L if and only if (η_q, A) is a soft ideal(filter) of L.

Proof. The proof is similar to the proof of Theorem 4.1. □

Theorem 4.3. Let μ be a fuzzy set of L and (η_{q_k}, A) an q_k-soft set over L with $A = (0, 1]$. Then μ is a fuzzy ideal(filter) of L if and only if (η_{q_k}, A) is a soft ideal(filter) of L for all $t \in A$.

Proof. The proof is similar to the proof of Theorem 4.1. □

Theorem 4.4. Let μ be a fuzzy set of L and (η, A) an $\in$-soft set over L with $A = (0, \frac{1-k}{2}]$. Then the following conditions are equivalent:

(i) μ is an $(\in, \in \vee q_k)$ fuzzy ideal(filter) of L;

(ii) (η, A) is a soft ideal(filter) over L.

Proof. Let μ is an $(\in, \in \vee q_k)$ fuzzy ideal of L. For any $t \in A$ and $x, y \in \eta(t)$, we have $x_t \in \mu$ and $y_t \in \mu$, that is, $\mu(x) \geq t$, $\mu(y) \geq t$. By Theorem 2.7(i), we obtain $\mu(x \vee y) \geq \mu(x) \sqcap \mu(y) \sqcap \frac{1-k}{2} = t$ and $\mu(x \wedge y) \geq \mu(x) \sqcap \mu(y) \sqcap \frac{1-k}{2} = t$ so $\mu(x \vee y) \geq t$ and $\mu(x \wedge y) \geq t$ that is, $(x \vee y)_t \in \mu$ and $(x \wedge y)_t \in \mu$, i.e., $x \vee y, x \wedge y \in \eta(t)$. Hence $\eta(t)$ is a sublattice of L for all $t \in A$. For any $t \in A$, $x \in \eta(t)$ and $y \in L$ with $y \leq x$, we have $x_t \in \mu$, that is, $\mu(x) \geq t$. Since μ is an $(\in, \in \vee q_k)$ fuzzy ideal of L, we get $\mu(y) \geq \mu(x) \sqcap \frac{1-k}{2} = t$, which implies that $y \in \eta(t)$. Therefore $\eta(t)$ is an ideal of L for all $t \in A$ i.e., (η, A) is a soft ideal over L.

Conversely, assume that (η, A) is a soft ideal over L. If there exist $a, b \in L$ such that $\mu(a \vee b) < \mu(a) \sqcap \mu(b) \sqcap \frac{1-k}{2}$, then we can choose $t \in A$ such that $\mu(a \vee b) < t \leq \mu(a) \sqcap \mu(b) \sqcap \frac{1-k}{2}$, which implies $a_t \in \mu$ and $b_t \in \mu$, but $(a \vee b)_t \notin \mu$, that is, $a_t \in \eta(t)$ and $b_t \in \eta(t)$, but $(a \vee b)_t \notin \eta(t)$. This is a contradiction. Therefore $\mu(x \vee y) \geq \mu(x) \sqcap \mu(y) \sqcap \frac{1-k}{2}$ for all $x, y \in L$. Similarly it can be shown that $\mu(x \wedge y) \geq \mu(x) \sqcap \mu(y) \sqcap \frac{1-k}{2}$ Hence μ is an $(\in, \in \vee q_k)$ fuzzy sublattice of L. If there exist $a, b \in L$ with $b \leq a$, such that $\mu(b) < \mu(a) \sqcap \frac{1-k}{2}$, then we can choose $t \in A$ such that $\mu(b) \leq t < \mu(a) \sqcap \frac{1-k}{2}$. Then $a_t \in \mu$, but $b_t \notin \mu$ which is a contradiction. Therefore $\mu(y) \geq \mu(x) \sqcap \frac{1-k}{2}$ with $y \leq x$ for all $x, y \in L$. Hence μ is an $(\in, \in \vee q_k)$ fuzzy ideal of L.

The proof of the filter case is similar. □

Theorem 4.5. Let μ be a fuzzy set of L and (η, A) an $\in$-soft set over L with $A = (\frac{1-k}{2}, 1]$. Then the following conditions are equivalent:

(i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$ fuzzy ideal(filter) of L;

(ii) (η, A) is a soft ideal(filter) over L.

Proof. Let μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$ fuzzy ideal of L. For any $t \in A$ and $x, y \in \eta(t)$, we have $x_t \in \mu$,$y_t \in \mu$, that is, $\mu(x) \geq t$, $\mu(y) \geq t$. By Theorem 2.9, we obtain $t \leq \mu(x) \sqcap \mu(y) \leq \mu(x \vee y) \sqcup \frac{1-k}{2} = \mu(x \vee y)$ and $t \leq \mu(x) \sqcap \mu(y) \leq \mu(x \wedge y) \sqcup \frac{1-k}{2} = \mu(x \wedge y)$. So $\mu(x \vee y) \geq t$ and $\mu(x \wedge y) \geq t$ that is, $(x \vee y)_t \in \mu$, $(x \wedge y)_t \in \mu$, i.e., $x \vee y$, $x \wedge y \in \eta(t)$. Hence $\eta(t)$ is a sublattice of L for all $t \in A$. For any $t \in A$, $x \in \eta(t)$ and $y \in L$ with $y \leq x$, we have $x_t \in \mu$, that is, $\mu(x) \geq t$. By Theorem 2.9, we get $\mu(y) \sqcup \frac{1-k}{2} \geq \mu(x) \geq t$, which implies that $y \in \eta(t)$. Therefore $\eta(t)$ is an ideal of L for all $t \in A$ i.e., (η, A) is a soft ideal over L.
Conversely, assume that (η, A) is a soft ideal over L. If there exist $a, b \in L$ such that $\mu(a \vee b) \sqcup \frac{1-k}{2} < \mu(a) \sqcap \mu(b)$, then we can choose $t \in A$ such that $\mu(a \vee b) \sqcup \frac{1-k}{2} < t \leq \mu(a) \sqcap \mu(b)$, which implies $a_t \mu$ and $b_t \mu$, but $(a \vee b)_t \notin \mu$, that is, $a_t \in \eta(t)$ and $b_t \in \eta(t)$, but $(a \vee b)_t \notin \eta(t)$. This is a contradiction. Therefore $\mu(x \vee y) \sqcup \frac{1-k}{2} \geq \mu(x) \wedge \mu(y)$ for all $x, y \in L$. Similarly it can be shown that $\mu(x \wedge y) \sqcup \frac{1-k}{2} \geq \mu(x) \wedge \mu(y)$ Hence μ is an $(\overline{\in}, \overline{\in} \vee \overline{q})$ fuzzy sublattice of L. If there exist $a, b \in L$ with $b \leq a$, such that $\mu(b) \sqcup \frac{1-k}{2} < \mu(a)$, then we can choose $t \in A$ such that $\mu(b) \sqcup \frac{1-k}{2} \leq t < \mu(a)$. Then $a_t \in \mu$, but $b_t \notin \mu$ which is a contradiction. Therefore $\mu(y) \sqcup \geq \mu(x)$ with $y \leq x$ for all $x, y \in L$. Hence μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$ fuzzy ideal ideal of L.

The proof of the filter case is similar. □

Theorem 4.6. Let μ be a fuzzy set of L and (η_q, A) an q-soft set over L with $A = (\frac{1-k}{2}, 1]$. Then the following conditions are equivalent:

(i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$ fuzzy ideal(filter) of L;

(ii) (η_q, A) is a soft ideal(filter) over L.

Proof. The proof is similar to the proof of Theorem 4.5. □

Theorem 4.7. Let μ be a fuzzy set of L and (η_{q_k}, A) an q_k-soft set over L with $A = (\frac{1-k}{2}, 1]$. Then the following conditions are equivalent:

(i) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$ fuzzy ideal(filter) of L;

(ii) (η_{q_k}, A) is a soft ideal(filter) over L.

Proof. The proof is similar to the proof of Theorem 4.5. □

Theorem 4.8. Given $\alpha, \beta \in (0, 1]$ and $\alpha < \beta$, Let μ be a fuzzy set of L and (η, A) an $\in$-soft set over L with $A = (\alpha, \beta]$. Then the following conditions are equivalent:

(i) μ is a fuzzy ideal(filter) with thresholds $(\alpha, \beta]$ of L;

(ii) (η, A) is a soft ideal(filter) over L.

Proof. Let μ be a fuzzy ideal with thresholds $(\alpha, \beta]$ of L. For any $t \in A$ and $x, y \in \eta(t)$. Then $\mu(x) \geq t$, $\mu(y) \geq t$. By Definition 2.13, we have
$\alpha < t = t \sqcap \beta \leq \mu(x) \sqcap \mu(y) \sqcap \beta \leq \mu(x \vee y) \sqcup \alpha$, and $\alpha < t = t \sqcap \beta \leq \mu(x) \sqcap \mu(y) \sqcap \beta \leq$

$\mu(x \wedge y) \sqcup \alpha$, which implies that $(x \vee y)_t \in \mu$ and $(x \wedge y)_t \in \mu$ so $x \vee y \in \eta(t)$ and $x \wedge y \in \eta(t)$. For any $t \in A$, $x \in \eta(t)$ and $y \in L$ with $y \leq x$, we have $x_t \in \mu$, that is, $\mu(x) \geq t$. Since μ is a fuzzy ideal with thresholds $(\alpha, \beta]$ of L, we get $\mu(y) = \mu(y) \sqcup \alpha \geq \mu(x) \sqcap \beta \geq t \sqcup \alpha = t$. which implies that $y \in \eta(t)$. Therefore $\eta(t)$ is an ideal of L for all $t \in A$ i.e., (η, A) is a soft ideal over L.
Let μ be a fuzzy subset of L such that (η, A) is a soft ideal over L. If there exist $a, b \in L$ such that $\mu(a \vee b) \sqcup \alpha < \mu(a) \sqcap \mu(b) \sqcap \beta$ and $\mu(a \wedge b) \sqcup \alpha < \mu(a) \sqcap \mu(b) \sqcap \beta$, then we can choose $t, s \in A$ such that $\mu(a \vee b) \sqcup \alpha < t \leq \mu(a) \sqcap \mu(b) \sqcap \beta$ and $\mu(a \vee b) \sqcup \alpha < s \leq \mu(a) \sqcap \mu(b) \sqcap \beta$ which implies $a_t, a_s \in \mu$ and $b_t, b_s \in \mu$, but $(a \vee b)_t \notin \mu$ and $(a \wedge b)_s \notin \mu$ that is, $a_t, b_t \in \eta(t)$ and $a_s, b_s \in \eta(t)$, but $(a \vee b)_t \notin \eta(t)$ and $(a \wedge b)_s \notin \eta(t)$. This is a contradiction. Therefore $\mu(x \vee y) \sqcup \alpha \geq \mu(x) \sqcap \mu(y) \sqcap \beta$ and $\mu(x \wedge y) \sqcup \alpha \geq \mu(x) \sqcap \mu(y) \sqcap \beta$ for all $x, y \in L$, that is, μ is a fuzzy sublattice with thresholds $(\alpha, \beta]$ of L. If there exist $a, b \in L$ with $b \leq a$, such that $\mu(b) \sqcup \alpha < \mu(a) \sqcap \beta$, then we can choose $t \in A$ such that $\mu(y) \sqcup \alpha < t \leq \mu(a) \sqcap \beta$. Then $x_t \in \mu$ but $(y)_t \notin \mu$, that is, $x_t \in \eta(t)$ but $(y)_t \notin \eta(t)$. This is a contradiction. Therefore $\mu(y) \sqcup \alpha \geq \mu(x) \sqcap \beta$ for all $x, y \in L$. Therefore, μ is a fuzzy ideal with thresholds $(\alpha, \beta]$ of L.

The proof of the filter case is similar. □

Corollary 4.1. Let μ be a fuzzy subset of a lattice L and (η, A) an $\in$-soft set over L with $A = (0, 1]$. Then the following conditions are equivalent:

(i) μ is a fuzzy ideal of L if and only if the $\in$-soft set (η, A) is a soft ideal(filter) over L for all $t \in A$.

(ii) μ is an $(\in, \in \vee q_k)$-fuzzy ideal of L if and only if the $\in$-soft set (η, A) is a soft ideal(filter) over L for all $t \in (0, \frac{1-k}{2}]$.

(iii) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy ideal(filter) of L if and only if the$\in$-soft set (η, A) is a soft ideal(filter) over L for all $t \in (\frac{1-k}{2}, 1]$.

Corollary 4.2. Let μ be a fuzzy subset of a lattice L and (η_q, A) (resp.(η_{q_k}, A)) an $q(resp.q_k)$-soft set over L with $A = (0, 1]$. Then the following conditions are equivalent:

(i) μ is a fuzzy ideal of L if and only if the $q(resp.q_k)$-soft set $(\eta_q, A)(resp.(\eta_{q_k}, A))$ is a soft ideal(filter) over L for all $t \in A$.

(ii) μ is an $(\in, \in \vee q_k)$-fuzzy ideal(filter) of L if and only if the $q(resp.q_k)$-soft set $(\eta_q, A)(resp.(\eta_{q_k}, A))$ is a soft ideal(filter) over L for all $t \in (0, \frac{1-k}{2}]$.

(iii) μ is an $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy ideal of L if and only if the $q(resp.q_k)$-soft set $(\eta_q, A)(resp.(\eta_{q_k}, A))$ is a soft ideal(filter) over L for all $t \in (\frac{1-k}{2}, 1]$.

Remark 4.1. The following example shows that there exist a set of parameters A and a fuzzy set μ in L such that

(i) μ is neither a fuzzy ideal(filter) and an $(\in, \in \vee q_k)$-fuzzy ideal(filter) nor $(\overline{\in}, \overline{\in} \vee \overline{q_k})$-fuzzy ideal(filter) of L,

(ii) An $\in$-soft set (η, A) over L is a soft ideal(filter) over L,

(iii) An q-soft set (η_q, A) over L is a soft ideal(filter) over L.

(iv) An q_k-soft set (η_{q_k}, A) over L is a soft ideal(filter) over L.

Example 4.1. Let $L = \{0, 1, 2, 3, 4\}$ be a lattice with the following diagram: Let μ

FIGURE 5. Hasse diagram of the lattice L

be a fuzzy subset in L defined by $\mu(0) = 0.9$, $\mu(1) = 1$, $\mu(2) = 0.9$, $\mu(3) = 0.4$ and $\mu(4) = 0.5$. Let (η, A) be an $\in$-soft set over L where $A = (0.2, 0.8]$. Then

$$\eta(t) = \begin{cases} L & \text{if} \quad 0.2 < t \leq 0.4, \\ \{0,1,2\} & \text{if} \quad 0.4 < t \leq 0.8. \end{cases}$$

which are ideals of L. Hence (η, A) is a soft ideal over L. But μ

- is not a fuzzy ideal of L,
- is not an $(\in, \in \vee q_{0.1}$-fuzzy ideal of L.
- is not an $(\overline{\in}, \overline{\in} \vee \overline{q_{0.1}})$ fuzzy ideal of L.

Let (η_q, B) be an q-soft set over L where $B = (0.4, 0.9]$. Then

$$\eta_q(t) = \begin{cases} \{0,1,2\} & \text{if} \quad 0.4 < t \leq 0.6, \\ L & \text{if} \quad 0.6 < t \leq 0.9. \end{cases}$$

which are ideals of L. Hence (η_q, B) is a soft ideal over L. But μ

- is not a fuzzy ideal of L,
- is not an $(\in, \in \vee q_{0.1})$-fuzzy ideal of L.
- is not an $(\overline{\in}, \overline{\in} \vee \overline{q_{0.1}})$ fuzzy ideal of L.

Let $(\eta_{q_{0.1}}, C)$ be an $q_{0.1}$-soft set over L where $C = (0.1, 0.7]$. Then

$$\eta_{q_{0.1}}(t) = \begin{cases} \{0,1,2\} & \text{if} \quad 0.1 < t \leq 0.5, \\ L & \text{if} \quad 0.5 < t \leq 0.7. \end{cases}$$

which are ideals of L. Hence $(\eta_{q_{0.1}}, C)$ is a soft ideal over L. But μ

- is not a fuzzy ideal of L,
- is not an $(\in, \in \vee q_{0.1})$-fuzzy ideal of L.
- is not an $(\overline{\in}, \overline{\in} \vee \overline{q_{0.1}})$ fuzzy ideal of L.

REFERENCES

[1] *H. Aktaş, N. Çağman*, Soft sets and soft groups, Information Sciences, **177**(2007), No.13 , 2726-2735.

[2] *M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabira*, On some new operations in soft set theory, Computers and Mathematics with Applications, **57**(2009), No. 9, 1547-1553.

[3] *S.K. Bhakat, P. Das*, $(\in, \in \vee q)$-fuzzy subgroups, Fuzzy Sets Syst., **80**(1996), 359-368.

[4] *G. Birkhoff*, Lattice theory, American Mathematical Society, Providence, RI, 1984.

[5] *D. Chen, E.C.C. Tsang, D.S. Yeung, X. Wang*, The parameterization reduction of soft sets and its applications, Computers and Mathematics with Applications, **49**(2005), 757-763.

[6] *P. Das*, Fuzzy groups and level subgroups, J. Math. Anal. Appl., **85**(1981), 264-269.

[7] *B.A. Davey, H.A. Priestley*, Introduction to Lattices and Order, Cambridge University Press, Cambridge, 1990.

[8] *F. Feng, Y.B. Jun, X. Zhao*, Soft semirings, Computers and Mathematics with Applications **56**(2008), No. 10, 2621-2628.

[9] *W.L. Gau, D.J. Buehrer*, Vague sets, IEEE Transactions on Systems, Man and Cybernetics **23**(1993), No. 2, 610-614.

[10] *J.A. Goguen*, L-fuzzy sets, J. Math. Anal. Appl. **18**(1967), 145-174.

[11] *M.B. Gorzalzany*, A method of inference in approximate reasoning based on interval-valued fuzzy sets, Fuzzy Sets and Systems **21**(1987), 1-17.

[12] *Y.B. Jun*, Soft BCK/BCI-algebras, Computers and Mathematics with Applications **56** (2008), 1408-1413.

[13] *Y.B. Jun, C.H. Park*, Applications of soft sets in ideal theory of BCK/BCI-algebras, Information Sciences **178** (2008), 2466-2475.

[14] *Y.B. Jun, K.J. Lee, C.H. Park*, Soft set theory applied to ideals in d-algebras, Computers and Mathematics with Applications 57 **(3)** (2009), 367-378.

[15] *Y.B. Jun, K.J. Lee, A. Khan*, Soft ordered semigroups, Mathematical Logic Quarterly **56** (2010), No. 1, 42-50.

[16] *O. Kazancı, Ş. Yılmaz, S. Yamak*, Soft sets and Soft BCH-algebras, Hacettepe J. Math. Stst. **39** (2010), No. 2, 205-217.

[17] *O. Kazancı, B. Davvaz*, More on fuzzy lattices,(accepted).

[18] *Z. Kong, L. Gao, L. Wang, S. Li*, The normal parameter reduction of soft sets and its algorithm, Computers and Mathematics with Applications **56** (2008), No. 12, 3029-3037.

[19] *P.K. Maji, R. Biswas, A.R. Roy*, Soft set theory, Computers and Mathematics with Applications **45**, (2003), 555-562.

[20] *P.K. Maji, A.R. Roy, R. Biswas*, An application of soft sets in a decision making problem, Computers and Mathematics with Applications **44**, (2002), 1077-1083.

[21] *D. Molodtsov*, Soft set theory - first results, Computers and Mathematics with Applications **37**, (1999), 19-31.

[22] *E.K.R. Nagarajan, G.Meenambigai*, An application of soft sets to lattices, Kragujevac Journal of Mathematics, **35**, (2011), No. 1, 75-87.

[23] *Z. Pawlak*, Rough sets, International Journal of Information and Computer Sciences **11**, (1982), 341-356.

[24] *A. Rosenfeld*, Fuzzy groups, J. Math. Anal. Appl. **35**, (1971), 512-517.

[25] *U.M. Swamy, D. Viswanadha Raju*, Fuzzy ideals and congruences of lattices, Fuzzy Sets Syst. **95**, (1998), 249-253.

[26] *A. Tepavacevic, G. Trajkovski*, L-fuzzy lattices: an introduction, Fuzzy Sets Syst. **123**, (2001), 209-216.

[27] *Z. Xiao, K. Gong, S.S. Xia, Y. Zou*, Exclusive disjunctive soft sets, Computers and Mathematics with Applications **59**, (2010), 2128-2137.

[28] *L.A. Zadeh*, Fuzzy sets, Information Control **8**, (1965), 338-353.