FAULT DIAGNOSIS IN ANALOG CIRCUITS BASED ON PARAMETRIC DEVIATION OF COMPONENTS COMPUTATION

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Lucrarea prezintă o metodă de diagnosticare a defectelor circuitelor electrice analogice rezistive. Se presupun cunoscute topologia circuitului şi valorile nominale ale componentelor. Pornind de la valorile măsurate ale potenţialelor nodurilor circuitului supus testării şi utilizând metoda nodală de analiză a circuitelor şi teoria grafurilor, se determină abaterea componentelor de la valorile nominale.

This paper proposes a new approach for analog circuit fault diagnosis. We assume that the network topology and the nominal values of circuit components are known. Starting from measured values of nodes potentials of the circuit under test and using the nodal analysis method and the graph theory, the component deviations from the nominal values are determined.

Keywords: analog test, faulty circuit, diagnosis, graph.

1. Introduction

The diagnosis of faults in analog circuits is a challenging problem with limited results currently available and no general theory seems to have been established. Analog fault diagnosis has been addressed by two general methods: simulate-before-test (SBT) and simulate-after-test (SAT). The first is based on the use of fault dictionary, which contains responses from simulations of the circuit for different predefined faults. The second uses measurements to compute parameters of the circuits solving a set of fault diagnosis equations. All computations occur after measurements are acquired.

The main factors that make analog circuit diagnosis difficult can be summarized as follows: analog circuits are frequently nonlinear, include noise and have parameter values that vary widely, the complexity of today's analog circuits and their many parameters, as well as the limited accessibility to their internal components, the unknown values for the actual component (which differ from the nominal values) etc.

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Faults in analog circuits can be categorized as catastrophic and parametric. Catastrophic faults are open and short circuits, caused by sudden and large variations of components. The parametric faults are reported to the circuit functionality. Thus, the value of parameters deviates continuously with time or with environmental conditions to an unacceptable value.

Analog fault diagnosis usually consists of three stages which respectively address three important problems in the analog testing and diagnosis:

- fault detection – during that we have to find out if the circuit under test is faulty or not;
- fault location – which has as purpose to identify where the faulty parameters are;
- parameter evaluation – to tell how much the parameter deviations from nominal values are.

Starting from the nodal analysis method and graph theory [1], in the next sections, it is presented a diagnosis method for resistive analog circuits based on parametric deviation of component computation.

2. Method description

We consider a linear circuit with *b* branches and *n* nodes.

Let $\boldsymbol{g} = (g_1, ..., g_b)$ the branch conductance vector, $\boldsymbol{v}_{n-1}$ – is the node voltage vector, and $\boldsymbol{i}_{sc}$ – is the node current vector. Then, according to nodal analysis method [5], the circuit can be expressed by:

$$\boldsymbol{Y}_{n-1} \cdot \boldsymbol{v}_{n-1} = \boldsymbol{i}_{sc} \tag{1}$$

Equation (1) can be also written:

$$\boldsymbol{A} \cdot \boldsymbol{Y}_b \cdot \boldsymbol{A}^T \cdot \boldsymbol{v}_{n-1} = \boldsymbol{i}_{sc} \tag{2}$$

where $\boldsymbol{A}$ – is the reduced node-branch incidence matrix, $\boldsymbol{Y}_b = \text{diag}(\boldsymbol{g})$ is a diagonally matrix with the dimension *bxb*, having on the main diagonal the branch conductance of the circuit, and $\boldsymbol{A}^T$ is transpose of matrix $\boldsymbol{A}$.

We assumed that each circuit branch deviates with Δg_k, k = 1,...,b, then the equation (1) becomes:

$$\left(\boldsymbol{Y}_{n-1} + \Delta\boldsymbol{Y}_{n-1}\right)\left(\boldsymbol{v}_{n-1} + \Delta\boldsymbol{v}_{n-1}\right) = \boldsymbol{i}_{sc} \tag{3}$$

or, from (2) it results:

$$\left[\boldsymbol{A}\left(\boldsymbol{Y}_b + \Delta\boldsymbol{Y}_b\right)\boldsymbol{A}^T\right]\cdot\left(\boldsymbol{v}_{n-1} + \Delta\boldsymbol{v}_{n-1}\right) = \boldsymbol{i}_{sc} \tag{4}$$

where $\boldsymbol{\Delta Y}_b = \mathrm{diag}(\Delta g_1, ..., \Delta g_b)$.

Let us denote $\boldsymbol{v}'_{n-1} = \boldsymbol{v}_{n-1} + \Delta\boldsymbol{v}_{n-1}$, the measured potentials. From equation (4) we have:

$$\left(\boldsymbol{A}\boldsymbol{Y}_b\boldsymbol{A}^T + \boldsymbol{A}\cdot\Delta\boldsymbol{Y}_b\cdot\boldsymbol{A}^T\right)\cdot\boldsymbol{v}'_{n-1} = \boldsymbol{i}_{sc} \tag{5}$$

$$\left(\boldsymbol{A}\cdot\Delta\boldsymbol{Y}_b\cdot\boldsymbol{A}^T\right)\cdot\boldsymbol{v}'_{n-1} = \boldsymbol{i}_{sc} - \boldsymbol{A}\boldsymbol{Y}_b\boldsymbol{A}^T\cdot\boldsymbol{v}'_{n-1} \tag{6}$$

The equation (6) is a n-1 equation system with b unknowns (Δg_k, k = 1..b). This can be written as:

$$\boldsymbol{M}\cdot\boldsymbol{x}_b = \boldsymbol{y} \tag{7}$$

where $\boldsymbol{M}$ is a matrix with the dimensions (n-1) x b, $\boldsymbol{y}$ is the right hand side of equation (6) with the dimensions (n-1) x 1, and $\boldsymbol{x}_b$ is the vector of unknown variables Δg_k, k = 1,...,b.

$$\boldsymbol{M} = \boldsymbol{A}\otimes\left(\boldsymbol{A}^T\cdot\boldsymbol{v}'_{n-1}\right) \tag{8}$$

The operand $\otimes$ means an element by element multiplication of each column from $\boldsymbol{A}$ with the concordant element from vector ($\boldsymbol{A}^T.\boldsymbol{v}'_{n-1}$).

Equation system (7) is compatible if only if rang[$\boldsymbol{M}$] = rang[$\boldsymbol{M}$ $\boldsymbol{y}$].

With n-1 < b and max(rang[$\boldsymbol{M}$]) = n-1 the matrix $\boldsymbol{M}$ is partitioned as:

$$\left[\boldsymbol{M}'\quad\boldsymbol{M}''\right]\cdot\begin{bmatrix}\boldsymbol{x}'\\ \boldsymbol{x}''\end{bmatrix} = \boldsymbol{y} \tag{9}$$

where $\boldsymbol{M}'$ is a square matrix of dimensions n-1 made up of the columns of $\boldsymbol{M}$ corresponding to a tree of circuit. Equation (7) becomes:

$$\boldsymbol{M}'\cdot\boldsymbol{x}'+\boldsymbol{M}''\cdot\boldsymbol{x}'' = \boldsymbol{y} \tag{10}$$

$$\boldsymbol{x}' = \boldsymbol{M}'^{-1}\cdot\left(\boldsymbol{y} - \boldsymbol{M}''\cdot\boldsymbol{x}''\right) \tag{11}$$

We assumed that all the faults are located in the branches of the tree represented by matrix $\boldsymbol{M}'$, than $\boldsymbol{x}'' = \boldsymbol{0}$ and from (11), it results:

$$\boldsymbol{x}' = \boldsymbol{M}'^{-1} \cdot \boldsymbol{y} = \begin{bmatrix} \Delta g_1 \\ \Delta g_2 \\ \vdots \\ \Delta g_{n-1} \end{bmatrix} \tag{12}$$

For this work we developed an application which computes all the circuit trees, starting from the reduced node-branch incidence matrix. For each circuit tree we solve equation (12) with concordant matrix ***M***'.

It can be observed than in case of a single fault, the faulty component deviation has the same value for all circuit trees which contained it, thus the location is facilitated.

3. Example

Let be the circuit described in Fig.1:

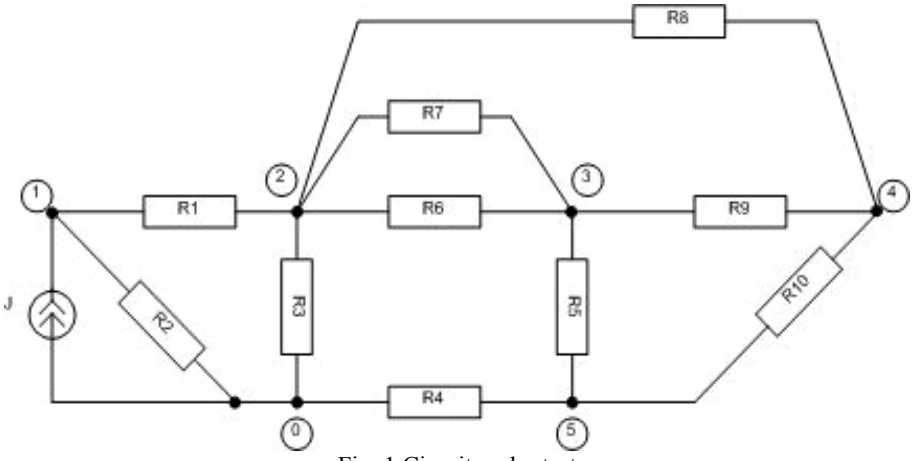

Fig. 1 Circuit under test

with the next values $R_k = k \cdot 10\ \Omega$, k = 1..10, J = 1 A.
The node potentials of the circuit for component nominal values are:

$$\boldsymbol{v}_{n-1}^{fd} = \begin{bmatrix} 12.436 & 8.653 & 6.632 & 6.479 & 3.589 \end{bmatrix}^T \tag{13}$$

The reduced node-branch incidence matrix [6] is:

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \tag{14}$$

and the vector of injected current in nodes is:

$$\boldsymbol{i}_{sc} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \tag{15}$$

We consider the occurrence of one fault in branch 5, trough modification of resistor value R5 from 50 Ω to 23 Ω.

After a simulation under Spice [10], the node potentials of the circuit are:

$$\boldsymbol{v}_{n-1}^{cd} = \begin{bmatrix} 12.303 & 8.455 & 5.978 & 6.346 & 4.121 \end{bmatrix} \tag{16}$$

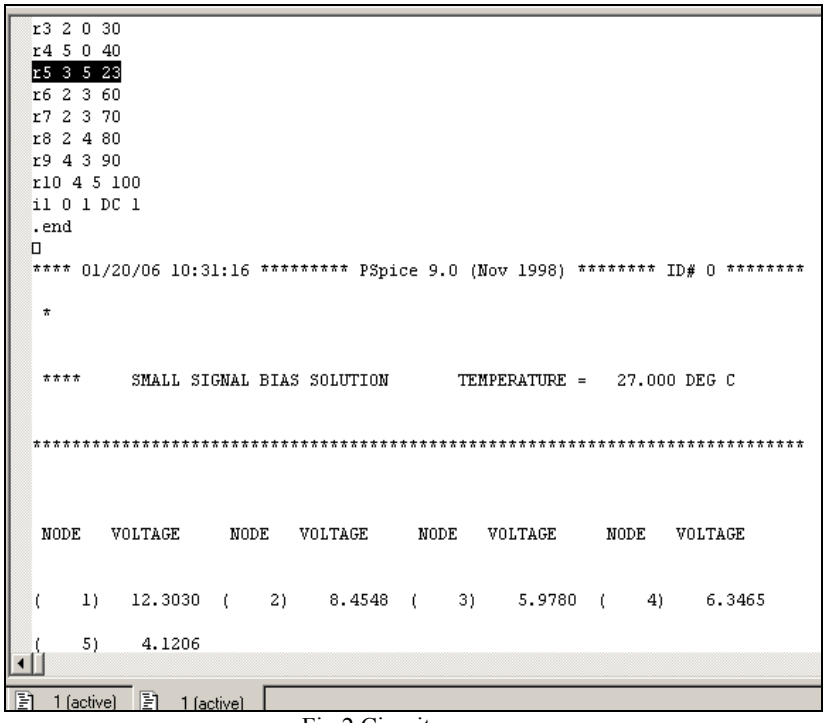

Fig.2 Circuit response

With $\boldsymbol{v}^{cd} \neq \boldsymbol{v}^{fd}$ the occurrence of the fault is made.

The diagnosis equation (7) becomes:

$$\boldsymbol{M}_{(5\times10)} \cdot \boldsymbol{x}_{b(10\times1)} = \boldsymbol{y}_{(5\times1)} \tag{17}$$

$$\begin{bmatrix} 3.848 & 12.303 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3.848 & 0 & 8.455 & 0 & 0 & 2.477 & 2.477 & 2.109 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.857 & -2.477 & -2.477 & 0 & -0.368 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2.109 & 0.368 & 2.225 \\ 0 & 0 & 0 & 4.121 & -1.857 & 0 & 0 & 0 & 0 & -2.225 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \\ x_9 \\ x_{10} \end{bmatrix} = \begin{bmatrix} 0.0000 \\ -0.0001 \\ 0.0436 \\ 0.0000 \\ -0.0436 \end{bmatrix} \tag{18}$$

The rank of matrix $\boldsymbol{M}$ is

$$rang(\boldsymbol{M}) = 5$$

By means of described method, implemented in Matlab [9], for all circuit trees it determine $\boldsymbol{M}'$ and it solve equation (12), resulting the nodes of faulty branch and the new value of the resistor.

Hence:

$R_5^n = 22.995\ \Omega$

closed to the assumed value ($R_5 = 23\ \Omega$).

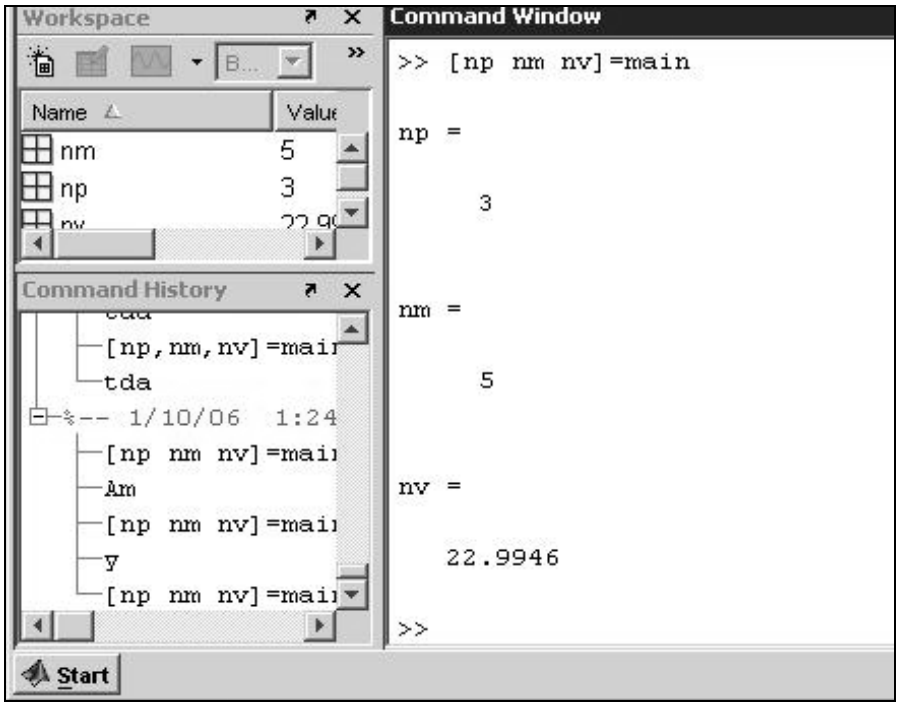

Fig. 3 Capture with application response

The method is valid both for parametric faults (soft) and for catastrophic faults (open and short circuits). Moreover by means of this approach we can determine multiple faults (up to *n-1* faults), with the condition that those will be occurred on the same tree. But the locating of this tree circuit is more difficult.

4. Conclusions and future work

This paper has proposed a new diagnosis approach for analog linear resistive circuits. It consists of the formulation of diagnosis equation using nodal analysis method, the determination of all tree circuit and to solve the diagnosis equation for each tree circuit.

This method is efficient in both cases of faults: slight and large deviation of parameters values.

In the future we plan to improve the algorithm by taking into account other circuit elements (capacitors, coils, commanded sources, etc), as well as for other functioning states.

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