INFLUENCE OF THE FAILURE-ON-DEMAND PARAMETER UPON CONTINUITY OF SUPPLY IN ELECTRIC DISTRIBUTION SYSTEMS

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Evaluarea corectă a comportamentului sistemelor electrice de distribuție reprezintă o sarcină dificilă, din cauza răspunsului complex al sistemelor la apariția defectelor. În cadrul analizei de fiabilitate, sunt abordate probleme care apar la insularizarea sistemelor. Rezultatele studiului efectuat asupra unui sistem-test sunt prezentate sub forma histogramelor funcțiilor densității de probabilitate ale indicilor de fiabilitate, prin utilizarea unei tehnici eficiente bazate pe funcțiile caracteristice ale acestora.

The proper evaluation of the distribution systems behavior is a particularly difficult task, due to the complexity of the distribution system response to failures. The issues concerning system islanding, which have a major impact on reliability results, are overviewed and included in the overall reliability model. Results of reliability analysis are shown on a test system by computing the probability density function of reliability indices, by using a fast and efficient technique based on the characteristic functions.

Keywords: distribution systems, failure-on-demand, continuity of supply, power quality

1. Introduction

The ever-increasing adoption of the new technologies for small-scale generation and the trend towards the adoption of distributed resources is modifying the characteristics of distribution systems [1], [2]. This work is aimed at making a comprehensive evaluation of the probability density functions (PDFs) of a set of reliability indices applied to electricity distribution systems which contains dispersed generation in the instance of micro-grid [3]. The surveyed indices include frequency and duration of the interruptions, and power and energy not supplied. In previous article [4], care has been taken for adopting viable test-system as benchmark for future upgrades of the simulation techniques. The complexity of the issue under study is increased by the addition of a new parameter –failure on demand (FOD)– within the reclosure scheme of the test-

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system. Thus, by considering the number of occurrences and the duration of the interruptions as random variables (RVs), the indices become RVs [5-8].

For correctly evaluate the reliability parameters, an analysis must be performed in order to identify the appropriate time intervals for system response to contingencies [9]. The IEC 61508 - Part 1 defines four safety integrity levels to accommodate a wide range of risk reduction or safety integrity that the safety-related systems will have to achieve. Table 1 shows the Safety Integrity Levels (SILs) for safety related systems operating in a demand mode of operation and in a continuous/ high demand mode of operation.

Target failure measures are shown for each of the four SILs to ensure that the hardware safety integrity is achieved [10].

Table 1

Safety Integrity Levels & target failure measures [11]

SAFETY INTEGRITY LEVEL	DEMAND MODE OF OPERATION Probability of failure to perform its design function on demand PFDavg	CONTINUOUS / HIGH DEMAND MODE OF OPERATION Probability of a dangerous failure per year
4	$\geq 10^{-5}$ to 10^{-4}	$\geq 10^{-5}$ to 10^{-4}
3	$\geq 10^{-4}$ to 10^{-3}	$\geq 10^{-4}$ to 10^{-3}
2	$\geq 10^{-3}$ to 10^{-2}	$\geq 10^{-3}$ to 10^{-2}
1	$\geq 10^{-2}$ to 10^{-1}	$\geq 10^{-2}$ to 10^{-1}

The standard IEC 61508 defines PFD as the average probability of failure to perform its design function on demand (average probability of dangerous failure on demand of the safety function according to), i.e. the probability of unavailability of the safety functions leading to dangerous consequences [12].

The main contribution of this paper reside in the new approach of FOD as part of the reliability criterion assessment using Monte Carlo method, in the case of DG embedded traditional distribution network.

2. The analytical simulation technique

The reliability analysis performed in this section deals with reliability indices [13] assumed to be random variables (RVs) due to the randomness of the faults occurring in the time interval $(0,T)$. Fault occurrences are assumed as independent random events and times to failure as exponentially-distributed RVs, so that the number of occurrences of a fault follows a Poisson distribution [14]. The probability of occurrence of a fault during the restoration after a preceding fault is negligible. The power interrupted is deterministic and independent of the fault occurrence. Switching and repair times are assumed to be RVs with Gamma

Probability Density Function (PDF) $f_{\tau}(x)=\frac{x^{\alpha-1}}{\beta^{\alpha}\Gamma(\alpha)}e^{-x/\beta}$ with shape parameter α, scale parameter β, average value $\mu = \alpha\,\beta$ and variance $\sigma^2 = \alpha\,\beta^2$.

Reliability analysis is carried out by considering a single feeder and M feeders with similar reliability profile. The fast and efficient technique proposed is used for performing numerical simulations. This technique adopts a characteristic functions-based approach in which the direct convolution of the PDFs is avoided by resorting to the properties of the compound Poisson process and to the use of the direct and inverse Discrete Fourier Transforms. A key feature of the method is the possibility of handling any type of probability distribution, including multi-modal PDFs, over any time interval of analysis, providing the whole PDF of local and global reliability indices. The results accuracy is checked out by comparing average value and variance of the numerically computed reliability indicators to average value and variance computed with the analytical method.

Restoration modes (Table 2) are scanned by using the index h = 1,…, H_U, H_U+1,…, H_U+H_D, where H_U = 4 refers to the first four modes and H_D = 4 refers to the fifth to last mode. The index h is applied to the interrupted power C_{int} and to the probability of successful restoration p_R. The Gamma RVs representing the switching time τ_S and the repair time τ_R are used to form the vector $\boldsymbol{\tau} = [\tau_S, \tau_R, \tau_S, \tau_R, \tau_S, \tau_R, \tau_S, \tau_R]^T$. The subscripts U and D represent the upstream and downstream network, respectively. The total energy not supplied is **w** for a single fault occurrence and **W** for all fault occurrences.

Table 2

Probabilities of successful restoration (restoration mode with initial letter U for faults in the upstream network, D for faults in the downstream network)

Restoration mode	Successful island formation	Successful S_U switching	Successful S_D switching	Successful B_D/B_U selective operation	Interrupted power C_{int} (p.u.)	Probability of successful restoration p_R
U1	yes	yes	---	---	C_U	$p_I\,p_U$
U2	yes	no	---	---	C_U	$p_I\,(1-p_U)$
U3	no	yes	yes	---	$C_U + C_D$	$(1-p_I)\,p_U$
U4	no	no	no	---	$C_U + C_D$	$(1-p_I)\,(1-p_U)$
D1	---	---	yes	yes	C_D	$p_S\,p_D$
D2	---	---	yes	no	C_D	$p_S\,(1-p_D)$
D3	---	---	no	yes	$C_U + C_D$	$(1-p_S)\,p_D$
D4	---	---	no	no	$C_U + C_D$	$(1-p_S)\,(1-p_D)$

Results are shown by first taking into account the upstream and the downstream networks separately. The energy not supplied for a *single* fault occurrence has expected value

$$\mu_{\mathbf{w}U} = \sum_{h=1}^{H_U} p_R^{(h)} C_{int}^{(h)} \mu_{\boldsymbol{\tau}^{(h)}} \ , \quad \mu_{\mathbf{w}D} = \sum_{h=H_U+1}^{H_U+H_D} p_R^{(h)} C_{int}^{(h)} \mu_{\boldsymbol{\tau}^{(h)}} \tag{1}$$

and second raw moment

$$m_{2\mathbf{w}U} = \sum_{h=1}^{H_U} p_R^{(h)} \alpha_{\boldsymbol{\tau}^{(h)}} \left(\alpha_{\boldsymbol{\tau}^{(h)}} + 1\right) \left(C_{int}^{(h)} \frac{\mu_{\boldsymbol{\tau}^{(h)}}}{\alpha_{\boldsymbol{\tau}^{(h)}}} \right)^2 , m_{2\mathbf{w}D} = \sum_{h=H_U+1}^{H_U+H_D} p_R^{(h)} \alpha_{\boldsymbol{\tau}^{(h)}} \left(\alpha_{\boldsymbol{\tau}^{(h)}} + 1\right) \left(C_{int}^{(h)} \frac{\mu_{\boldsymbol{\tau}^{(h)}}}{\alpha_{\boldsymbol{\tau}^{(h)}}} \right)^2 \tag{2}$$

For *all* fault occurrences in the time interval $(0,T)$, the total energy not supplied $\mathbf{W}_U$ has expected value $\mu_{\mathbf{W}_U} = \lambda_U T \mu_{\mathbf{w}_U}$ and variance $\sigma^2_{\mathbf{W}_U} = \lambda_U T m_{2\mathbf{w}_U}$, while $\mathbf{W}_D$ has expected value $\mu_{\mathbf{W}_D} = \lambda_D T \mu_{\mathbf{w}_D}$ and variance $\sigma^2_{\mathbf{W}_D} = \lambda_D T m_{2\mathbf{w}_D}$. The total energy not supplied $\mathbf{W}$ for the whole system with M equal feeders is found by the composition of independent events in the two networks, so that its expected value is $\mu_{\mathbf{W}} = M\left(\mu_{\mathbf{W}_U} + \mu_{\mathbf{W}_D}\right)$ and its standard deviation is $\sigma_{\mathbf{W}} = \sqrt{M\left(\sigma^2_{\mathbf{W}_U} + \sigma^2_{\mathbf{W}_D}\right)}$. The total duration of the interruptions $\mathbf{d}$ is easily derived from the above results, with expected value $\mu_{\mathbf{d}} = \mu_{\mathbf{W}} / C_{tot}$ and standard deviation $\sigma_{\mathbf{d}} = \sigma_{\mathbf{W}} / C_{tot}$.

An important value characterizing the PDF and the Cumulative Distribution Function (CDF) is the probability p_0 that no faults occur during the time interval of analysis $(0,T)$, which does not depend on the effectiveness of the switching operations. The value p_0 is the amplitude of the Dirac pulse in the origin of the PDF and the initial step value of the corresponding CDF. For a single feeder where both C_U and C_D are positive, any fault leads to an interruption, so that $p_0 = e^{-\lambda_U T} e^{-\lambda_D T}$. If $C_D = 0$ and $C_U > 0$, the interruptions depend on the occurrence of faults on the upstream network or faults on the downstream network for which the selective operation of B_D and B_U fails, so that $p_0 = e^{-\lambda_U T} e^{-(1-p_S)\lambda_D T}$. If $C_U = 0$ and $C_D > 0$, the interruptions depend on the occurrence of faults on the downstream network or faults on the upstream network for which the island formation fails, so that $p_0 = e^{-(1-p_I)\lambda_U T} e^{-\lambda_D T}$. The case of M equal feeders corresponds to $p_0^{(M)} = p_0^M$.

The whole PDF of the total energy not supplied is obtained by using the characteristic functions approach, where the Gamma PDFs of switching and repair times are represented by using their characteristic function $\Phi_{\boldsymbol{\tau}}(\omega) = (1 + j\omega\beta)^{-\alpha}$ and second characteristic function $\Psi_{\boldsymbol{\tau}}(\omega) = \ln(\Phi_{\boldsymbol{\tau}}(\omega)) = -\alpha \ln(1 + j\omega\beta)$. For the sake of brevity, the argument ω is omitted in the sequel. For a single fault occurrence, the energy not supplied in a single restoration mode is represented, for $h = 1,\ldots H_U+H_D$, by a Gamma PDF with shape factor $\alpha_{\mathbf{w}^{(h)}} = \alpha_{\boldsymbol{\tau}^{(h)}}$, scale factor $\xi_{\mathbf{w}^{(h)}} = C_{int}^{(h)} \mu_{\boldsymbol{\tau}^{(h)}} / \alpha_{\boldsymbol{\tau}^{(h)}}$ and characteristic functions $\Phi_{\mathbf{w}^{(h)}} = \left(1 + j\omega\xi_{\mathbf{w}^{(h)}}\right)^{-\alpha_{\mathbf{w}^{(h)}}}$ and $\Psi_{\mathbf{w}^{(h)}} = \ln\left(\Phi_{\mathbf{w}^{(h)}}\right)$. A single occurrence of a fault in a feeder of the upstream network (or downstream network) leads to a characteristic function computed as the weighted sum of the characteristic functions of the various restoration modes:

$$\Phi_{\mathbf{w}_U} = \sum_{h=1}^{H_U} p_R^{(h)} \Phi_{\mathbf{w}^{(h)}} \ , \quad \Phi_{\mathbf{w}_D} = \sum_{h=H_U+1}^{H_U+H_D} p_R^{(h)} \Phi_{\mathbf{w}^{(h)}} \tag{3}$$

The energy not supplied for all occurrences of the fault in the time interval (0,*T*) is given by the random sums $\mathbf{W}_U = \sum_{i=1}^{\mathbf{n}} \mathbf{w}_U$ and $\mathbf{W}_D = \sum_{i=1}^{\mathbf{n}} \mathbf{w}_D$, where **n** is the Poisson-distributed RV representing the number of fault occurrences. With **n** mutually independent RVs having equal PDFs, the above random sums correspond to a compound Poisson process, providing the second characteristic functions $\Psi_{\mathbf{W}_U} = \lambda_U T(\Phi_{\mathbf{w}_U} - 1)$ and $\Psi_{\mathbf{W}_D} = \lambda_U T(\Phi_{\mathbf{w}_D} - 1)$. The compound Poisson process replaces the direct convolution of the PDFs. The second characteristic functions are additive, so that considering all faults over all feeders leads to the second characteristic function of the total energy not supplied $\Psi_{\mathbf{W}} = M(\Psi_{\mathbf{W}_U} + \Psi_{\mathbf{W}_D})$ and to $\Phi_{\mathbf{W}} = e^{\Psi_W}$.

3. Application on a test -system

The reliability analysis performed in this paper assumes that some areas of the system under analysis can be aggregated into a few equivalent networks, each of which containing the components subject to the same interruption in case of fault. These networks are represented by their equivalent reliability parameters [14].

The effect of a DG unit on reliability is investigated by considering the feeder to which the DG unit is connected (Fig.1) and representing two equivalent networks with their failure rates, namely, λ_U for the *upstream* network and λ_D for the *downstream* network, and with the corresponding load powers C_U and C_D. The two equivalent networks can be connected to other supply sources during the system restoration after a fault by closing the switches S_U and S_D. Then, the service restoration after a fault in the downstream network requires opening the circuit breaker B_D and performing manual operations.

For a fault in the upstream network, the service restoration requires performing manual operations. The presence of the local DG unit can assist the restoration process, by opening the circuit breaker B_S and connecting the DG unit to the downstream network.

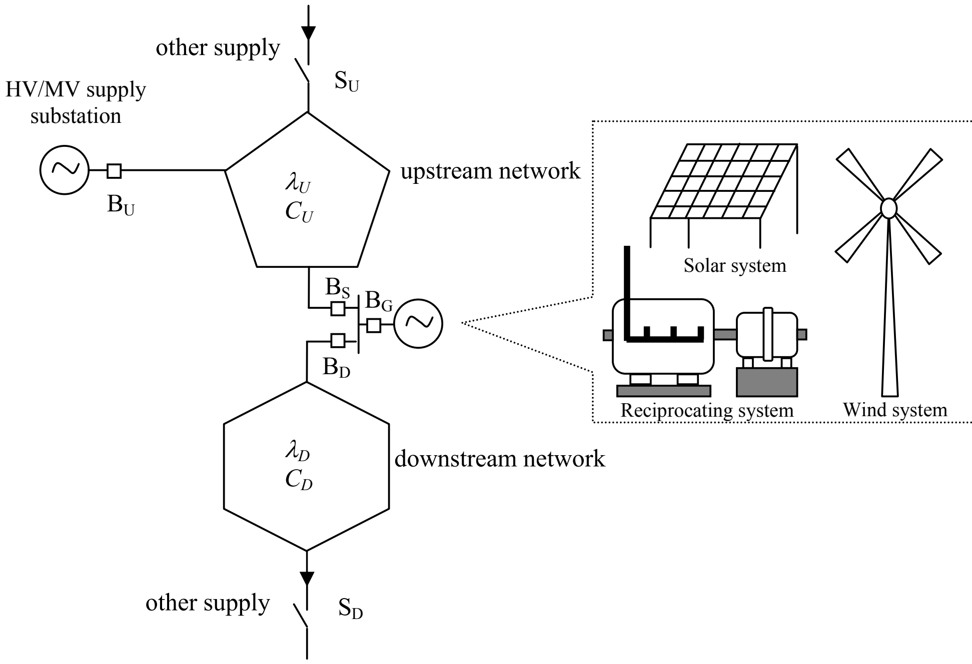


Fig. 1. Scheme of the test system for reliability study of a distribution system with DG

Reliability analysis takes into account two types of indices (local and global). Assuming the power P_k to be delivered to load point k during normal operation, the following local indices are defined: f_k (frequency of the interruptions), d_k (duration of the interruptions), $P_k^{NS} = f_k P_k$ (power not supplied), $E_k^{NS} = d_k P_k$ (energy not supplied), $d_k^{NS} = d_k / f_k$ (average duration of the interruptions). The global indices are defined for the whole electrical network, representing the overall system reliability. The global indices can be built by computing a weighted average of the load point indices, using as weights the numbers of customers or the power supplied to the load points in normal conditions and, for large distribution systems, approach the Normal shape. Local indices depend on the occurrence of a few failures at the specified load point in the time interval of analysis, so that they can have uncommon PDFs (e.g., multi-modal) and their PDFs are more difficult to compute. For space limitations, the analysis reported in this paper will refer only to the most challenging computation of the local indices.

Faults are assumed to be independent random events, with negligible probability of simultaneous faults. The number of occurrences of a fault in a specified time interval is represented by the RV **n**, with Poisson distribution. The multi-phase service restoration is assumed to have a random restoration time for each phase, independent of the fault and of the restoration phase of the same fault.

The RV $\boldsymbol{\tau}$ is used to represent the restoration time. Generally speaking, there is a multitude of distribution functions which are used to represent $\boldsymbol{\tau}$.

The exponential PDF is sometimes used for its simplicity, but the Gamma distribution has been proven to be better to represent the real behavior of the restoration times and will be used in the present work.

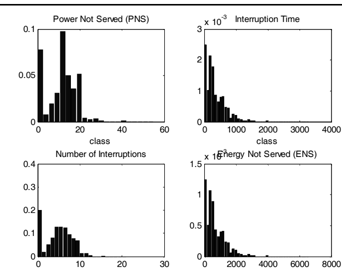
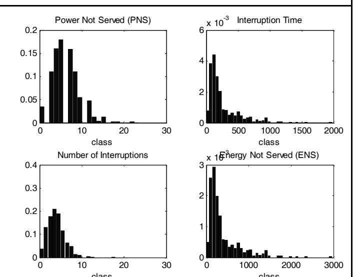
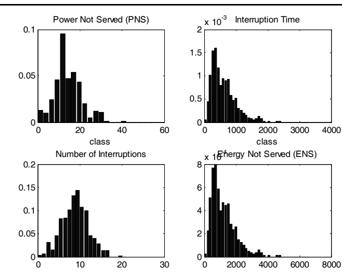
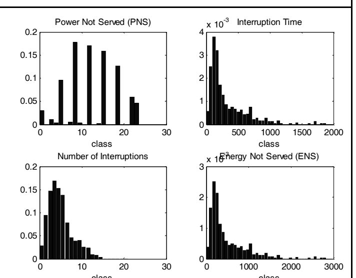
4. Simulation results

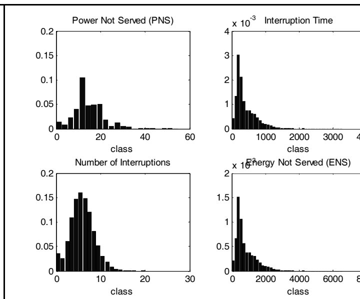
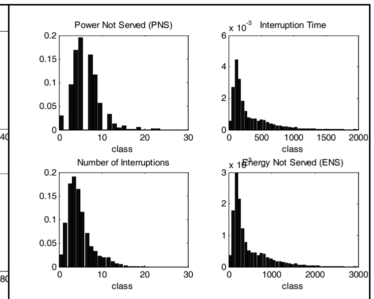
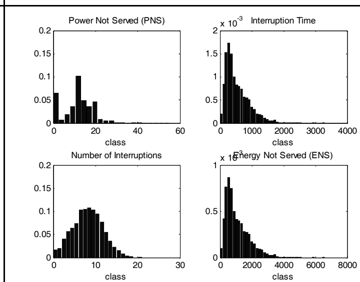
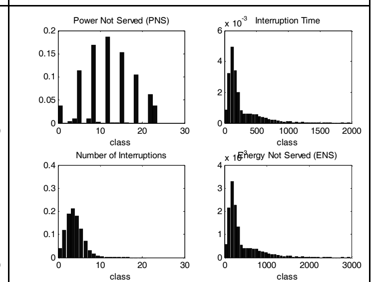
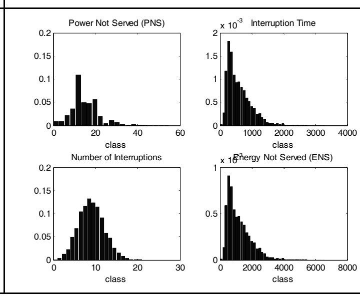
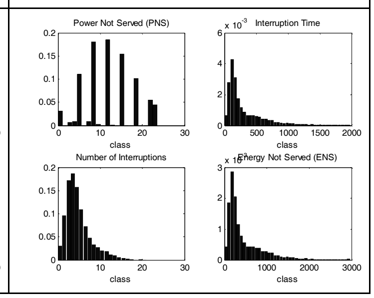
The output of the program looks as a series of histograms as indicated in Table 2. In order to further clarify the program output, a table containing principal indices of the network has been realized.

It shows the variables (PNS, Interrupted Time, Number of Interruptions and ENS), differentiated upon the number of extractions made by the program.

Table 2

Histograms of the simulated parameters PDFs'

No. of extractions	P_{sel}	P_{swU}	P_{swD}	Upstream network	Downstream network
500	0.95	0.9	0.05		
	0.05	0.05	0.05		

5000	0.95	0.9	0.9		
	0.05	0.05	0.9		
	0.05	0.05	0.05		

The outcome of the tests shows that the number of extractions has a limited influence upon the final result, excepting the case in which a more sensitive approach of the PDFs is required. The FOD parameter proved to be essential for the behavior of PNS. Thus, if the security-based function is not performing properly, increases the uncertainty in PNS prediction to a large number of classes, making the prediction of consumers' interruption a more difficult task than usual.

4. Mean values of the reliability indices calculated with the analytical method

In order to validate the results obtained from Matlab-based software, the expected values of the reliability indices have been calculated by using an analytical method. Assuming n_b as the number of branches belonging to the faulted (left or right) side of the network including the load point under test, the expressions of the expected values for the k-th load point are:

- *number of interruptions*;

- *power not supplied*;
- *interruption time*: for temporary faults;
- *energy not supplied*.

Table 3

Values for program variables, depending on the positioning on the network

Extractions	P_{sel}	P_{swU}	P_{swD}	P_{NS}	N_{IN}	T_{INT}	E_{NS}
500	0,95	0,9	0,9	9	3	83,3201	166,6403
				6	6	140,2934	210,4401
	0,05	0,9	0,9	6	6	779,5486	1559,1
				10,5	3	65,0715	97,6
	0,95	0,05	0,9	4	6	157,6459	315,2918
				14	4	115,9714	315,2918
	0,95	0,9	0,05	12	6	**805,6612**	**1611,3**
				4,5	3	124,4048	186,6
	0,95	0,05	0,05	10	5	395,7078	791,4157
				7,5	5	457,2395	685,8592
	0,05	0,05	0,9	0	3	185,3443	370,6886
				10,5	3	185,3443	278,0164
	0,05	0,05	0,05	12	**11**	550,2374	1100,5
				17,5	5	395,8561	593,8
5000	0,95	0,9	0,9	16	**8**	309,0325	618,0649
				3	2	39,2620	58,8929
	0,05	0,9	0,9	0	1	32,0254	64,0509
				3,5	1	32,0254	48,0382
	0,95	0,05	0,9	10	5	135,9129	88,8105
				6	4	88,8105	133,2157
	0,95	0,9	0,05	4	2	41,2667	82,5334
				4,5	3	115,2650	82,5334
	0,95	0,05	0,05	10	5	150,5077	341,0154
				0	0	0	0
	0,05	0,05	0,9	0	5	128,2006	256,4011
				17,5	5	128,2006	192,3008
	0,05	0,05	0,05	14	7	160,6119	321,2239
				7,5	5	**509,1320**	**763,6980**

Table 3 shows that for the analysis performed upon the test-system, some discrepancies occur between the values obtained with 500 extractions and 5000 extractions. These differences depend on the number of permanent faults occurred during the reliability simulation in the time period under study. Bolded parameters correspond to the biggest values for each parameter (column).

6. Conclusions

A framework study for distribution systems has been implemented to compute the reliability indices by taking into account various parameters. The method proved to be effective to give not only the *power not served* and *number of interruptions*, but also the *time of the interruptions* and *energy not served*. PFD, as parameter of power quality, has a greater influence in the event of multiple occurrences within the network. Thus, based on the simulation performed it is observable that the biggest values of E_{NS} and T_{INT} appear when at least two failure safety devices does not perform accordingly in the case of fault occurrence, or selectivity request. The interest for the PDF is due to the possibility of computing particular information such as the tail probability, which could be useful for defining additional indicators such as the probability of exceeding a given value, important for both the regulation of competitive electricity markets and computing various penalty schemes. The software is scalable, which constitutes a major advantage for further refining of the desired outcome. The future development of the proposed software will include the possibility of choosing between different reliability patterns of various distributed generation technologies.

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