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SPH METHOD IN APPLIED MECHANICS

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În această lucrare, autorul prezintă fundamente ale metodei SPH (Smoothed Particle Hydrodynamics) şi aplicaţii ale acesteia în mecanica aplicată. Metoda SPH este o metodă numerică Lagrangiană, fără reţea (mesh), folosită pentru a modela probleme care implica deformaţii sau distorsiuni mari. Mai întai, în lucrare, sunt prezentate exemple de utilizare ale metodei SPH, întro manieră comparativă cu metoda elementelor finite (MEF).Apoi, metoda SPH este folosită pentru simularea caracteristicilor post-impact ale norului debris.

In this paper, the author presents some fundamentals of the SPH (Smoothed Particle Hydrodynamics) method, and its applications in applied mechanics. SPH is a meshless Lagrangian numerical technique used to model problems, where large mesh distortions occur. First examples referring to SPH method are presented in a comparative way with the finite element method (FEM). Then, SPH method is used for simulation of the debris cloud post impacting characteristics.

Keywords: SPH (smoothed particle hydrodynamics), FEM, debris cloud, impact

1. Introduction

Many authors and specialists consider that Smoothed Particle Hydrodynamics is a numerical method of simulation invented by Lucy [2] in 1977. The first applications of this method were connected to cosmological problems. The method was extended to fluid simulation, especially with free-surface by Monaghan [4] in 1992, and to other fields. The field of applied mechanics is the last one, but it is extensively studied and significant advances have been made.

The last preoccupations are focused upon the coupling of this numerical method with standard numerical procedures, such as the finite element method and other meshless techniques because they offer new possibilities to solve complex problems in engineering at nano, micro and macro scales.

Thus, about Smoothed Particle Hydrodynamics (SPH) we could say that, comparatively with Finite Element Method (FEM), it is a new numerical technique, but which has a quick development. Nowadays, this method is used in

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many scientific fields. The advantages seem to be grater then disadvantages from a lot points of view, especially in some fields.

In applied mechanics, SPH method appears to be powerful and useful for those problems that involve large displacements. For solving impact problems, the SPH method is more suitable one than others, like FEM.

Until now, in our country, the SPH method is practically not used, but, due to its possibility to be efficient in some problems, it will be surely used more and more in the future.

Many special programs exist, but next to these, the SPH method is implemented in LS-Dyna and Autodyne programs.

2. Fundamentals of the SPH method

The SPH method belongs to the meshless methods, so the investigated domain is represented by a number of nodes, representing the particles of this domain with their material characteristics. Each particle represents an interpolation point on which the material properties are known.

The problem solution is given by the computed results, on all the particles, using an interpolation function. We can say that the fundamentals of SPH theory consist in interpolation theory; all the behavior laws are transformed into integral equations. The kernel function gives a weighted approximation of the field variable (function) in a point (particle). A function $A(r)$ can be thus estimated by the relation:

$$A(r) = \int A(r')W(r-r',h)dr' \tag{1}$$

where the function $W(r-r',h)dr'$ is the kernel function, which has two main properties:

$$\text{a) } \int W(r-r',h)dr' = 1, \tag{2}$$

$$\text{b) } \lim_{h->0} \mathrm{W(r-r',h)} = \delta(\mathrm{r-r'}), \tag{3}$$

δ is the Dirac delta function and h is the smoothing length. An intuitive representation of this parameter can be seen in the Fig. 1.

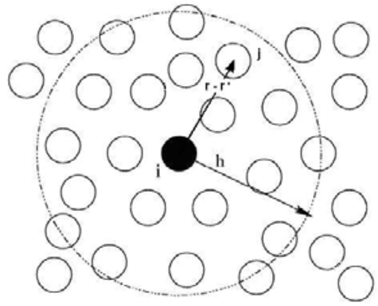


Fig. 1 The smoothing lengh *h*

The smoothing lengh defines a domain containing particles in interaction with particle *i*. The form of the smoothing function W(r,h) = W(r/h) is presented in figures 2-a and 2-b.

Different kernel function can be used: Gaussian, polynomial, spline etc. The most used function is the cubic B-spline one. Such a function has the form:

$$W(r,h)=\frac{\sigma}{h^n}\begin{cases}(1-\frac{3}{2}s^2+\frac{3}{4}s^3) & 0\le s\le 1\\ \frac{1}{4}(2-s)^3 & 1\le s\le 2\\ 0 & s>2\end{cases} \tag{4}$$

where $s = r/h$, n is the number representing the spatial dimension (1, 2 or 3) and σ is a constant which can have the value: 2/3, 10/7π or 1/π, depending on the space with one, two or three dimensions. In fact, the kernel function is a delta or Dirac function with some specific requirements.

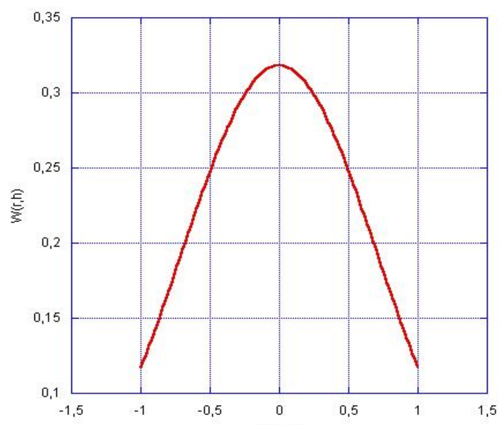


Fig. 2-a Kernel function – general representation

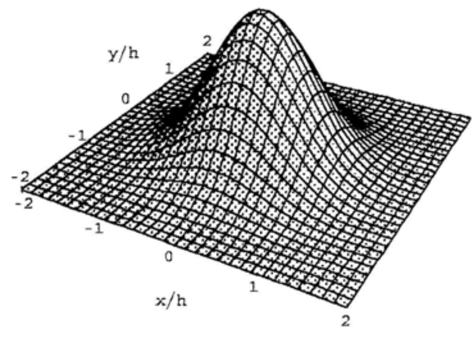


Fig. 2-b Grafical representation of 2D-Kernel function

Full theoretical details of the mathematical derivation of the kernel approximation for a continuous function and also aspects regarding the particle forces, treatment of the boundaries and many other aspects are beyond the target of this paper.

3. SPH - FEM. Problems and results

One of the problems presented and solved by SPH and FEM is the longitudinal impact of two identical bars of aluminum. The 3D model with finite elements is presented in figure 3.

This model consist in 2626 nodes and 1250 elements (solid with 8 nodes) for each bar. The impact velocity was 100 m/s or 0.01 cm/μs, for each bar coming from oposite directions. The bar dimensions were 1cm x 1 cm x 3 cm.

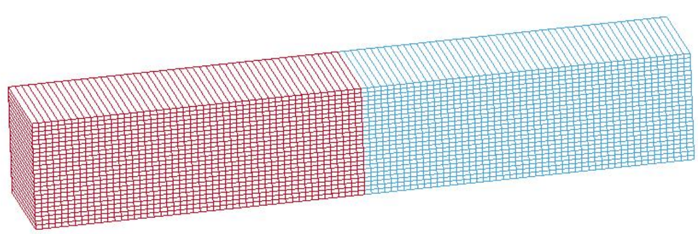


Fig. 3 The FEM 3D model of two impacting bars

For the same problem, a 2D model with SPH is presented in figure 4. Each bar was modeled with 2700 nodes and SPH elements.

Fig. 4 The SPH 2D model of two impacting bars

Both analyses were carried out with LS-Dyna program. Very close results were obtained, some of them being about identically as values. Figures 5 and 6 show the field of von Mises stress, corresponding to the two different models.

As we can see, looking at Figures 5 and 6, the maximum values of von Mises stresses are the same.

Also, we should notice that the used mesh of SOLID elements and the density of SPH elements are not the best, but they have the same scale.

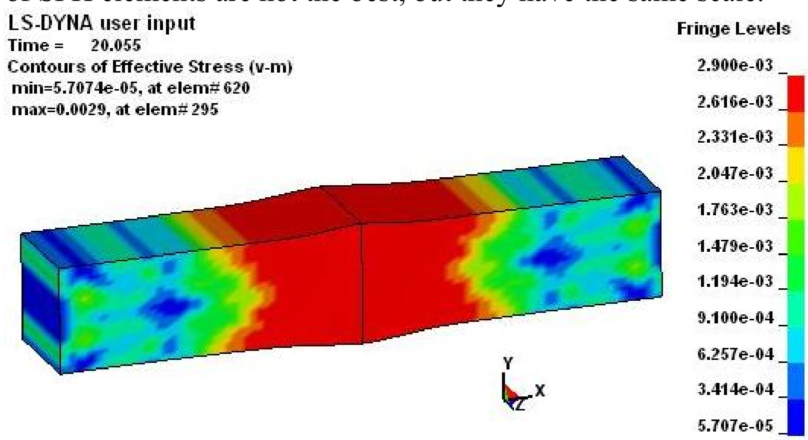


Fig. 5 Von Mises stress field, using FEM 3D model

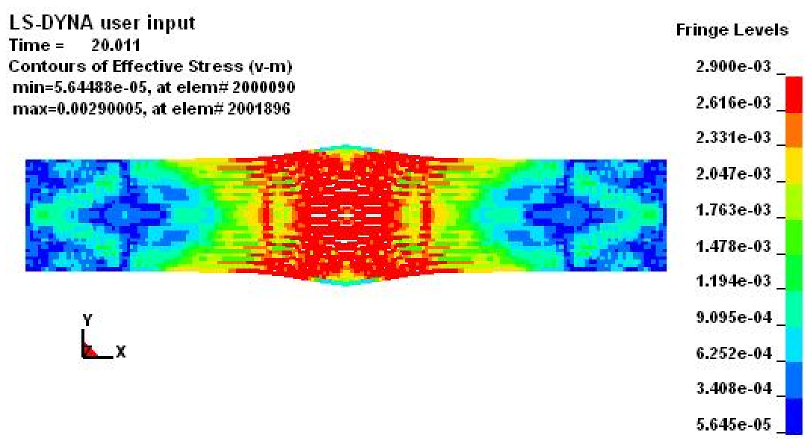


Fig. 6 Von Mises stress field, using SPH 2D model

Another comparative test, made using 2D models with SHELL elements and SPH elements, was an impact problem of a bar with a rigid wall.

The model with SHELL elements is presented in Figure 7. The model has 10800 elements and 11041 nodes for a bar with dimension 0.5cm x 3.0cm x

9.0cm. In Figure 8, von Mises stress field is illustrated, for an analysis time of 20 microseconds.

The model with SPH elements, for the same problem, is presented in Figure 9. This model has the same number of nodes (particles). The sum of masses, attributed to each particle, represents the mass of the bar. In Figure 10 von Mises stress field is presented, for the same analysis time. The maximum value of von Mises stress is practically the same in this problem too.

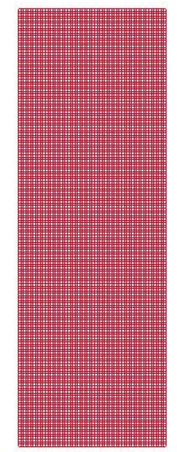


Fig. 7 Shell element model for impacting with a rigid wall

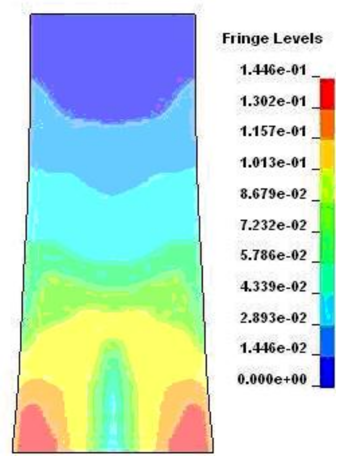


Fig. 8 Von Mises stress field of SHELL elements

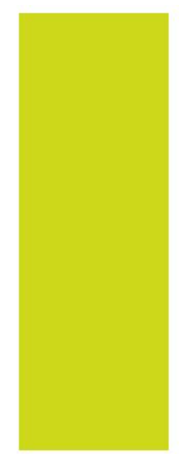


Fig. 9 SPH element model for impacting with a rigid wall

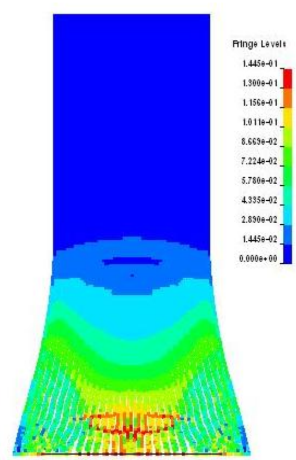


Fig. 10 Von Mises stress field of SPH elements

The von Mises stress values, presented in Figures 8 and 10 are very close, the error being negligible one. A general conclusion drawn from the results of the problems presented above, is that the SPH method yields results that are sufficiently reliable in order to be used in a more complex numerical simulation.

In all numerical simulation [6], [7], a unit measure system, having the folowing fundamental measures: mass [g], time [s]10^{-6} and length [cm], was used. So, the stress values, which appear in figures 5, 6, 8 and 10 are expressed in a derived measure unit [g/cm(s10^{-6})^2] or, they would have to be multiplied with 10^5 for an expresion in [N/mm^2].

4. About post impacting debris cloud and its simulation

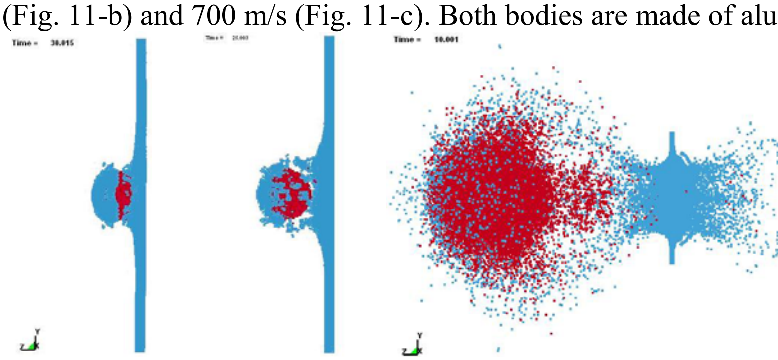
A concentration of particles or fragments in a defined region of space is what the specialists often name a debris cloud. A debris cloud is formed by a certain single source, which, in all cases, is represented by an impact at high velocity.

Piekutowski's studies of debris cloud include a flash X-ray of the normal impact of aluminum spheres with bumper plates. All the observations confirm the numerical study by SPH [3], [5].

The impact velocity range is from 1 to 15 km/s [1], [3]. Most fragments are in the range from 1 to 10 mm. Any impact between an orbital aircraft with these particles is a dangerous phenomenon. The effects of such post impacting debris cloud have to be studied too.

In the case of a normal impact between a sphere and a plate, both of aluminum, at different velocity, debris cloud occurs starting with one level of impact velocity that depends on material and geometric characteristics of the impacting bodies.

In Figures 11-a to 11-c, an impact between a sphere of 6 mm diameter and a plate of 1 mm thickness is presented, for three velocities: 300 m/s (Fig. 11-a), 500 m/s (Fig. 11-b) and 700 m/s (Fig. 11-c). Both bodies are made of aluminum.



a) b) c)

Fig. 11 The state of materials after impact at 30, 25 and 10 microseconds, respectively

As we can see in Figure 11, at the velocity of 300 or 500 m/s, the debris cloud does not occur. For the velocity of 700 m/s, debris cloud appears. The same problem was studied, but with a sphere of steel and the modeling made with FEM (Figure 12), and SPH (Figure 13).

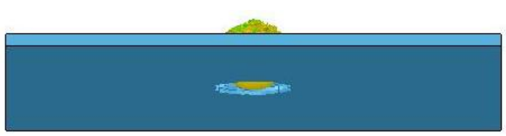


Fig. 12 FEM model

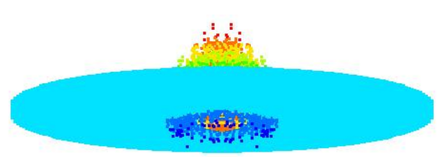


Fig. 13 SPH model

Looking at Figures 12 and 13 we can see a very different aspect: in the case of FEM modeling, debris cloud is not described, but SPH modeling presents this phenomena. This observation is also valid just in the case when both impacting bodies are made of aluminum.

As the sphere displacement is concerned (UZ for example, being the impact direction), in the case of low carbon steel sphere, a very good agreement can be noticed, the error being less than 4.5% in the perforation time.

5. Conclusions

The SPH method can be used for solving complex and unexpected problem of applied mechanics. There are many aspects when the SPH method is better than FEM and surely vice versa.

For problems like high velocity impact, impact with special materials (ceramics, glass etc), direct interaction between solid-fluid and others, the SPH method is a powerful numerical tool.

Referring to debris cloud, the researches are going on, in our preoccupation being the analysis of many other aspects like hole dimension, target deformation, impact-induced stresses, residual velocity of the projectile and of the cloud particles, material failure mode, the post-penetration debris cloud characteristics, the particle effect upon the second plate (target) etc.

Another aspect for future research is the impact with a thicker plate, when the thermal effects should be taken into account.

R E F E R E N C E S

[1] *Buruchenko, S., K.,* Smoothed Particle Hydrodynamics: Some Results, Вычислительңые, теҳнологии, Том 7, No. 1, 2002
[2] *Lucy, L., B.,* A numerical approach to the testing of the ssion hypothesis. Astrophysical, Journal, (82):1013{1024, 1977.
[3] *Ma, W., L., Pang, B., J., Zhang, W., Gai, B., Z.,* Numerical Simulation of Major Debris-Cloud Features Produced by Projectile Hypervelocity Impact on Bumper, Hypervelocity Impact Research Center, Harbin Institute of Technology, P.O. Box 3020, No.2, Yikuang Street, Harbin 150080, P. R. China
[4] *Monaghan, J., J.,* Smoothed particle hydrodynamics, In Annu. Rev. Astron. Astrophys., 1992, s. 543-574
[5] *Tanaka, K.,* Numerical Study on the High Velocity Impact Phenomena by Smoothed Particle, Hydrodynamics (SPH), 1-1-1 Umezono, Centtral 2 Tsukuba, Ibaraki 305-8568 JAPAN
[6] * * * AUTODYN Documentation
[7] * * * LS-DYNA Documentation