MOTION ANALYSIS OF AN UNDER-CONSTRAINED MULTI-ROBOTS COLLABORATIVE TOWING SYSTEM BASED ON STIFFNESS

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For the under-constrained multi-robots collaborative towing system that multiple serial mechanisms cooperatively tow a payload by cables, the motion accuracy of the system tends to be affected by the weak stiffness characteristics. Firstly, the dynamic model of the system was attained according to the spatial relationship. Subsequently, this paper analyzed and built the dynamic and static stiffness models. Then the influence of stiffness on the motion performance, especially the feasible region, was analyzed. At last, the feasible regions of the system were simulated with and without considering stiffness; the results suggest that stiffness has a great influence on the motion of the system. The conclusion lays a foundation for further research on the accuracy compensation control, trajectory planning and stability analysis of the system.

Keywords: multi-robots; towing system; kinematics; stiffness

1. Introduction

Utilizing multiple serial mechanisms to pull the same object collaboratively and locate its position and pose, which is defined as multi-robots collaborative towing system, has many usages in practice. The multi-robots collaborative towing system includes multiple cranes, cables and payloads. With cranes as driving part and cables as medium, movement and force are delivered through the changes of the position of crane lifting point and the length of cables to implement the transformation movement of the payloads' position and pose. There are big differences between this system and the traditional cable driven parallel robot. The main difference is that the essence of the cable driven parallel robot is a single robot, which leads to low load carrying capacity, small workspace and poor reconfigurability; while the multi-robots collaborative towing

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system can solve the above problems effectively and handle the position and pose of big and overweight loads.

We know that in order to implement the fully constraint of the lifted object, 7 single cable-driven mechanisms are needed to constitute the multi-robots collaborative towing system[1]. Although the study of fully constrained system has been relatively mature, in order to reduce the complexity of the system’s design and manufacture, the under-constrained parallel towing system by using less tandem lifting mechanisms can be considered as a better plan. The cable-driven parallel robots have aroused the interest of many researchers since its appearance in 1980s[2-5]. In practical use, the cables should be always in taut state in order to make sure the cable-driven system can control the payloads, so during the specific actual motion control, the cable tension and optimization[6-8], trajectory planning and optimization[9-13] shall be calculated in real time.

The multi-robots collaborative towing system not only has the crane structure with relatively large stiffness, but also includes flexible cables. For this system, the cable is in tension under the safe working condition, so it’s inevitable for the cable to deform under tension, and even fracture when the external force is too large. The stiffness change of the system will affect the positioning precision and its stability, and it can also cause the change in the system’s feasible areas simultaneously under heavy load condition. That requires the proper simplification on this complicated system to determine its equivalent stiffness. The towing system is composed of multiple tandem parts in accordance with their usage requirements. When establishing the stiffness model, the system model can be divided into multiple sub-models in accordance with the character that the stiffness conforms to the linear model, and then establish the stiffness model of each sub-model respectively. Finally, the stiffness of the multi-robots collaborative towing system can be obtained by coupling.

2. System Dynamics Modeling

Figure 1 shows *m* cranes tow a payload of *n*-DOF through the cable, each crane is connected with the hoisted payload through one cable. It can be assumed that cranes can meet the usage requirements, so no further studies will be carried out on the single crane. P_i（i=1,2,…m）is the connection point of the cable and crane, and the towing system realize the change of the payload position and pose by changing the position of P_i and the length of the cable. With the establishment of the reference coordinate system *O-XYZ* and the body coordinate system *O'-X'Y'Z'* which is built on the payload’s center of mass, the position of the payload center of mass *O'* on the reference coordinate system can be considered as:

$$\boldsymbol{r} = [x\,y\,z]^{\mathrm{T}} \tag{1}$$

Supposing the payload's quality is M, so the gravity is represented by the unit spinor of zero pitch in the global coordinate system as

$$\boldsymbol{G} = -\boldsymbol{Mg}[\boldsymbol{i} \;\; \boldsymbol{r}\times\boldsymbol{i}]^{\mathrm{T}}, \quad \boldsymbol{i} = [0 \;\; 0 \;\; 1]^{\mathrm{T}} \tag{2}$$

The velocity and angular velocity of the payload in the reference coordinate are $\boldsymbol{v}$ 、 $\boldsymbol{\omega}$, so

$$\boldsymbol{v} = \dot{\boldsymbol{r}} = [\dot{x} \;\; \dot{y} \;\; \dot{z}]^{\mathrm{T}}, \; \boldsymbol{\omega} = \begin{bmatrix} \dot{\gamma} & \dot{\beta} & \dot{\alpha} \end{bmatrix}^{\mathrm{T}} \tag{3}$$

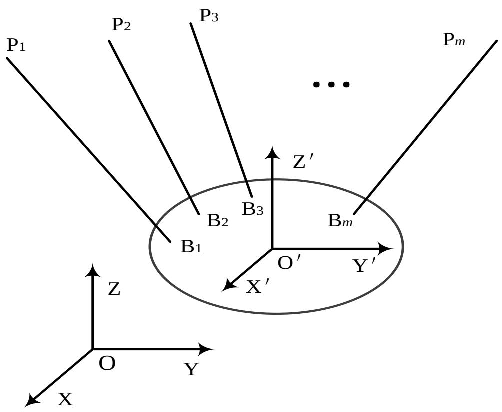


Fig. 1. Schematic diagram of the system

The inertial matrix of the payload in the body coordinate system is ***I***, so the inertial matrix ***I'*** in reference coordinate system is

$$\boldsymbol{I}' = \boldsymbol{R}\boldsymbol{I}\boldsymbol{R}^{\mathrm{T}} \tag{4}$$

where ***R*** represents the rotation transform matrix of the body coordinate system relative to the reference coordinate system.

$$\boldsymbol{R} = \begin{bmatrix} \mathrm{c}\alpha\mathrm{c}\beta & \mathrm{c}\alpha\mathrm{s}\beta\mathrm{s}\gamma - \mathrm{s}\alpha\mathrm{c}\gamma & \mathrm{c}\alpha\mathrm{s}\beta\mathrm{c}\gamma + \mathrm{s}\alpha\mathrm{s}\gamma \\ \mathrm{s}\alpha\mathrm{c}\beta & \mathrm{s}\alpha\mathrm{s}\beta\mathrm{s}\gamma + \mathrm{c}\alpha\mathrm{c}\gamma & \mathrm{s}\alpha\mathrm{s}\beta\mathrm{c}\gamma - \mathrm{c}\alpha\mathrm{s}\gamma \\ -\mathrm{s}\beta & \mathrm{c}\beta\mathrm{s}\gamma & \mathrm{c}\beta\mathrm{c}\gamma \end{bmatrix} \tag{5}$$

where γ 、 β 、 α are body coordinate system rotation angle of X axis, Y *axis and* Z axis relative to the reference coordinate system, c represents cos, and s represents sin. Supposing the tension of each cable is $T_1, T_2, \ldots, T_m$, using Newton-Euler equation for load, the following equation can be obtained.

$$\begin{bmatrix} M\boldsymbol{I_3} & 0 \\ 0 & \boldsymbol{I}' \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{v}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{\omega}\times\boldsymbol{I}'\boldsymbol{\omega} \end{bmatrix} = \boldsymbol{G} + [e_1 \;\; e_2 \ldots e_m]\boldsymbol{T}_{m\times 1} \tag{6}$$

where M is load quality. $\boldsymbol{I}_3$ is 3×3 unit matrix. $\boldsymbol{T} = [T_1 \; T_2 \ldots T_m]$, m dimensional column matrix constituted by the tension of m cables.

By deforming the equation (7), the kinetic equation of the load is,

$$\boldsymbol{AT} = \boldsymbol{C} \tag{7}$$

where ***C*** is *n* dimensional column matrix, which represents the sum of all the external force spinors (including the load gravity, inertial force etc.). Matrix ***A*** meets,

$$\boldsymbol{A} = [\boldsymbol{e}_1\ \boldsymbol{e}_2 \ldots \boldsymbol{e}_m] \in \boldsymbol{R}^{n\times m} \tag{8}$$

Matrix ***A*** is constituted by *m* spinors $\boldsymbol{e}_i$, each $\boldsymbol{e}_i$ is,

$$\boldsymbol{e}_i = \frac{1}{\|\boldsymbol{l}_i\|_2}\begin{bmatrix} \boldsymbol{l}_i \\ (\boldsymbol{R}\boldsymbol{B}_i)\times \boldsymbol{l}_i \end{bmatrix} \tag{9}$$

$$l_i = \|\boldsymbol{P}_i - \boldsymbol{B}_i\| \tag{10}$$

where $\boldsymbol{P}_i$（i=1,2,…m） is the position vector of point P_i in the reference frame. $\boldsymbol{B}_i$（i=1,2,…m） is the position vector of the connection point B_i between the cable and load in the global coordinate system.

3. Analysis on the Stiffness of the Towing System

3.1. Analysis on the Static Stiffness of the Towing System

The static stiffness of the multi-robots collaborative towing system can be considered as the coupling of the static stiffness of multiple cranes and cables. As an example, take 3 industrial robots that collaboratively tow the same mass point load to discuss the static stiffness of the system.

This configuration is a special feature of Figure 2. The two ends of the rope are connected to the end of the robot and the particle respectively. The joint P_i can rotate freely and its coordinates are (x_i, y_i, z_i). The coordinates (x, y, z) are the location point of the lifted object. Next, the stiffness of cable-driven system will be analyzed as per the basic definition of the stiffness.

3.1.1 The Static Stiffness of Robots in the Towing System

The robot model is simplified to be beam elements structure, and it satisfies the material continuity, uniformity and isotropy. The robot deformation equation is established under the condition that material obeys the Hooke's Law and the deformation of the robot end is calculated considering the static stiffness.

$$|\boldsymbol{w}_i| = -\frac{T_i L_{\mathrm{B}i}^3}{3E I_i} \quad i = 1, 2, \cdots, n \tag{11}$$

where $\boldsymbol{w}_i = (x_i - x_{\mathrm{LB}i}, y_i - y_{\mathrm{LB}i}, z_i - z_{\mathrm{LB}i})$ is the deformation vector of the end of the robot i in space, $\boldsymbol{L}_{\mathrm{B}i} = (x_{\mathrm{LB}i}, y_{\mathrm{LB}i}, z_{\mathrm{LB}i})$ is the robot's arm length vector. E_i is the robot's elastic modulus. I_i is the polar moment of inertia of the mechanical arm's section.

Because the force and the arm of force are vertical, the deformation vector of the robot end and the robot's arm length vector are vertical, and their product is zero. The force and the arm of the force are in the same plane, so their mixing product is zero. The new position equations of the robot end are:

$$\begin{cases} \boldsymbol{w_i} \times \boldsymbol{L}_{\mathbf{B}\boldsymbol{i}} = 0 \\ \begin{vmatrix} x_{i\mathrm{N}} - x_{\mathrm{LB}i} & y_{i\mathrm{N}} - y_{\mathrm{LB}i} & z_{i\mathrm{N}} - z_{\mathrm{LB}i} \\ x_{\mathrm{LB}i} & y_{\mathrm{LB}i} & z_{\mathrm{LB}i} \\ x_{\mathrm{LB}i} - x_{\mathrm{D}} & y_{\mathrm{LB}i} - y_{\mathrm{D}} & z_{\mathrm{LB}i} - z_{\mathrm{D}} \end{vmatrix} = 0 \\ (x_{i\mathrm{N}} - x_{\mathrm{LB}i})^2 + (y_{i\mathrm{N}} - y_{\mathrm{LB}i})^2 + (z_{i\mathrm{N}} - z_{\mathrm{LB}i})^2 = 0 \end{cases} \tag{12}$$

where $(x_{i\mathrm{N}}, y_{i\mathrm{N}}, z_{i\mathrm{N}})$ is the end point coordinates of the robot *i* after deformation.

The end point coordinates of robot deform, so the robot stiffness can be obtained. The stiffness equation of the robot end is:

$$K_{\mathrm{R}i} = \frac{T_i}{\sqrt{(x_{i\mathrm{N}} - x_i)^2 + (y_{i\mathrm{N}} - y_i)^2 + (z_{i\mathrm{N}} - z_i)^2}} \tag{13}$$

where $K_{\mathrm{R}i}$ is the stiffness of the towing system robots.

The static stiffness of robot changes, the position of the joint between the robot end and the cable also changes because of the connection between the robot end and the cable. The static stiffness of the system can be obtained as per the static stiffness of cable.

3.1.2. The Static Stiffness of Cables in the Towing System

The deformation of cables occurs in the length of each cable and points to the lifted object. After deformation, the cable is stretched, and it is subject only to tension but not compression. Through the calculation of the static stiffness of cables, the new load position can be obtained.

For the convenience of establishing model, the following assumptions are made:

(1) the connections of the cable with the robot end and the lifted object are ideal universal joints;

(2) the deformation of the cable is linear elastic deformation;

(3) the cable can only sustain tension but not bending moment

The deformation equation of each cable under tension:

$$\Delta L_i = \frac{T_i L_i}{E_i A_i} \quad i = 1, 2, \cdots, n \tag{14}$$

where L_i is the length of cable *i*, ΔL_i is the deformation of cable *i* under tension, and A_i is the cross section of robot *i*.

The static stiffness of the cable can be obtained if the length of the cable changes. The static stiffness of equation of each cable is:

$$K_{\mathrm{L}i} = \frac{T_i}{\Delta L_i} \quad i = 1, 2, \cdots, n \tag{15}$$

where $K_{\mathrm{L}i}$ is the stiffness of the towing system.

The towing system static stiffness is the coupling of robot static stiffness and cable static stiffness. Therefore, the total stiffness of the cable-driven system can be obtained by combining the robot static stiffness and cable static stiffness.

$$K = \sum_{i=1}^{n} \frac{K_{\mathrm{L}i} K_{\mathrm{R}i}}{K_{\mathrm{L}i} + K_{\mathrm{R}i}} \tag{16}$$

Because of the influence of the towing system stiffness, the position of the lifted object and the feasible region of the system are changed simultaneously.

3.2 Analysis on the Dynamic Stiffness of the Towing System

Dynamic stiffness reflects the system's resistance to deformation under dynamic load. Usually, violent vibration will occur when the vibration frequency of external excitation is close to the natural frequency of system structure. Therefore, system's dynamic stiffness is measured by the natural frequency of system structure. The purpose of dynamic stiffness study is to find the natural frequency of the under-constrained multi-robots collaborative towing system and avoid the happening of sympathetic vibration, and then further to study the natural frequency influence on the system by the mechanism's amplitude and response.

3.2.1 The Establishment of Robot Dynamic Stiffness Model

Take the isolation method to study robot dynamic stiffness, when the single robot is connected with the cable, the component force of the gravity will be passed to the robot end by cable. The force of a single robot end can be decomposed as perpendicular to and along the axial direction. When the cable tightly-coupled multi-robots towing system lifts objects, not only the robot end is subjected to the lifted object gravity transmitted along the cable, but also the robot base is subjected to the torque. The robot position is definite at any moment, so the robot can be simplified as the cantilever beam structure and its structure changes at different times as the change of robot's position and pose. The deformation is very small during the vibration when the force on the robot end is decomposed to be perpendicular to the axis, so the structure of the cable tightly-coupled multi-robots towing system can be seen as changeless. It can be assumed that the tension F will not change due to the change of structure when vibration occurs. It can be suggested form the knowledge of material mechanics that its static deflection curve is as follows when the robot end is subject to the lateral force F.

$$y(\xi) = y_{\mathrm{m}}[\frac{3}{2}(\frac{\xi}{L})^2 - \frac{1}{2}(\frac{\xi}{L})^3] \tag{17}$$

where y_{m} is the maximum deflection of the free end, that is:

$$y_{\mathrm{m}} = \frac{FL^3}{3EI} \tag{18}$$

Therefore, the equation stiffness of cantilever beam relative to m is:

$$k_{\mathrm{RV}} = \frac{3EI}{L^3} \tag{19}$$

The force model can be simplified as pull rod when the force that transferred to the robot end by cable is decomposed into axial force.

$$k_{\mathrm{RH}} = \frac{EA}{L} \tag{20}$$

Because the vertical axial force model and the axial force are perpendicular, its joint stiffness is:

$$k_{\mathrm{R}} = \sqrt{k_{\mathrm{RH}}{}^2 + k_{\mathrm{RV}}{}^2} \tag{21}$$

3.2.2 Cable Stiffness Model

The cable can be seen as a spring in the elastic deformation region of the cable (the cable's safe stress range), so the quality-spring motion model can be established as shown in the figure 2.

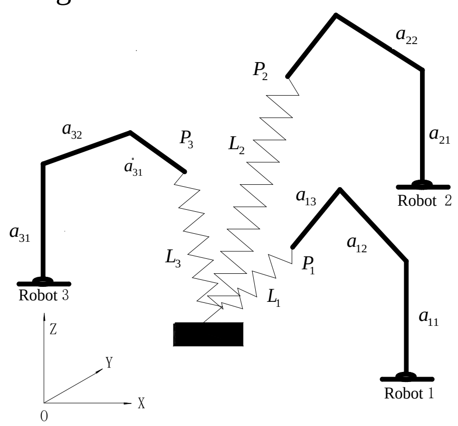


Fig.2. Schematic diagram of cable stiffness model

Because the cable can only withstand unidirectional tension, the towing system can be seen analogously as the rod vibration system with damping, and its stiffness is:

$$k_s = \frac{EA}{L} \tag{22}$$

The deformation of the cable can be decomposed into horizontal and vertical. The horizontal stiffness is k_{sH}, and the vertical stiffness is k_{sV}, so:

$$k_s = \sqrt{k_{sH}^2 + k_{sV}^2} \tag{23}$$

(1) Horizontal Space Stiffness Model

The horizontal deformation model is shown as Fig. 3.

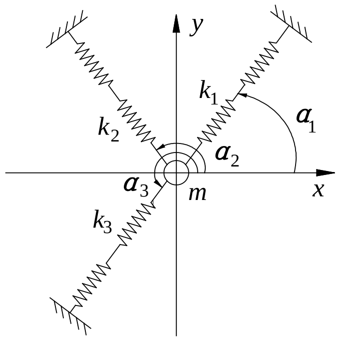


Fig.3. Schematic diagram of horizontal deformation model

Assume the load is a mass point, and the deformation of two springs during the vibration can be independent of each other. When the load has the displacement on x and y direction, the deformation of the three springs can be projected as $x\cos\alpha_1$, $y\sin\alpha_1$, $x\cos\alpha_2$, $y\sin\alpha_2$, $x\cos\alpha_3$, $y\sin\alpha_3$.

So the elastic forces F_1, F_2, F_3 of the spring can be obtained:

$$\begin{cases} F_1 = -k_1(x\cos\alpha_1 + y\sin\alpha_1) \\ F_2 = -k_2(x\cos\alpha_2 + y\sin\alpha_2) \\ F_3 = -k_3(x\cos\alpha_3 + y\sin\alpha_3) \end{cases} \tag{24}$$

The projections of F_1, F_2, F_3 on the x and y axes are respectively k_1, k_2, k_3.

$$\begin{aligned} F_x &= -\sum_{i=1}^{3} k_i(x\cos\alpha_i + y\sin\alpha_i)\cos\alpha_i \\ F_y &= -\sum_{i=1}^{3} k_i(x\cos\alpha_i + y\sin\alpha_i)\sin\alpha_i \end{aligned} \tag{25}$$

Therefore, the system's differential equation of motion on the x and y axes can be established:

$$\begin{aligned} m\ddot{x} + k_{11}x + k_{12}y = 0 \\ m\ddot{y} + k_{21}x + k_{22}y = 0 \end{aligned} \tag{26}$$

In these equations:

$$k_{11} = \sum_{i=1}^{3} k_i \cos^2 \alpha_i \qquad k_{12} = k_{21} = \sum_{i=1}^{3} k_i \sin \alpha_i \cos \alpha_i \qquad k_{22} = \sum_{i=1}^{3} k_i \sin^2 \alpha_i$$

The cable stiffness and robot stiffness are tandem, so its equivalent stiffness factor is:

$$\frac{1}{k_i} = \frac{1}{k_{\mathrm{L}i}} + \frac{1}{k_{\mathrm{R}i}} \qquad i = 1, 2, \cdots, n \tag{27}$$

(2) Vertical Space Stiffness Model

The vertical stiffness of cable and robot are paralleled in the vertical direction, and the parallel stiffness is obtained.

Its stiffness is:

$$\begin{aligned} k = k_1 + k_2 + k_3 \\ m\ddot{z} + kx = 0 \end{aligned} \tag{28}$$

4. Analysis on the Towing System Based on Stiffness

The feasible region of the under-constrained multi-robots collaborative towing system can be seen as the set of positions that the load can reach. As per the geometrical relationship in figure 2, the position coordinates of the load can be obtained:

$$\begin{cases} (x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2 = L_1^2 \\ (x_2 - x)^2 + (y_2 - y)^2 + (z_2 - z)^2 = L_2^2 \\ (x_3 - x)^2 + (y_3 - y)^2 + (z_3 - z)^2 = L_3^2 \end{cases} \tag{29}$$

Meanwhile, the cable tension T_i can be obtained as per the kinetic equation. Supposing the stiffness coefficient of the cable $K_{\mathrm{L}i}$ is known, the deformation of each cable can be worked out. Since the static stiffness of the towing system is coupling of each part's stiffness, the error of lifted object's position can be seen as coupling of each part's deformation. Considering the static stiffness of the towing system, analyzing the deformation and calculating the load's position, the set of positions that the load can reach is the feasible region when the stiffness of the towing system is considered.

In consideration of the system's stiffness, when $n(3 \le n < 6)$ robots lift one load, the load position equation in the system is:

$$\sqrt{(x_i - x_\mathrm{p})^2 + (y_i - y_\mathrm{p})^2 + (z_i - z_\mathrm{p})^2} = L_i + \frac{T_i}{K_{\mathrm{L}i}} \qquad i = 1, 2, \cdots, n \tag{30}$$

In the under-constrained multi-robots collaborative towing system, since each robot has deformed, the feasible region of the robots changes, which means the position of the joint P_i between the cable and the robot end is different. Each cable is different in force and stiffness, so their deformations are different. Coupling these two deformations, the changes of the towing system feasible region after considering the stiffness can be obtained.

Put the coordinates of the deformed robot end joint point and the length of the cable deformation into equation (31), the lifted object position can be worked out when $n \geq 3$. With further calculation, the towing system's feasible region after considering the stiffness can be obtained with Monte Carlo method.

5. Simulation Analysis

Put 3 robots on the circle with a radius of 900mm uniformly, and place the robots' bases on the work platform with height 1100mm. The platform of the placement of 3 robots is as show in figure 4.

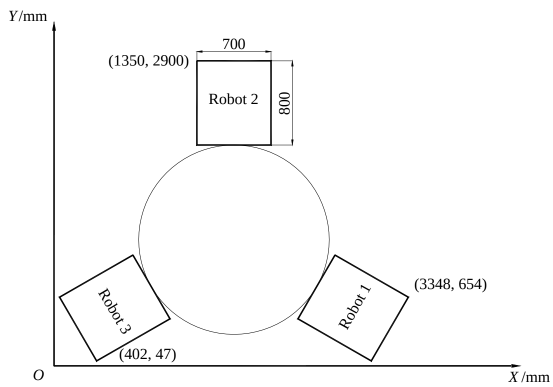


Fig. 4. Schematic diagram of robot working position

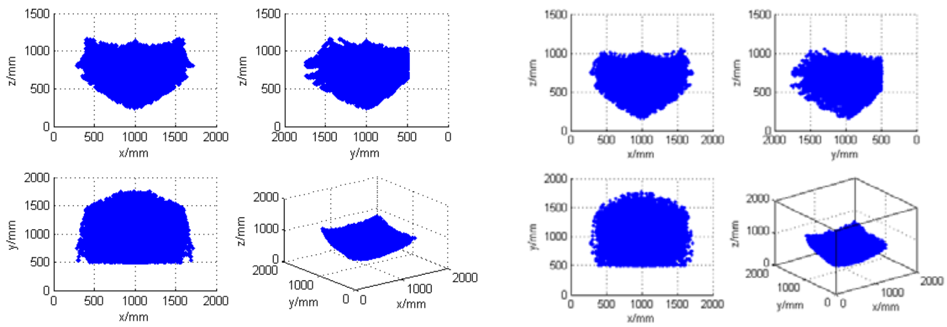
In figure4, the 3 robots' vertical stroke is 400 mm, so the 3 robots' actual height at work is the sum of the height of working stand and the robots' vertical stroke. Firstly, the feasible region of the robot end is determined in the reference. Then, the gravity of the load and the length of the cable are determined, the feasible region of the multi-robots collaborative towing system is calculated in both cases that with and without considering the stiffness.

In the simulation experiment, the Monte Carlo method is used to generate the random end-point of the robot. This method is a random simulation method. The random end-point combination of the robot is generated according to the uniform distribution probability in the workspace of the robot. With the increasing number of sampling samples, the whole space of the end-point of the robot will be covered. Therein, the robots' stroke in the x direction is 800mm, and in the y direction is 600mm. The elasticity modulus E_i is 207 MPa, and the polar moment of inertia I_i is 8.33×106mm^4. When the stiffness of cable-driven system is not considered, the system’s feasible region is as show in figure5(a).

The feasible region in figure5(a) is like an inverted cone, and bulges in the direction close to each robot. The position points of the lifted object are dense in the center and loose around.

When the stiffness of cable-driven system is 3N/mm, the simulation result of the towing system feasible region is as shown in figure 5(b). Since the influence of the system stiffness has been considered, the feasible region changes. It can be seen that the shape of the feasible region has hardly changed, but the spatial position has changed, and the main change is in the Z direction. The changes of the extreme values in feasible region are as shown in table 1.

It can be obtained from table 1 that the maximum and minimum values of the towing system feasible region with considering the stiffness have changed obviously, and the maximum and minimum position points have deviated from their original position. The feasible region of the system varies greatly in Z direction, mainly because the gravity of the lifted object makes the rope deform greatly in Z direction, but not in X-Y direction.



(a) stiffness is no considered (b) stiffness is considered

Fig.5. Diagram of feasible region of towing system

Table 1

The Extreme Value of Feasible Region

Working Condition	Direction	Min(mm)	Max(mm)
Without Considering the Stiffness	x	285	1714.8
	y	500	1741.4
	z	239.1	1167
With Considering the Stiffness	x	283	1711
	y	499.9	1761.4
	z	163.9	1048.1

6. Conclusions

In this paper, the system’s kinetic model can be established according to the space geometric position relations, which can be used as basis for studying the effect of stiffness on motion. The static stiffness model and dynamic stiffness model of the under-constrained multi-robots collaborative towing system have been established, and the influence of stiffness on system movement can be analyzed on the base of stiffness model. Through the comparison of the system's two feasible regions that with and without considering the stiffness, it can be seen that stiffness has a great influence on the under-constrained multi-robots collaborative towing system.

In a word, the analysis and establishment of the stiffness model of the under-constrained multi-robots collaborative towing system has laid the foundation for the further accuracy compensation control, trajectory planning and stability analysis.

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