U.P.B. Sci. Bull., Series D, Vol. 73, Iss. 2, 2011 ISSN 1454-2358

MATHEMATICAL MODELLING OF A BIOLOGICAL WASTEWATER TREATMENT PROCESS. CASE STUDY: THE WASTEWATER TREATMENT STATION OF ROMÂNOFIR S.A. TRADING CO. - TĂLMACIU

Grigore VLAD[1], Dan Niculae ROBESCU[2], Cătălina Raluca MOCANU[3]

Monitorizarea şi controlul sistemelor din staţia de epurare a apelor uzate ***ROMÂNOFIR S.A. Trading Co.*** *se bazează pe o structură ierarhică muti-agent distribuită. În timpul operării normale a staţiei de epurare a apelor uzate unităţile de monitorizare şi control decid, pe baza unui set de parametri colectaţi de controlere, cu care parametri vor trebuie să funcţioneze în scopul de a majora performanţele de epurare dorite în condiţiile date. În scopul de a regla controlerele PID s-a elaborat un model matematic a procesului biologic. Modelul matematic creat se bazează pe patru ecuaţii de masă fundamentale care conduc la dezvoltarea procesului prin separarea efectelor. Rezultatele experimentale ţi determinările analitice din laborator ale parametrilor biologice de funcţionare au fost utilizaţi pentru calibrarea modelului şi pentru ajustarea anumitor coeficienţi ai modelului.*

The monitoring and control system of ***ROMÂNOFIR S.A. Trading Co.****'s wastewater treatment plant is based on a hierarchical multi-agent distributed structure. During the normal operation of the wastewater treatment plant, the structure of the Data Acquisition System has at least mentioned the set of tuning parameters for the controllers with which these ones will have to operate in order to provide the adjustment performances under the given conditions. In order to tune the PID controllers, a mathematical model of the biological process has been drawn up. Themodel is based on four fundamental mass balance equations aiming at developing the processes by separating the effects. The experimental results and the laboratory analytical determinations of the biological step functioning parameters have been used for calibrating the model and for adjusting certain model coefficients.*

Keywords: biological treatment, mathematical modelling, simulation

[1] Eng. S.C. ICPE BISTRIŢA S.A, Romania, e-mail: vlad@icpebn.ro
[2] Prof., University POLITEHNICA Bucharest, Power Engineering Faculty, Romania, e-mail: robescu@hydrop.pub.ro
[3] PhD.eng. University POLITEHNICA Bucharest, Power Engineering Faculty, Bucharest, Romania, e-mail: r.mocanu@yahoo.com

1. Introduction

Mathematical modelling is an important preliminary step for implementing the wastewater treatment processes guiding systems. Measurement accuracy as well as the testing of the control algorithms and check of the performances have to be evaluated for determination of the linearised structure of the model used in designing the control algorithm,. Mathematical modelling remains the most efficient research method, even though it sometimes leads to abstract models that only approximately describe the structure of these processes. The input-state-output models developed in this paper are the most widely used in the simulation technique and in biological processes control, as they are directly compatible to the efficient simulation languages, amongst which MATLAB environment holds the most significant position.

As performed in such cases, a mixed identification of the parameters will be made, namely a model structure is set up out of the partial knowledge of the process and the parameters of the model are experimentally determined. Within such an approach, the analytical identification provides information prior to the experimental identification procedure by means of which the parameters are estimated. This modelling manner is realistic and blends the physical comprehension of the process with the experimental determination of certain parameters.

2. Model elaboration

In this case, the principle scheme of Figure 1 was considered for the mathematical modelling the biological wastewater treatment process.

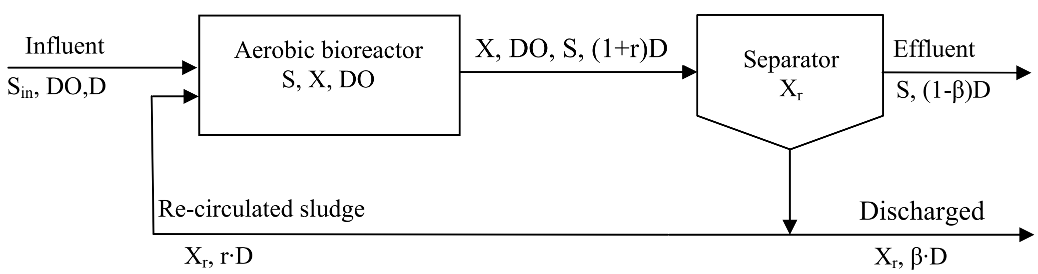


Fig. 1. The block-scheme of the process

The process is mainly constituted by two sequential tanks, an aerated tank and a settler. In the aerated tank (also called the aerobic biological reactor), bacteria and other micro-organisms feed on the organic matter contained in the incoming wastewater that is to be discharged in the tank, thus reducing the level of degradation. The clarifier tank or settler is a gravitational sedimentation tank, where the sludge and the clean residual water are separated. Part of the settled active sludge is re-circulated from the settler into the aeration reactor, so that the content (population) of micro-organisms in the aerobic reactor could be maintained at an appropriate level, in order to continue the wastewater treatment process. The excess of active sludge is discharged from the biological wastewater treatment system.

The provisions below tackle with a simplified model of the installation for the biological wastewater treatment plant. The model is based on the following hypotheses:

- the aeration tank is incessantly stirred and mixed;
- no bio-reaction is taking place in the settler and the sludge is the only component re-circulated in the aerobic biological reactor;
- the oxygen and substrate concentrations are neglected along the re-circulation flow;
- the exit flow rate from the aeration tank equals the sum between the exit flow rate from the separator and the flow rate of re-circulated sludge.

For model determination purposes there are four fundamental equations, which aim at carrying out the processes by separating the effects [1], [2]:

1) the equation linked to the balance of the active sludge at the level of the aeration tank – the process of increasing the active mass of the sludge :

$$\frac{dX(t)}{dt} = \mu(t)X(t) - D(t)(1+r)X(t) + rD(t)X_r(t) \tag{1}$$

2) the equation linked to the mass balance of the substrate – the process of degradation of the carbon-based organic compounds :

$$\frac{dS(t)}{dt} = -\frac{\mu(t)}{Y}X(t) - D(t)(1+r)S(t) + D(t)S_{in} \tag{2}$$

3) the equation linked to the mass balance of the oxygen in the water mass – the oxygen consumed by the bio-chemical degradation of the organic matters, the oxygenation process performed by the oxygen transfer from the air supplied with specific equipment in the water :

$$\begin{aligned}\frac{dD_0(t)}{dt} &= -K_0\frac{\mu(t)}{Y}X(t) - D(t)(1+r)DO(t) + \\ &+ D(t)DO_{in} + \alpha W[DO_{\max} - DO(t)]\end{aligned} \tag{3}$$

4) the equation linked to the balance of the active sludge at the level of the settling tank :

$$\frac{dX_r(t)}{dt} = D(t)(1+r)X(t) - D(t)(\beta + r)X_r(t) \tag{4}$$

In the above equations, the following notations were used::

$X(t)$ – biomass,
$S(t)$ – substrate,
$DO(t)$ – dissolved oxygen,
DO_{max} – maximum amount of dissolved oxygen,
$Xr(t)$ – recycled biomass,
$D(t)$ – dilution rate (the ratio between the flow rate of the influent and the volume of the aeration tank),
S_{in} and DO_{in} – concentrations of dissolved oxygen and of substrate in the mass of the influent,
Y – biomass yield factor,
α – oxygen transfer rate,
W – aeration rate,
K_0 – model constant,
r – the ratio between the re-circulated flow rate and the influent flow rate,
β - the ratio between the waste flow rate and the influent flow rate.

The biomass growth rate μ, is a complex function, which depends on physical, chemical and biological factors. Several analytical laws for modelling this parameter have been suggested. The most popular one is the Monod law [3], however in this case it is supposed that μ depends on the concentrations of substrate, on the concentrations of dissolved oxygen and on a few kinetic parameters, according to the model suggested by Olsson [1]:

$$\mu = \mu_{\max} \frac{S(t)}{K_S + S(t)} + \frac{DO(t)}{K_{DO} + DO(t)} \tag{5}$$

where

μ_{max} - represents the maximum value of the specific growth rate,
k_S – affinity constant, which expresses the dependence of the degradation level on the concentration of pollutant S,
K_{DO} – saturation constants

3. Simulation of the Biological Wastewater Treatment Process

Matlab's toolbox, i.e. SIMULINK, was used to simulate the functioning of the process modelled by means of the four non-linear differential equations presented in the previous chapter. It is a software pack for modelling, simulating and analysing the dynamic systems, the pack being included in the Matlab programming environment. Linear, non-linear, continuous, discrete and hybrid systems can be modelled. SIMULINK has a user's graphic interface for creating models under the form of diagrams built out of blocks, according to techniques of drag and drop type by using the computer mouse. Drawing diagrams is thus simple and intuitive, almost as simple as drawing these diagrams directly on paper. In addition, the laborious mathematical formulation for solving differential equations systems is thus avoided.

The Simulink Model. Model Calibration

Figure 2 presents the SIMULINK model corresponding to the equations listed in the previous sub-section.

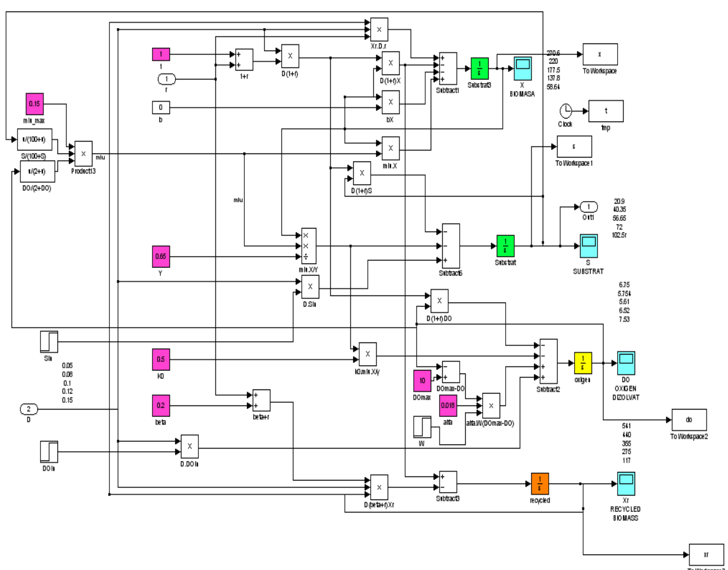


Fig. 2. The Simulink model for studying the behaviour in time of the biomass concentration, of the substrate concentration and of the dissolved oxygen concentration

The results of the measurements on the concentrations of dissolved oxygen and the results of the analytical determinations presented in Table 1 were used to calibrate the model and to adjust certain coefficients in the kinetic model of the process.

Table 1

The quality of the used / physically-chemically and biologically waste waters discharged from SC Românofir SA's platform

No.	Test performed	MU	Biological step entrance	Biological step exit
1.	pH	Unit. pH	7.17	6.52
2.	CCO-Cr	mg/l	363	65.12
3.	CBO_5	mg/l	86	7.54
4.	Total phosphorus	mg/l	5.11	4.51
5.	Total nitrogen	mg/l	55.8	54
6.	Ammonium	mg/lNH_4	7.48	8.25
		mg/l N	6.16	6.8
7.	TC	mg/l	57.2	57.2
8.	TIC	mg/l	10.9	32.6
9.	TOC	mg/l	46.3	20

A volume of 400m^3 of the biological compartment derives from the constructive characteristics of the combined tank.

According to the functional characteristics of the station, the maximum flow rate of wastewater per hour is 50m^3/h. The maximum flow rate of air is approx. 900m^3/h. As for the stationary regime conditions, the inverter was adjusted to 70%, meaning 630m^3/h, and the acting duration is 72% (it is operating for 2h and it is stopped for 45min.). A 450m^3/h medium air flow rate comes out then.

In the following, the model will be calibrated according to the measurements performed at a station input flow rate of 24m^3/h of wastewater. The flow rate of re-circulated sludge: approximately 20m^3/h. But the re-circulation is made by means of an airlift pump connected to the blower, so the medium flow rate re-circulated is: 20 x 0.72 = 14m^3/h, namely r = 14:24 (the entrance flow rate) = 0.6 (60%).

The mathematical model used here is a simplified model of the ASM1 type [5], based on the total concentration of biodegradable organic substance expressed by the BOD (the bio-chemical oxygen consumption). The conversion into mg of BOD can be achieved if a conversion coefficient that differs on the various components is used for the organic substance. Seeing that both the entrance concentration and the exit one are taken into account, the information resulted from the CBO_5 laboratory tests will be used. Under these circumstances, the following comes out for the table values: *Sin*= 86 mg/l.

According to the constructive data of the station, the following are considered: $W = 450$ m^3/h, $D = 24/400$ h^{-1} = 0.06h^{-1}.

The results of the simulation were obtained by using the following values for the coefficients of the kinetic model parameters: $Y = 0.65$; $\beta = 0.2$; K_{DO} = 2mg/l; $K_0 = 0.5$; k_S = 60mg/l; DO_{max} =10mg/l; DO_{in} = 0.5mg/l.

As for the oxygen transfer rate from the air, as a result of the measurements on the dissolved oxygen DO concentration, on the air flow rate W and as a result of calibrating the model on these measurements, a value α = 0,02 m^{-3} comes out.

The following values linked to the stationary regime of the existed results yield from the set of equations representing the model of the process: DO = 6.36 mg/l, S = 7.6 mg/l, X = 120 mg/l and Xr = 240 mg/l.

In order to obtain a value of the organic substance at the output that should be close to the one provided in Table 1, the adjustment of the μ_{max} coefficient to the 0.28 mg/l value was needed.

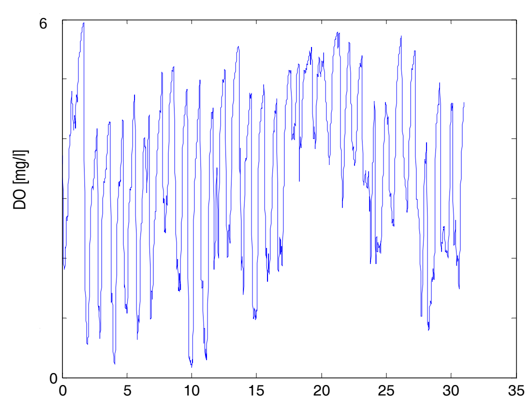


Fig. 3. The evolution of the dissolved oxygen concentration for 31 days

By using the experimental data for the concentration of dissolved oxygen for 31 days, by processing these data and by comparing the dynamics resulted from the transitory regimes comprised in these experimental data to the answers obtained by simulation, the dynamic model related to the dissolved oxygen concentration was validated.

Figure 3 presents the results of the measurements for the concentration of dissolved oxygen (DO). The measurements were performed at 15 minutes time intervals. On the graph, time is represented in days and the dissolved oxygen concentration in mg/l.

In order to compare the results of the simulations with the results obtained experimentally, the evolution of the dissolved oxygen concentration as a response to the modification of the air flow rate supplied by the blower in the aeration tank was extracted. On analysing the functioning of the biological process and the model obtained for it, the conclusion that may be drawn is that the dynamics of modifying the dissolved oxygen concentration as a response to a step-type modification of the flow rate of the air is the fastest one. The step-type modification is in fact the easiest to achieve on the flow rate of air introduced by the blower into the aeration pit.

Furthermore, on analysing the response of the process model for the dissolved oxygen (DO) concentration to a step-type modification of the inserted air flow rate, (*W*), one may notice that it is a response characteristic to a first order system.

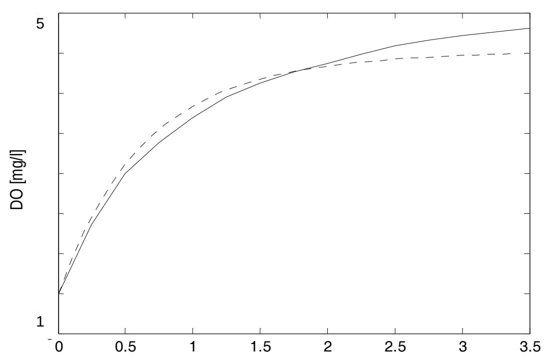
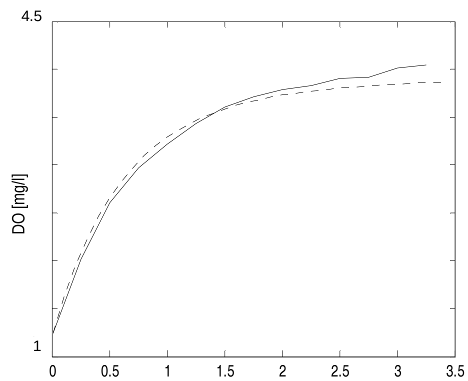


Fig. 4. The simulated and experimental step response on W for the DO at the time moment t_1

Seeing the foregoing, one can extract from the graph presented in Figure 3 a section that shows a typical response for a first order system with the fastest dynamics. Such an answer can be noticed in the measurements comprised in the 1832-1845 time interval, namely from the moment t_1=19.08 days since the beginning of the measurements. As specified, the time interval between two measurements is 15 minutes. The response extracted from the experimental measurements and the response obtained by simulation were represented on the graphic (Figure 4). Time is represented in hours and the dissolved oxygen concentration in mg/l. The response obtained from experimental measurements is represented by a continuous line and the simulated response is represented by a dotted line.

The dynamics of the two responses is noticed to be very close and if analysing further, this dynamics corresponds to that of a system with a a time constant of approx. 0.8 hours, that is 48 minutes. This value falls into the domain specified in the literature for the time constant corresponding to the dynamics of the dissolved oxygen, literature that asserts it lasting dozens of minutes. In order to obtain an amplitude of the simulated response close to the one derived from measurements, the coefficient corresponding to the flow rate of transferring oxygen from the air was modified at the following value: $\alpha = 0.02\ m^{-3}$.

The procedure presented above was repeated for the measurements ranged between the 1253 and 1266 interval, namely from moment $t_2 = 13.05$ days since the beginning of the measurements. The results are presented in Figure 5.



Fig, 5. The simulated and experimental step response on W for the DO at the time moment t_2

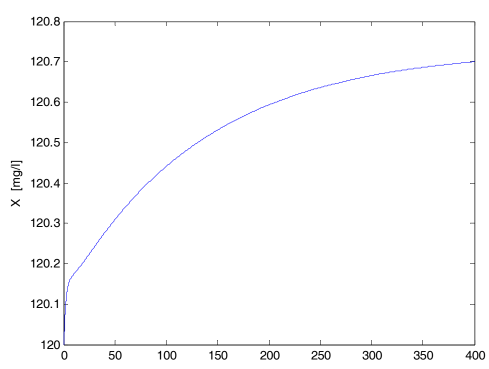


Fig. 6. The variation of the biomass concentration at a 10 % step on the W

One may notice that this time the dynamics of the simulated response is even closer to the one obtained from the experimental measurements.

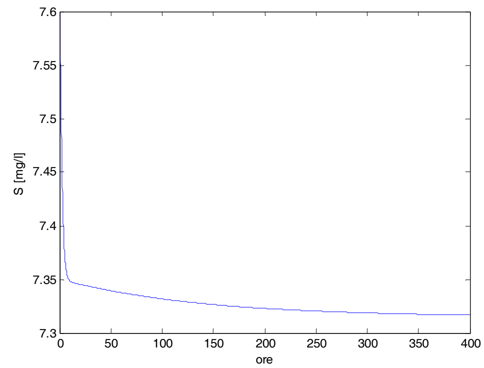


Fig. 7. The variation of the substrate concentration at a 10 % step on the W

The variation in time of the biomass concentration and of the substrate concentration for a 10 % step on the air flow rate *W* is presented in Figure 6 and Figure 7, respectively. At the beginning, one can notice a sudden change (however constant in terms of time, i.e. for approx. 1h) of the values, due to the supplementary contribution of oxygen, followed by a slow evolution (however constant in terms of time, i.e. for approx. 120h), due to the enhancement of the biomass concentration. A weak sensitivity of the substrate concentration as compared to the air flow rate may also be noticed.

Figures 8 and 9 show the variation in time of the biomass concentration and of the substrate concentration for a 10 % step on the re-circulation flow rate.

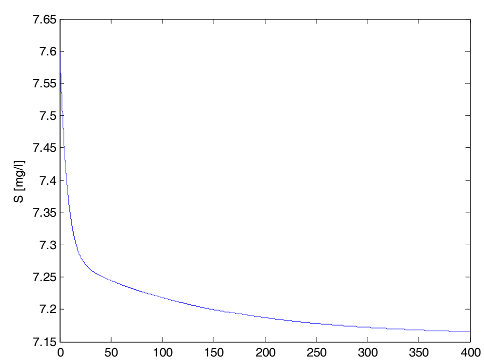


Fig. 8. The variation of the substrate concentration at a 10 % step on *r*

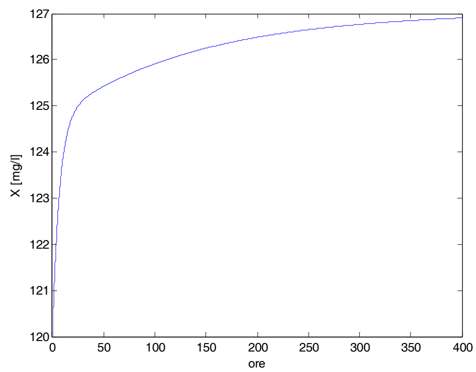


Fig. 9. The variation of the biomass concentration at a 10 % step on *r*

One can notice the change, a slower response in the first phase as compared to a step response on the air flow rate; in exchange, a greater sensitivity of the substrate concentration can be noticed related to the re-circulation flow rate.

4. Conclusions

For the dynamic process modelling from the biological process within S.C. ROMÂNOFIR S.A.'s wastewater treatment station, a model based on the principle of the mass balance for the substrate, for the biomass and for the dissolved oxygen was drawn up. This model enabled the performance of a complete analysis of the biological part of wastewater treatment process behaviour under various operating conditions. The programme for simulating the biological process with active sludge in the aeration tank led to the variation curves of the biomass, substrate concentrations and of the oxygen dissolved in the water mass. This variation of the measurements in time corresponds to the reality of the experimentations.

Acknowledgement

The work has been partial co-funded by the Sectoral Operational Programme Human Resources Development 2007-2013 of the Romanian Ministry of Labour, Family and Social Protection through the Financial Agreement POSDRU/89/1.5/S/62557.

R E F E R E N C E S

[1] *F. Nejjari, A. Benhammou, B. Dahhou and G. Roux*, Non-linear multivariable adaptive control of an activated sludge wastewater treatment process, Int. J. Adapt. Control Signal Process., **Vol. 13**, pp. 347-365, 1999

[2] *S. Caraman, Mihaela Sbarciog, M. Barbu*, Predictive Control of a Wastewater Treatment Process, International Journal of Computers, Communications & Control, **Vol. II**, No. 2, pp. 132-142, 2007

[3] *Diana Robescu, A. Verestoy, S. Lanyi, D. Robescu*, Modelarea şi simularea proceselor de epurare (Mathematical modelling and numerical simulation of wastewater processes), Editura Tehnică, Bucureşti, 2004 (in Romanian).

[4] *U. Jeppsson*, Modelling Aspects of wastewater treatment processes, Ph.D. thesis, Dept. of Industrial Electrical Eng. and Automation, Lund University, Sweden, 1996.

[5] *Diana Robescu, Dan Robescu, Raluca Mocanu, Corina Moga*, Modelling of attached growth biological wastewater treatment processes, Scientific Bulletin of University Politehnica Bucharest, Series C – Electrical engineering, **vol 69**, nr 4/2007, pp 593 – 599, ISSN 1454-234x

[6] *Catalina Raluca Mocanu*, Influence de l'hydrodynamique de l'aérateur canal sur les processus d'épuration des eaux usées, Scientific Bulletin of University Politehnica Bucharest, Series D – Mechanical engineering, **vol 68**, nr 1/2006, pp 43 – 52

[7] *Dan Robescu, Catalina Raluca Mocanu*, Oxygen dispersion modelling in the water transport pipes, Scientific Bulletin of University Politehnica Bucharest, Series D – Mechanical engineering, **vol 68**, nr 1/2006, pp 35 – 42

[8] *Henze M.,Gujer W.,Takashi M., Mark van Loosdrecht*, Activated Sludge Models ASM1, ASM2, ASM2D AND ASM3, IWA Publishing, Alliance House, London SW1H 0QS, UK, 2002