COMPUTATION METHOD FOR ESTABLISHING THE CONTOUR OF A NEW TYPE OF PROFILED ROTOR

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Lucrarea prezintă o metodă de calcul folosită pentru determinarea coordonatelor (x_i, y_i) care definesc profilul exterior al unui nou tip de rotor. Forma profilului exterior a rotorului este calculată doar pentru un cadran al sistemului de axe xOy şi este repetată pentru restul cadranelor.

S-a obținut un program de calcul ce realizează profilul exterior al rotorului, având la bază unele dimensiuni ale acestuia, cum ar fi raza rotorului şi înălțimea pistonului. Coordonatele astfel obținute permit realizarea desenului rotorului şi realizarea lui pe un echipament CNC.

Un ansamblu realizat din doi astfel de rotori montați intr-o carcasă constituie o maşina de lucru (pompă sau ventilator)

A computation method for establishing the coordinates (x_i, y_i) which define the contour of a new type of profiled rotor is presented in the paper; the shape of the outline is established for one of the quadrants of the {xOy} reference system and repeated for the other three quadrants.

Computational software was elaborated; a computational example of the rotor profile boudary is presented for several values of the rotor radius and of the piston height; the coordinates thus obtained allow drawing the rotor for automatic machining on a CNC equipment.

Keywords: rotating piston, profiled rotor outline

1. Establishing the contour of the cavity created by the rotating piston in the adjacent rotor

The cylindrical rotor is endowed with two rotating pistons and two cavities in which the pistons of the adjacent rotor penetrate . The shape of the cavities will be previously established and the shape of the pistons will follow them.

In theory, if the upper rotor (2) would be fixed (Fig. 1), the tip of the lower rotor piston (1) (the point A) would describe on its surface an arc of radius O_1A; actually, the rotor (2) is mobile and in this case the trajectory of the point A will not be an arc, but rather a curve whose trajectory shows the contour of the cavity. The curved trajectory is established in the following way:

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a) Two rotors which rotate with the same speed, in the direction indicated in Fig. 2, are considered. In order to draw the cavity profile, the radius $O_1A = R_r + z$ moves from the initial position ($\theta_i = 0°$) to the final one ($\theta = \theta_f$), meaning that the point A leaves the surface of the upper rotor.

The following notations are used:

R_r – rotor radius [m];

z – piston height [m];

$O_1A = R_c$ - casing radius [m].

b) Geometric coordinates (x_i, y_i) of the point A, which describes the trajectory of the piston tip, are established; the angle θ is subsequently divided into n steps, each of 1° in size (Table 1.).

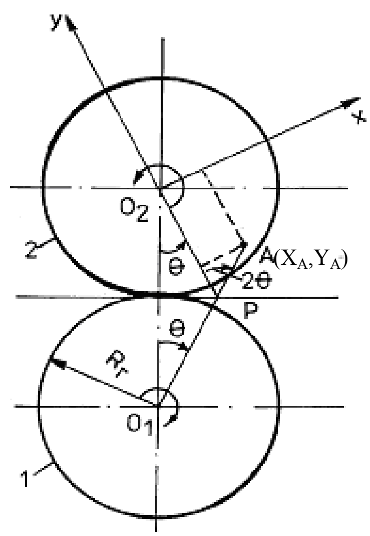

Fig. 1. Schema of computing notation for the outline cavity
1 – lower rotor; 2 – upper rotor;
R_r – rotor radius; θ – angle between the axis *oy* and the line OP.

Fig. 1 shows that $O_1A = R_c = R_r + z$; the coordinates of the point A can be thus written:

$$x_A = \text{PA}\sin 2\theta\ ; \tag{1}$$

$$y_A = -\text{O}_2\text{P} + \text{AP}\cos 2\theta\ ; \tag{2}$$

$$x_A = \left(R_c - \frac{R_r}{\cos\theta}\right)\sin 2\theta\ ; \tag{3}$$

$$y_A = -\frac{R_r}{\cos\theta} + \left(R_c - \frac{R_r}{\cos\theta}\right)\cos 2\theta \ . \tag{4}$$

Or:

$$x_A = \left(R_c - \frac{R_r}{\cos\theta}\right) 2\cos\theta\sqrt{1-\cos^2\theta} \ ; \tag{5}$$

$$y_A = 2R_c\cos^2\theta - 2R_r\cos\theta - R_c \ . \tag{6}$$

It is obtained that:

$$x_A = 2\left(R_C\cos\theta - R_r\right)\cdot\sqrt{1-\cos^2\theta} \ ; \tag{7}$$

$$y_A = 2\cos\theta\cdot\left(R_c\cos\theta - R_r\right) - R_c \ . \tag{8}$$

Relations (7) and (8) represent the parametric equations of the movement of the lower rotor piston tip (1) inside the upper rotor (2).

$$x_A = f(R_c, R_r, \theta) \ ; \qquad y_A = f(R_c, R_r, \theta) \ . \tag{9}$$

The 4th degree equation presented in [4] is obtained if angle θ is eliminated between the two equations (9) and (10):

$$\left(R_c\cdot\frac{R_r+\sqrt{R_r^{\,2}+2R_c(y_A+R_c)}}{2R_c} - R_r\right)^2\cdot \left[1-\left(\frac{R_r+\sqrt{R_r^{\,2}+2R_c(y_A+R_c)}}{2R_c} - R_r\right)^2\right] - \left(\frac{x_A}{2}\right)^2 = 0 \tag{10}$$

The value of angle θ successively increases with the step size of 1°, from an initial value ($\theta_i = 0°$) to a final value θ_f .

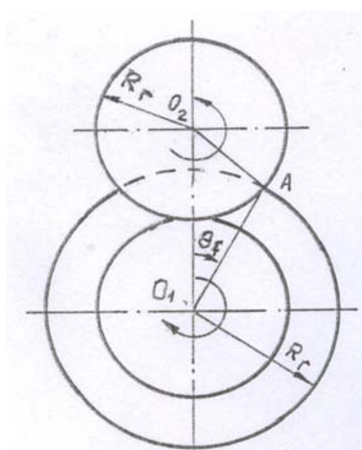

Fig. 2. Computing notations .

Fig. 2 shows that: $O_1A = R_c$; O_2 A $= R_r$; O_1O_2=2 R_r .

The cosine theorem is applied in the triangle O_1O_2 A (Fig.2) in order to establish the value of θ_f :

$$O_2A^2 = O_1A^2 + O_1O_2^2 - 2O_1O_A \cdot O_1O_2 \cos\theta_f \; ; \tag{11}$$

$$\cos\theta_f = \frac{R_c^{\,2} + (2R_r)^2 - R_r^{\,2}}{2 \cdot R_c \cdot 2R_r} \; ; \tag{12}$$

$$\cos\theta_f = \frac{R_c^{\,2} + 3R_r^{\,2}}{4R_c R_r} \; . \tag{13}$$

If the following values are chosen: R_c = 0,08 m, R_r =0,05 m, relation (13) leads to:

$$\cos\theta_f = \frac{0{,}08^2 + 3(0{,}05)^2}{4 \cdot 80 \cdot 50} = 0{,}86875 \, , \tag{14}$$

corresponding to an angle of $\theta_f = 29{,}7^0$.

The following values are introduced in (7) and (8) in order to obtain the coordinates of the points (x_i, y_i): $R_c = 80 \cdot 10^{-3}$ m ; $R_r = 50 \cdot 10^{-3}$ m and $\theta_f = 1°, 2° ... 29°$. The value used for computing the last point is $\theta_f = 29.7^\circ$.

A software based on the previously presented relations generated the values entered in Table 1.

Table 1

Values of the coordinates of the cavity points

Nr. crt.	θ_1	x_i [m]	y_i [m]	Nr. crt.	θ_1	x_i [m]	y_i [m]
0	0	0	-0.02	16	16	0.01482	-0.02828
1	1	0.00104	-0.02003	17	17	0.01549	-0.02930
2	2	0.00209	-0.02013	18	18	0.01612	-0.03038
3	3	0.00312	-0.02030	19	19	0.01669	-0.03151
4	4	0.00415	-0.02053	20	20	0.01722	-0.03268
5	5	0.00517	-0.02083	21	21	0.01769	-0.03390
6	6	0.00618	-0.02120	22	22	0.01811	-0.03517
7	7	0.00716	-0.02163	23	23	0.01847	-0.03647
8	8	0.00813	-0.02212	24	24	0.01877	-0.03782
9	9	0.00907	-0.02268	25	25	0.01902	-0.03920
10	10	0.00999	-0.02330	26	26	0.01920	-0.04062
11	11	0.01088	-0.02398	27	27	0.01932	-0.04207
12	12	0.01174	-0.02473	28	28	0.01937	-0.04355
13	13	0.01257	-0.02553	29	29	0.01936	-0.04506
14	14	0.01336	-0.02639	30	29,7	0.01931	-0.04613
15	15	0.01411	-0.02731				

Due to the fact that the positioning accuracy of a part on CNC equipment is about 10^{-5} m [2], five decimals are used in the computation.

Data provided in Table 1 lead to the point-by-point drawing of the curve AB (Fig. 3), which represents half of the outline of a cavity existing in each rotor.

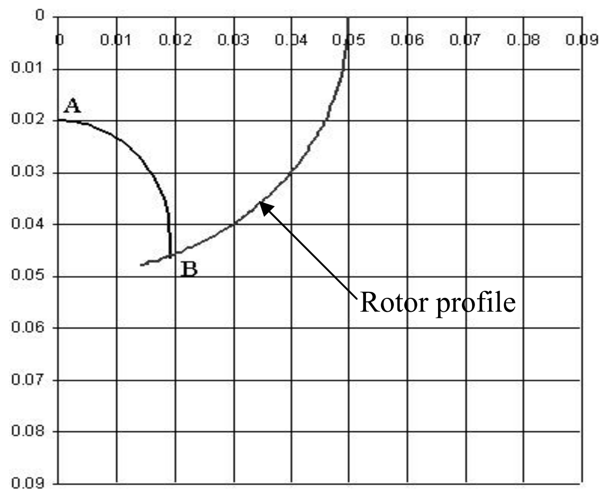

Fig. 3. The outline of the cavity machined in the rotor .

The shape of the rotor outline on the sectors BC and CD must be subsequently established (Fig. 4).

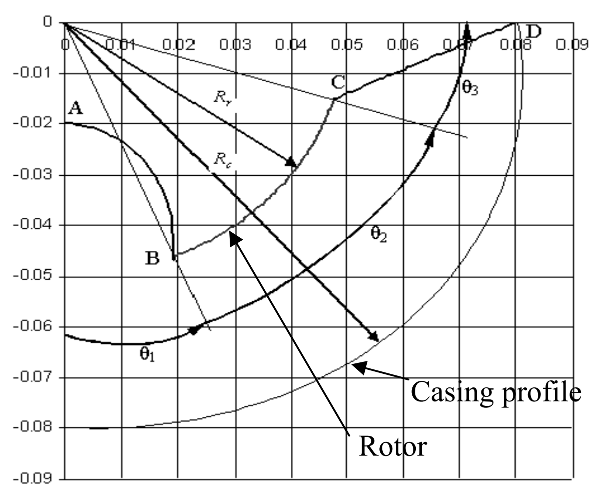

Fig. 4. The contour of the rotor profile and casing profile, drawn in the fourth quadrant of the $\{xOy\}$ system .

2. Establishing the coordinates of the points situated on a circular arc belonging to the architecture of the rotor

The arc BC (Fig. 4) belongs to a circle of radius R_r =0.05 m, and the central angle θ_2 equal to:

$$\theta_2 = 90^0 - \left(\theta_1^* + \theta_3\right) . \tag{15}$$

Table 1 is used in order to establish the value of $\theta_1 = \theta_1^*$ (particular value). The following coordinates result for θ_1=29.7°:

$$x_B = 0.01931 \text{ [m]} ; \quad y_B = -0.04613 \text{ [m]} . \tag{16}$$

The angle θ_1^* between the axis *oy* and the line OB (Fig. 4) will be equal to:

$$tg\theta_1^* = \frac{x_B}{y_B} = \frac{0.01931}{0.04613} = 0.41859 . \tag{17}$$

Thus: θ_1^*= 22.7°.

In order to establish the value of θ_3, it is observed that the point C founds itself at the intersection of line DC and circle of radius R_r =0.05 m. The coordinates of the point C will be computed as follows.

We consider that the rotating piston has a triangular shape, symmetric in respect to the *Ox* axis. Because the opening angle of the cavity equals $2 \cdot 22{,}7° = 45{,}4°$, the angle of the piston tip is chosen as equal to $50°$. The line DC will be accordingly bent with 25° towards the horizontal line (Fig. 5).

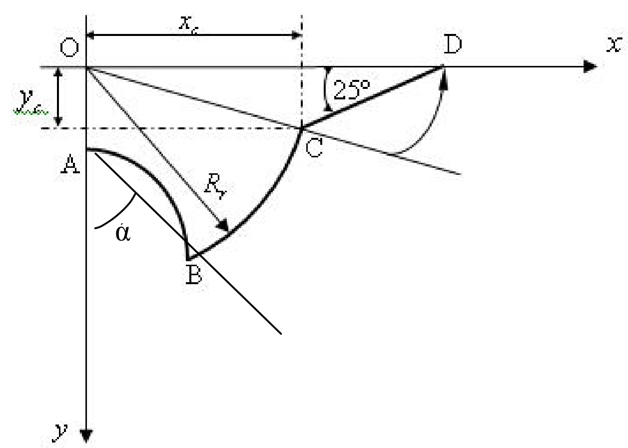

Fig. 5. The contour of the rotor profile in the fourth quadrant of the $\{xOy\}$ system.

If the cosine theorem is applied in the triangle OCD, the following result is obtained:

$OC^2= CD^2+OD^2-2CD\cdot OD\cos25°$; (18)

$R_r^{\ 2} = CD^2+(2R_r)^2-2CD\cdot 2R_c\cos25°$; (19)

$CD^2-144.96CD+3900=0$. (20)

It results:

$CD=35.695\ mm = 35.695\cdot 10^{-3}\ m$.

The coordinates of the point C will be:

x_c= OD-CDcos25°=80-35.695·0.906=47.649 mm= $47.649\cdot 10^{-3}$ m ; (21)

y_c=-CDsin25°=-35.695·0.4226=-15.085 mm= $-15.085\cdot 10^{-3}$ m; (22)

$$tg\theta_3 = \frac{y_c}{x_c} = \frac{15.085}{47.649} = 0.31658\ ; \qquad (23)$$

θ_3=17.55°.

Thus:

θ_2=90°-(22.7 + 17.55)=49.75° . (24)

The angle θ_2 changes accordingly from θ_1=22.7° to 72.45°.

The coordinates of the points situated on the circle arc BC of the rotor outline can be obtained from the following equations:

$x = R_r \cos\theta_2$;

$y = -R_r \sin\theta_2$. (25)

The results of the computation are presented in Table 2.

Fig. 4 shows that the first values of the angle θ_2 are "superposed" on the values of θ_1 in the range of 22°-30°, but the computed results are different because they regard the arc BC.

Table 2

Coordinates of the points situated on a circle of radius R_r

Nr. crt	θ_2	x_i [m]	y_i [m]	Nr. crt	θ_2	x_i [m]	y_i [m]
1	22.7	0.01929	-0.04612	27	48	0.03715	-0.03345
2	23	0.01953	-0.04602	28	49	0.03773	-0.03280
3	24	0.02033	-0.04567	29	50	0.03830	-0.03213
4	25	0.02113	-0.04531	30	51	0.03885	-0.03146
5	26	0.02191	-0.04493	31	52	0.03940	-0.03078
6	27	0.02269	-0.04455	32	53	0.03993	-0.03009
7	28	0.02347	-0.04414	33	54	0.04045	-0.02938
8	29	0.02424	-0.04373	34	55	0.04095	-0.02867
9	30	0.025	-0.04330	35	56	0.04145	-0.02795
10	31	0.0257	-0.04285	36	57	0.04193	-0.02723
11	32	0.02649	-0.04240	37	58	0.04240	-0.02649
12	33	0.02723	-0.04193	38	59	0.04285	-0.02575
13	34	0.02795	-0.04145	39	60	0.04330	-0.025

14	35	0.02867	-0.04095	40	61	0.04373	-0.02424
15	36	0.02938	-0.04045	41	62	0.04414	-0.02347
16	37	0.03009	-0.03993	42	63	0.04455	-0.02269
17	38	0.03078	-0.03940	43	64	0.04493	-0.02191
18	39	0.03146	-0.03885	44	65	0.04531	-0.02113
19	40	0.03213	-0.03830	45	66	0.04567	-0.02033
20	41	0.03280	-0.03773	46	67	0.04602	-0.01953
21	42	0.03345	-0.03715	47	68	0.04635	-0.01873
22	43	0.03409	-0.03656	48	69	0.04667	-0.01791
23	44	0.03473	-0.03596	49	70	0.04698	-0.01710
24	45	0.03535	-0.03535	50	71	0.04727	-0.01627
25	46	0.03596	-0.03473	51	72	0.04755	-0.01545
26	47	0.03656	-0.03409	52	72.45	0.04767	-0.01511

3. Establishing the coordinates of the points situated on the bank of the rotating piston

The sector CD of the profiled rotor contour constitutes a bank of the rotating piston; it is a line passing through the points C and D with known coordinates :

$$\mathrm{x_c} = 47.67 \cdot 10^{-3}\,\mathrm{m}\,; \qquad x_D = 0.08\,\mathrm{m}\,;$$

$$y_c = -15.11 \cdot 10^{-3}\,\mathrm{m}\,; \qquad y_D = 0\,\mathrm{m}\,.$$

The coordinates of the points situated on this line are obtained as the intersection of the line CD with the line of equation $y = mx$ passing through the origin of the coordinate system (Fig. 4) and having the direction of the casing radius (R_c). This line will describe the angle θ_3 step by step, from 72.4° to 90°. Consequently, m=tgθ_3.

The coordinates of the points situated on the line CD are obtained by solving the system of equations of the two lines:

$$\begin{cases} \dfrac{x - x_c}{x_D - x_c} = \dfrac{y - y_c}{y_D - y_c}; \\ y = tg\theta_3 \cdot x. \end{cases} \tag{26}$$

The computed results are presented in Table 3.

Table 3

Coordinates of the points situated on a side of the rotating piston

Nr. crt	θ_3	x_i [m]	y_i [m]	Nr. crt	θ_3	x_i [m]	y_i [m]
1	72.45	0.0476	-0.01507	11	82	0.0656	-0.00910
2	73	0.0494	-0.01424	12	83	0.0674	-0.00819
3	74	0.0512	-0.01394	13	84	0.0692	-0.00722
4	75	0.053	-0.01357	14	85	0.071	-0.00618

5	76	0.0548	-0.01313	15	86	0.0728	-0.00507
6	77	0.0566	-0.01262	16	87	0.0746	-0.00390
7	78	0.0584	-0.01205	17	88	0.0764	-0.00266
8	79	0.0602	-0.01141	18	89	0.0782	-0.00136
9	80	0.062	-0.01071	19	90	0.08	0
10	81	0.0638	-0.00994				

The last value presented in Table 3 is equal to 90°, corresponding to the fourth quadrant of the $\{xOy\}$ reference system.

The three tables can be unified in a single one. The values listed in that table allow the point-by-point drawing of the contour of the rotor profile in the fourth quadrant of the $\{xOy\}$ reference system ; this boundary is noted as ABCD (Fig. 6).

The sector A'B'C'D' is obtained symmetrically to the axis *Ox*; the whole contour of the rotor profile is obtained mirroring the profile ABCDC'B'A' around the *Oy* axis (Fig. 6).

The chosen constructive solution proposes a triangular shape for the piston, as the simplest possible shape; the shape of the piston may be parabolic or may be given by a curve defined through a series of points as well.

The computing mathematical model for establishing the shape of the rotor contour allowed the elaboration of a machining technology of the rotor [3] , [4] and subsequently to its manufacturing [5].

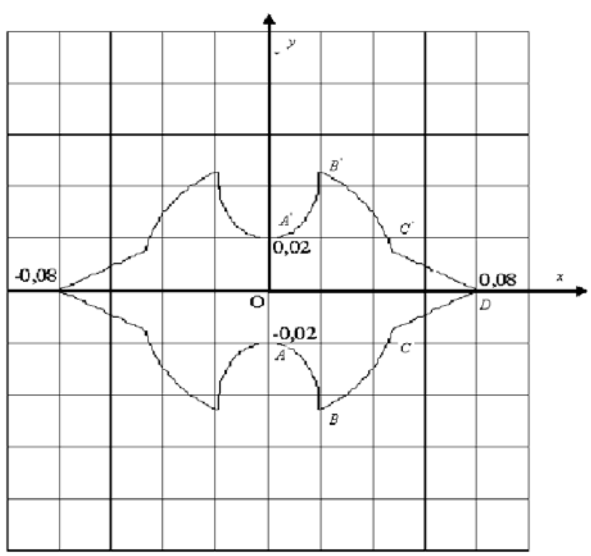

Fig. 6. The contour of the rotor profile.

4. Conclusions

1. The mathematical model developed for the computation of the coordinates of the rotor profile contour is a general one; it can be modified for different sizes of the rotor, new values of R_r and R_c being only needed.

2. After establishing the coordinates (x_i, y_i) a CNC program for the machining of the rotor may be further elaborated.

R E F E R E N C E S

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