RE-ORIENTATION OF ORTHOTROPIC AXES IN SHEET METAL USING A DEVELOPED METHOD BASED ON A SIMPLE SEMI GEOMETRICAL MODEL

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Experimental investigations demonstrate that when orthotropic sheet metal specimens are subjected to off-axis uniaxial tension, re-orientation of the axes occurs at moderate tensile plastic strain levels, while orthotropic symmetries are preserved. In this work this phenomenon has been investigated and simulated by using a developed method which is based on a simple semi geometrical model. In this model, a 3-elements mechanism with arbitrary angle represents two substructure textures of a cold rolled sheet metal. An explicit formulation has been obtained to determine the rotation angle of the orthotropic symmetry axes of the sheet metal under off-axis uniaxial tension. Also a simple method has been proposed to determine the direction of the orthotropic axes rotation under off-axis uniaxial tension. It is shown that the proposed formulations can be used to calculate the magnitude and direction of the rotation of the orthotropic axes under this type of loading and the results show good agreement with the experimental data.

Keywords: Orthotropic axes, Semi geometrical method, Re-Orientation, Sheet metal

1. Introduction

The theory of anisotropic plasticity of materials is a well known topic with great technological significance. The first work on the orthotropic material with representation of a yield function was proposed by Hill in 1950 [1]. Anisotropy indicates the difference of a property or response, e.g. the yield stress or the stress-strain response of the material in different directions. Therefore it can be characterized by two fundamental characteristics, its intensity and its orientation [2]. Re-orientation of the orthotropic directions is a very significant feature that is related to the secondary anisotropic property and its investigation is the major concern of this paper. Although both the intensity and orientation of the orthotropic axis are important, but most of the previous research works address the former subject. For example, extensive experimental works have been carried out in understanding the yield and flow behavior of the cold rolled anisotropic materials [3-4]. Also, a number of researches have been reported based on theoretical methods in this subject [5-13]. In addition, anomalous behavior

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observed in aluminum by Woodthrope and Pearce [14] motivated extensive studies on the yield functions [14-25].

There are also few works that address orientation of the ortotropic axis. For example, Kim observed that during the twisting of the cold drawn tubes, the orthotropic symmetry is maintained and the orthotropy axes are rotated in the twisting direction [26]. Boehler et al., Kim and Yin and Losilla et al. have shown that the cold rolled sheet metals under uniaxial off-axes tension loading remain approximately orthotropic but there is a large in-plane orientation of the orthotropic symmetric axes [27-29].

Bunge and Nielsen have used both experimental techniques and ODF measurement approaches to obtain the magnitude of the texture rotation of the orthotropic Aluminum cold rolled sheet specimen subjected to off-axis stretching [30].

Tugcu and Neale studied the orthotropic axes rotation using the orthogonal tensor R that specified the polar decomposition of the deformation with no experimental evidence [31]. Attempts have also been made to solve the problem of orthotropic re-orientation [32-33].

The main theoretical tool to explain orientation of orthotropic axes is the concept of plastic spin that at first suggested by Mandel [34]. This phenomenon is described and simulated by Dafalias using a simple theory of plasticity, which combines Hill's quadratic yield criterion for orthotropic sheet metals with the concept of plastic spin as an essential constitutive component for the orientational evolution of the anisotropic tensorial internal variables [2].

Tong et al. presented a simplified anisotropic plasticity theory which is used to explain the anisotropic flow behavior of the orthotropic polycrystalline sheet metals under off-axis uniaxial tension. Their theory was formulated in terms of the intrinsic variables of principal stresses and a loading orientation angle and its uniaxial tension. They acquired a suitable analytical formula of macroscopic plastic spin proposed for orthotropic sheet metals with preserved but rotated orthotropic symmetry axes under off-axis uniaxial tension [35]. Also Tong in the years 2005 and 2006 presented a phenomenological theory based on the plastic spin concept, Fourier series and concepts of microscopic polycrystalline plasticity for describing the anisotropic plastic flow of orthotropic polycrystalline aluminum sheet metals under plane stress [36, 37]. Although the models mentioned above are based on experimental or theoretical methods, their application in industry is difficult due to their complexity. Therefore, in this work, the orientation of orthotropic axis is described and simulated using a simple geometrical method and a 3-elements system of substructure textures of cold rolled sheet metal. For this purpose, an explicit analytical formulation is proposed to determine the rotation angle of the orthotropic symmetry axes of a sheet metal under off-axis uniaxial tension. Also, associated material anisotropic constants have been

obtained and compared with those given by Kim and Yin based on experimental tests [28]. Finally, a simple method is developed with associated equations to determine the direction of the orthotropic axes rotation.

2. Description of problem

Are the orthotropic symmetries preserved when orthotropic sheet metals are subjected to in-plane off-axis uniaxial tension loading where the direction of loading is fixed with respect to the initial orthotropic axes? Moreover, if orthotropic symmetries are preserved, do the orthotropic axes remain the same, or they rotate? If yes, towards which direction? Finally, how fast do they rotate in relation to the plastic strain induced by the non-coaxial loading? [2]

The above questions should be answered by experimental observations before developing a theory of anisotropic re-orientation. These answers can show whether the theoretical objective is worth pursuing or not.

Kim demonstrated that the answer of the first question is positive [26]. Kim and Yin's experiments [28] corroborate the results of Kim's experiment [26]. In the next step, they answered to the second and third questions. Meanwhile, if the answer to the third question is that the orthotropic axes do rotate but very slowly, again one may reason that for practical purposes, the orthotropic orientation may be assumed to remain fixed.

Kim and Yin performed an experimental method to study the cold rolled sheet metals anisotropy with tensile tests at different angles to the rolling direction [39]. They utilized variation of uniaxial yield stresses with tensile loading axis orientation which can be used to set up orthotropic symmetry. They selected cold rolled sheets of low carbon steel widely used in the automotive industry for the tests. This alloy has moderate initial orthotropy. To increase the degree of orthotropy, full size sheets were stretched along the rolling direction by 3 and 6 percent of tensile strains. Then tensile specimens were cut at an angle ψ from the rolling direction (R.D.). Fig. 1 presents a schematic diagram of the specimens. The R.D. and T.D. (Transverse Direction) are the initial orthotropic axes.
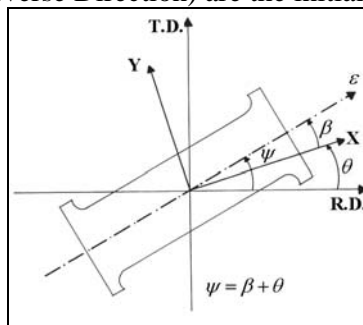

Fig 1. Schematic presentation of the tensile specimen and the different directions and related angles, ψ, β, and θ.

Three values of ψ were chosen at 30, 45 and 60 degrees which for each value of ψ, the specimens were subjected to a tensile second pre-strain ε of magnitude 1, 2, 5, and 10 percent along their axis. To investigate the possible re-orientation of the initial orthotropic directions R.D. and T.D. due to the mentioned pre-straining, small size specimens were cut from the pre-strained specimens at different angles and tested in tension. For tracing the evolution of orthotropic symmetries and orientation by following the "shift", with respect to the second pre-strain ε, the record of tensile yield stress distribution are carried out for the small specimens at each ε and ψ. It was shown that the answers of two questions mentioned at the beginning of this section are positive. Meanwhile, it was possible to investigate the evolution of the orientation of the orthotropic axes X and Y with respect to the second pre-strain ε. This can be done by calculating the shift of the symmetrical yield stress distribution using their angle β from the ε direction (see Fig. 1).

3. Modeling of magnitude of rotation of orthotropic axes

In this paper for the modeling of the orientation of the orthotropic axes of sheet metals due to the uniaxial off-axis tension (second pre-strain), a semi-geometrical model is used which is based on the concept of substructure textures (see Fig. 2).

Two arbitrary stripes of substructure texture of a second pre-strained specimen are simulated using the 3-element mechanism (see Fig. 2) in which one of the elements is grand and fixed and the other two elements can move. The angle between elements a and b is arbitrary and it is not necessary to be at 45 degree. The horizontal element AC shown in Fig. 2 indicates the orthotropic axis and its rotation represents the orientation of orthotropic axes under uniaxial off-axis tension and the element AB is fixed.
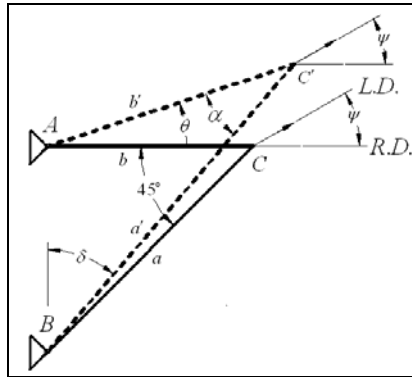

Fig. 2. Semi-geometrical model which is based on the concept of substructure textures and is used to modeling of the orientation of the orthotropic axes.

Two joints of A and B have only one degree of freedom i.e. AC can solely rotate around joint A and therefore BC can rotate around joint B. Joint C has three degrees of freedom i.e. both AC and BC can rotate around C and joint C can move both horizontally and vertically. When second plastic pre-strain is applying, both AC and BC start yielding contemporaneously, according to Fig. 2. Consequently, the system reaches a new position which is shown with dashed lines in Fig. 2. In this simulation, the symbols of ψ, θ and ε are equivalent to Kim and Yin's [28] experimental quantities and are defined in Fig. 1. Due to the second pre-strain experiments of Kim and Yin's the elements of BC and AC are subjected to strains of ε_1^p and ε_2^p respectively. With the assumption of $\overline{AB} = 1$, following equations can be obtained:

$$a' = \sqrt{2}\left(1+\varepsilon_1^p\right)$$
$$b' = \left(1+\varepsilon_2^p\right)$$
(1)

$$\theta b' = \left(\frac{\pi}{4}-\delta\right)a'$$
(2)

All the geometrical parameters in these equations are defined in Fig.s 1 and 2. By the substitution of Equation (2) into (1), it can be written as:

$$\theta = \frac{\sqrt{2}\left(\frac{\pi}{4}-\delta\right)\left(1+\varepsilon_1^p\right)}{\left(1+\varepsilon_2^p\right)}$$
(3)

And from triangle of ABC' it can be shown that:

$$a'\sin\alpha = \cos\theta$$
(4)

By substitution of Equation (1) into (4), it can be written as:

$$\sin\alpha = \frac{\cos\theta}{\sqrt{2}\left(1+\varepsilon_1^p\right)}$$
(5)

And from triangle of BCC' it can be demonstrated that:

$$\frac{\sin\left(\frac{\pi}{4}-\delta\right)}{\frac{a'}{2}} = \frac{\sin\left(\pi-2\left(\frac{\pi}{4}-\delta\right)\right)}{a}$$
(6)

Moreover, the substitution of Equation (2) in the above relation leads to:

$$\cos\left(\frac{\pi}{4}-\delta\right) = \frac{1}{1+\varepsilon_1^p}$$
(7)

The result of the combining Equations (3) and (7) is:

$$\theta = \sqrt{2}\frac{1+\varepsilon_1^p}{1+\varepsilon_2^p}\arccos\left(\frac{1}{1+\varepsilon_1^p}\right)$$
(8)

And with the assumption of

$$\varepsilon_2^p = K_2\varepsilon, \varepsilon_1^p = K_1\varepsilon$$
(9)

In addition, by substitution of Equation (9) into (8) gives:

$$\theta = \sqrt{2}\,\frac{1+K_1\varepsilon}{1+K_2\varepsilon}\arccos\left(\frac{1}{1+K_1\varepsilon}\right) \tag{10}$$

Where at above equations K_1 and K_2 are material constants.

Equation (10) should satisfy two following initial and boundary conditions:

If $\varepsilon = 0$ then $\theta = 0$, which is satisfied in Equation (10).

      For any orthotropic sheet metal beside the initial orthotropic axes, there is a direction, ψ_{eq} or equivalent angle that if the loading is applied in this direction, the initial orthotropic axes do not show any rotation. Also, there is a limit for the second pre-strain and if it is reached, the orthotropic axes do not rotate; and therefore, one of the orthotropic axes coincide with the secondary loading direction, in other words, this boundary condition for $\psi < \psi_{eq}$ leads to $\theta(\varepsilon_L) = \psi$

and for $\psi > \psi_{eq}$ results in $\theta(\varepsilon_L) = \dfrac{\pi}{2} - \psi$.

The second boundary condition for $\psi < \psi_{eq}$ in Equation (10) is obtained as:

$$\psi = \sqrt{2}\,\frac{1+K_1\varepsilon_L}{1+K_2\varepsilon_L}\arccos\left(\frac{1}{1+K_1\varepsilon_L}\right) \tag{11}$$

Where strain limit is shown with ε_L . Above equation can be re-written as:

$$K_2 = \frac{1}{\psi\varepsilon_L}\left[-\psi + \sqrt{2}\left(1+K_1\varepsilon_L\right)\arccos\left(\frac{1}{1+K_1\varepsilon_L}\right)\right] \tag{12}$$

And by substitution of Equation (12) into Equation (10) and for simplicity with the assumption of $K_1 = K$, it is easily shown than:

$$\theta = \frac{\sqrt{2}\left(1+K\varepsilon\right)\arccos\left(\dfrac{1}{1+K\varepsilon}\right)}{1+\dfrac{\varepsilon}{\psi\varepsilon_L}\left[-\psi + \sqrt{2}\left(1+K\varepsilon_L\right)\arccos\left(\dfrac{1}{1+K\varepsilon_L}\right)\right]} \tag{13}$$

The above equation is valid for $\psi < \psi_{eq}$ and for $\psi > \psi_{eq}$, the same procedure leads to:

$$\frac{\pi}{2} - \psi = \sqrt{2}\,\frac{1+K_1\varepsilon_L}{1+K_2\varepsilon_L}\arccos\left(\frac{1}{1+K_1\varepsilon_L}\right) \tag{14}$$

As a result:

$$K_2 = \frac{1}{\left(\dfrac{\pi}{2}-\psi\right)\varepsilon_L}\left[\left(\psi - \frac{\pi}{2}\right) + \sqrt{2}\left(1+K_1\varepsilon_L\right)\arccos\left(\frac{1}{1+K_1\varepsilon_L}\right)\right] \tag{15}$$

By substitution of Equation (15) into (10) and again for simplicity assuming that $K_1 = K$, it is easily shown that for $\psi > \psi_{eq}$:

$$\theta = \dfrac{\sqrt{2}\,(1+K\varepsilon)\arccos\left(\dfrac{1}{1+K\varepsilon}\right)}{1+\dfrac{\varepsilon}{\left(\dfrac{\pi}{2}-\psi\right)\varepsilon_L}\left[(\psi-\dfrac{\pi}{2})+\sqrt{2}\,(1+K\varepsilon_L)\arccos\left(\dfrac{1}{1+K\varepsilon_L}\right)\right]} \tag{16}$$

Above relation, have two distinct and independent constants, which consist of, K and ε_L which can be determined using experimental data. In practice, K_1 and K_2 used in Equation (11) are replaced by K and ε_L in Equation (13). An advantage of this replacement is that the value of ε_L, can be determined easily using experimental data and therefore, only one unknown constant (K) is remained which can be calculated using data fitting of the experimental tests.

4. Re-orientation direction of the orthotropic axes under uniaxial off-axis tension loading

In the problem of re-orientation of the orthotropic axes under uniaxial off-axis tension, both the magnitude and direction of the orientation should be determined.

Dafalias [2] has discussed on the direction of the re-orientation of the orthotropic axes under uniaxial off-axis tension using the relation which Hill represented in 1950 [1] in the form of $\tan^2\psi_{eq}=(g+2h-1)/(f+2h-1)$ where f, g and h are normalized coefficients of Hill's quadratic criteria. Dafalias showed that for the reported values of f, g and h by Kim and Yin [28], $\psi_{eq}=44.68°$.

In this investigation, we propose a simple relationship for the problem of orientation direction of orthotropic axes under off-axis uniaxial tension. To determine the relation for ψ_{eq}, we suppose that the yield stress for the initial sheet in ψ_{eq} is σ so the following equilibrium equations are satisfied:

$$\begin{aligned}\sigma\sin^2\psi_{eq}&=\sigma^Y_{T.D.}\\\sigma\cos^2\psi_{eq}&=\sigma^Y_{R.D.}\end{aligned} \tag{17}$$

Where $\sigma^Y_{R.D.}$ and $\sigma^Y_{T.D.}$ are the yielding stress for initial sheet in the rolling and transverse directions, respectively. Dividing both sides of the equilibrium equations give:

$$\psi_{eq}=\arctan\sqrt{\dfrac{\sigma^Y_{T.D.}}{\sigma^Y_{R.D.}}} \tag{18}$$

The equivalent angles are obtained based on Equation (18) and Kim and Yin's [28] experimental data which is given in Table (1).

Variation of equivalent angle with initial pre-strain obtained using equation (18)

Initial pre-strain	0%	3%	6%
ψ_{eq} (equivalent angle)	45.22	44.6	43.9

By combination of Equation (13) and (18) the final relationship for orientation of orthotropic axes under off-axes uniaxial tension leads can be obtained as for $\psi < \psi_{eq}$:

$$\theta = \frac{\sqrt{2}\left(1+K\varepsilon\right)\arccos\left(\frac{1}{1+K\varepsilon}\right)sign\left(\psi_{eq}-\psi\right)}{1+\frac{\varepsilon}{\psi\varepsilon_L}\left[-\psi+\sqrt{2}\left(1+K\varepsilon_L\right)\arccos\left(\frac{1}{1+K\varepsilon_L}\right)\right]} \tag{19}$$

And for $\psi > \psi_{eq}$ as:

$$\theta = \frac{\sqrt{2}\left(1+K\varepsilon\right)\arccos\left(\frac{1}{1+K\varepsilon}\right)sign\left(\psi_{eq}-\psi\right)}{1+\frac{\varepsilon}{\left(\frac{\pi}{2}-\psi\right)\varepsilon_L}\left[\left(\psi-\frac{\pi}{2}\right)+\sqrt{2}\left(1+K\varepsilon_L\right)\arccos\left(\frac{1}{1+K\varepsilon_L}\right)\right]} \tag{20}$$

5. Results

The reorientation of the orthotropic axis of the sheet metal under uniaxial off-axis tension is obtained using Equations (19) and (20) with different material constants. The results are presented in Fig.s (3) to (5). These results are compared with Kim and Yin's [28] experimental data for a metal sheet by 3% initial pre-strain.
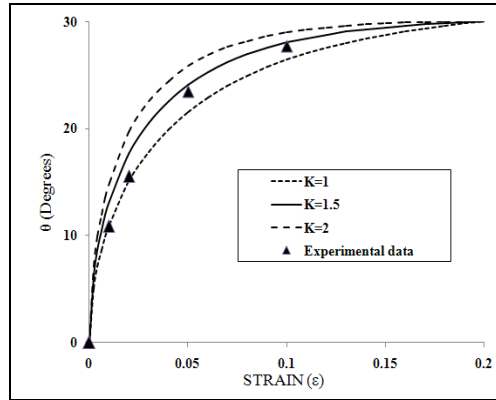

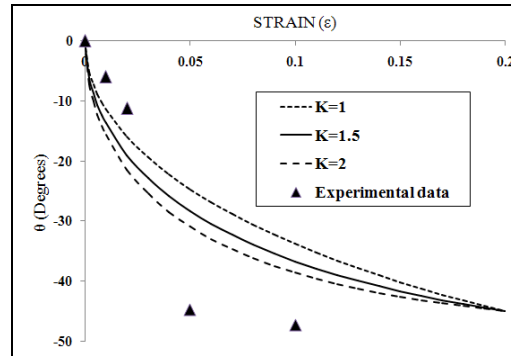Fig. 3. Rotation of the orthotropic axis for loading angles of 30 degree with $\varepsilon_L = 0.2$.

Fig. 4. Rotation of the orthotropic axis for loading angles of 45 degree with $\varepsilon_L = 0.2$.
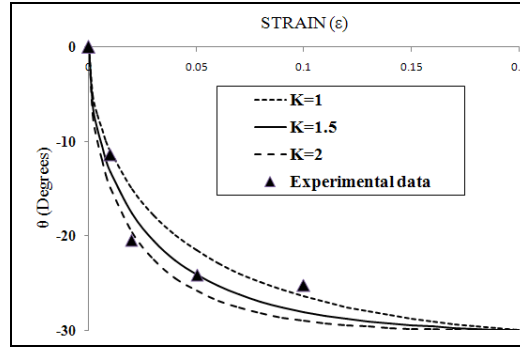

Fig. 5. Rotation of the orthotropic axis for loading angles of 60 degree with $\varepsilon_L = 0.2$.

The equations (19) and (20) are explicit formulation of the state of orientation of anisotropic orthotropy under off-axes uniaxial tension. The rotation of the orthotropic axis can be obtained using these equations based on two variables of loading angle (ψ) and secondary pre-strain ε . The secondary pre-strain consists of elastic and plastic strains but the former is neglected due to its low value. In addition to these variables, two other constants K and ε_L are required which can be obtained using experimental data with given initial anisotropy for any specific material.

Equations (13) and (16) give the value of the rotation of the axis and to obtain the direction of the rotation it is necessary to combine Equation (18) with them which lead to Equations (19) and (20).

To compare the experimental results, the constants of Equations (19) and (20) are extracted for the orthotropic sheet metal using Kim and Yin's experimental data [28].

The Fig.s (3), (4) and (5) show the rotation of the orthotropic axis for three loading angles of 30, 45 and 60 degrees respectively. These results are obtained using Equations (19) and (20) for low carbon sheet steel with 3% initial pre-strain. The material constants are given in each Fig. The agreement with experimental

data is good for loading angle of 30 degrees because the loading angle is smaller than the equivalent angle ($\psi < \psi_{eq}$), also, the difference between loading angle and equivalent angle is significant. For the same reason, for 60-degree loading angle the results compare well with the experimental data in spite of the fact that the loading angle is larger than the equivalent angle ($\psi > \psi_{eq}$). But there is a relatively large difference between the experimental data and the calculated values for the loading angle of 45-degrees. The main reason for this difference is that the loading angle is close to the equivalent angle, which we can consider that these two angles are coinciding.

The limit strain, ε_L, is an important parameter and its value is assumed to be the same for all loading conditions. $\varepsilon_L = 0.2$.

6. Discussion

Dafalias in the year 2000 in the course of a work on the subject of this paper elaborately discussed the re-orientation of orthotropic axes under uniaxial off-axes tension loading [2]. He entirely connected the problem of orthotropic axes re-orientation to plastic spin and also concept of rotation of substructure texture. We have been inspired by work of Dafalias [2] and his special approach regarding the concept of plastic spin and re-orientation of orthotropic axes, especially on 2-dimensional problems. He especially discussed about concept of equivalent angle and direction of re-orientation of orthotropic axes. However, the model proposed by Dafalias is difficult to use due to its complexity. On the contrary, the formulation presented in this paper is very simplest and very succinct and also applicable. Presented model in this work is on the basis of Kim and Yin's [39] experimental data and so Dafalias's work [2].

To make the proposed model more flexible, one can increase the number of the mechanism elements and this can be a future work for this research program. Also, to improve the model, the relation between the rotation tensor of the substructure texture of the sheet metal and stretching tensor should be taken into account. For achieving to this target, must be a fixed support in the modeling mechanism.

7. Conclusion

The orientation of orthotropic axis is described and simulated using a simple geometrical method. For this purpose, an explicit analytical formulation is proposed to determine the rotation angle of the orthotropic symmetry axes of a sheet metal under off-axis uniaxial tension. Also, associated material anisotropic constants have been obtained and compared with those given by Kim and Yin [28] based on experimental tests. Finally, a simple method is developed with associated equations to determine the direction of the orthotropic axes rotation.

The results show that using this model, the orientation and intensity of the orthotropic axis can be calculated for uniaxial off-axis loading with good agreement with experimental data. This model only requires one material constant and this adds to its flexibility and requires affordable number of tests.

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