INVERSE DYNAMICS OF COMPASS ROBOT ARM USING PRINCIPLE OF VIRTUAL WORK

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The principle of virtual work is used to study the motion of a system under the action of external and internal forces. General kinematics problem of systems of rigid bodies with constraints are presented in first part of the paper. If an internal joint force has to be found, a virtual displacement is considered in the system. Based on the principle of virtual work, the elaborated method solves the problem of calculus of input forces and joint forces of a compass robot arm, sketched as a serial chain.

Keywords: Dynamics, Joint force, Kinematics, Serial robot

1 Introduction

Considering the gravitational effects, the relevant objective of the multi-bodies dynamics is to determine the input torques or forces and the external and internal joint forces. Several methods have been applied to formulate the dynamics, which could provide the same results concerning these torques or forces. The first one is using the Newton-Euler procedure, the second one applies the Lagrange formalism with its multipliers and the third one is based on the principle of virtual work [1], [2], [3].

Difficulties commonly encountered in dynamics of multi-bodies systems include problematic issues such as: complex spatial kinematical structure with possess a large number of passive degrees of freedom, dominance of inertial forces over the frictional and gravitational components and the problem linked to the real-time control and the solution of inverse dynamics.

In the present paper, a recursive matrix method, already implemented in the inverse kinematics and dynamics of robots, is applied to the analysis of a serial mechanism. It has been proved that the number of equations and computational operations reduces significantly by using a set of matrices for dynamics modelling.

The general problem of kinematics for rigid body systems with constraints is first presented in that follows. In the second part of the paper, the application of

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the principle of virtual work is illustrated by determining of several internal forces or torques in the joints of the Compass robot arm.

2 Kinematics analysis

Let two bodies (T_i), (T_j) be with constrained motions by a coupling mechanism (Fig.1). The motion of (T_i) with respect the inertial reference frame $O_0x_0y_0z_0(T_0)$ is determined by the position vector $\vec{r}_0^{C_i} = \overrightarrow{O_0C_i}$ of mass center C_i and by the transformation matrix a_{i0} which gives the attitude of the frame $C_ix_iy_iz_i$ with respect the fixed reference frame. The number of degrees of freedom is reduced by some constrains which are imposed by coupling mechanisms. The relative motion of two bodies can be determined by the matrix a_{ij} which gives the relative attitude of (T_i) versus (T_j) and by position vector $\vec{r}_{ij} = \vec{r}_0^{C_i} - \vec{r}_0^{C_j}$.

Starting from the frame $C_jx_jy_{ji}z_j(T_j)$ and ending to the frame $C_ix_iy_iz_i(T_i)$, we can evaluate nine parameters $\alpha_{11} = \vec{i}_j^T\vec{i}_i$, $\alpha_{12} = \vec{j}_j^T\vec{i}_i$, $\alpha_{13} = \vec{k}_j^T\vec{i}_i$, $\alpha_{21} = \vec{i}_j^T\vec{j}_i$, $\alpha_{22} = \vec{j}_j^T\vec{j}_i$, $\alpha_{23} = \vec{k}_j^T\vec{j}_i$, $\alpha_{31} = \vec{i}_j^T\vec{k}_i$, $\alpha_{32} = \vec{j}_j^T\vec{k}_i$, $\alpha_{33} = \vec{k}_j^T\vec{k}_i$ giving the relative orientation of the mobile x_i, y_i, z_i with respect the frame of x_j, y_j, z_j axes [4]. These parameters constitute the contents of an orthogonal matrix of transformation a_{ij}. The projections in $C_ix_iy_iz_i$ of a vector $\vec{r}_j$ known in the space of $C_jx_jy_jz_j$ are given by the matrix relation $\vec{r}_i = a_{ij}\vec{r}_j$. The matrix a_{ij} could be easily determined using two absolute matrices $a_{ij} = a_{i0}a_{j0}^T$. Since all rotations take place successively about moving axes x_j, y', z_i, the rotation matrix $a_{ij} = a_3^ia_2^ia_1^i$ is given by three basic matrices $a_1^i = rot(x_j, \varphi_{1i})$, $a_2^i = rot(y', \varphi_{2i})$, $a_3^i = rot(z_i, \varphi_{3i})$.

Starting from the skew symmetric matrix $\tilde{\omega}_{i0} = a_{i0}\dot{a}_{i0}^T$, we obtain the known expression of the angular velocity

$$\vec{\omega}_{i0} = \dot{\varphi}_{1i}a_3^ia_2^i\vec{u}_1 + \dot{\varphi}_{2i}a_3^i\vec{u}_2 + \dot{\varphi}_{3i}\vec{u}_3, \tag{1}$$

where

$$\vec{u}_1 = [1 \quad 0 \quad 0]^T, \vec{u}_2 = [0 \quad 1 \quad 0]^T, \vec{u}_3 = [0 \quad 0 \quad 1]^T \tag{2}$$

are three orthogonal unit vectors.

Now, considering a kinematical chain $T_0, T_1, ..., T_j, ..., T_{k-1}, T_k, ..., T_i, ..., T_n$ the motions are given by skew symmetric matrices

$$\tilde{\omega}_{k0} = a_{k,k-1}\tilde{\omega}_{k-1,0}a_{k,k-1}^T + \omega_{k,k-1}\tilde{u}_3, \;\; \omega_{k,k-1} = \dot{\varphi}_{k,k-1}, \tag{3}$$

which are associated to the absolute angular velocities given by the recursive relations

$$\vec{\omega}_{k0} = a_{k,k-1}\vec{\omega}_{k-1,0} + \omega_{k,k-1}\vec{u}_3 . \tag{4}$$

Following relations give also the velocity $\vec{v}_{k0}$ of the joint O_k, the angular acceleration $\vec{\varepsilon}_{k0}^{A}$ and the acceleration $\vec{\gamma}_{k0}$ [5]

$$\begin{aligned} \vec{v}_{k0} &= a_{k,k-1}\vec{v}_{k-1,0} + a_{k,k-1}\tilde{\omega}_{k-1,0}\vec{r}_{k,k-1} + v_{k,k-1}\vec{u}_3 \\ \vec{\varepsilon}_{k0} &= a_{k,k-1}\vec{\varepsilon}_{k-1,0} + \varepsilon_{k,k-1}\vec{u}_3 + \omega_{k,k-1}a_{k,k-1}\tilde{\omega}_{k-1,0}a_{k,k-1}^{T}\vec{u}_3 \\ \vec{\gamma}_{k0} &= a_{k,k-1}\vec{\gamma}_{k-1,0} + a_{k,k-1}(\tilde{\omega}_{k-1,0}\tilde{\omega}_{k-1,0} + \tilde{\varepsilon}_{k-1,0})\vec{r}_{k,k-1} + 2v_{k,k-1}a_{k,k-1}\tilde{\omega}_{k-1,0}a_{k,k-1}^{T}\vec{u}_3 + \gamma_{k,k-1}\vec{u}_3 \end{aligned} \tag{5}$$

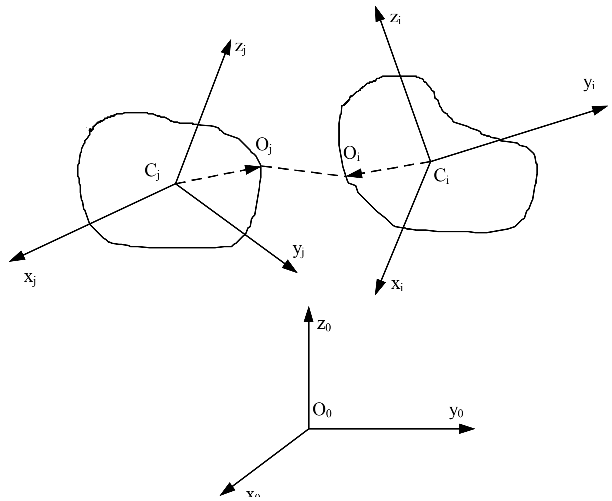

Fig.1 System of rigid bodies

3 Dynamics modeling

A solution of the dynamics problem of a multi-bodies mechanism provided with constraints can be developed based on the fundamental *principle of virtual work*. Knowing the position and kinematics state of each link as well as the external forces acting on the mechanism, we applies the principle of virtual work for the inverse dynamic problem in order to establish some matrix relations giving the input forces or torques required in a given motion, using a recursive procedure.

The force of inertia of a rigid body T_k, for example, and the resulting moment of the forces of inertia

$$\begin{aligned} \vec{f}_{k0}^{in} &= -m_k\left[\vec{\gamma}_{k0} + \left(\tilde{\omega}_{k0}\tilde{\omega}_{k0} + \tilde{\varepsilon}_{k0}\right)\vec{r}_k^{C}\right] \\ \vec{m}_{k0}^{in} &= -[m_k\tilde{r}_k^{C}\vec{\gamma}_{k0} + \hat{J}_k\vec{\varepsilon}_{k0} + \tilde{\omega}_{k0}\hat{J}_k\vec{\omega}_{k0}] . \end{aligned} \tag{6}$$

are determined with respect to the centre of joint O_k. On the other hand, the wrench of two vectors $\vec{f}_k^*$ and $\vec{m}_k^*$ evaluates the influence of the action of the weight $m_k\vec{g}$ and of other external and internal forces applied to the same element T_k of the mechanism, for example

$$\vec{f}_k^* = 9.81 m_k a_{k0}\vec{u}_3\,,\ \vec{m}_k^* = 9.81 m_k \tilde{r}_k^C a_{k0}\vec{u}_3\,. \tag{7}$$

Considering successive independent virtual motions of the mechanical system, *virtual displacements and velocities* should be compatible with the virtual motions imposed by all kinematical constraints and joints at a given instant in time.

The fundamental principle of the virtual work states that a mechanism is under dynamic equilibrium if and only if the virtual work developed by all external, internal and inertia forces vanish during any general virtual displacement, which is compatible with the constraints imposed on the mechanism.

Assuming that frictional forces at the joints are negligible, the virtual work produced by the forces of constraint at the joints is zero. So, the virtual powers contributed by active force $\vec{f}_{q,q-1}$ and actuator torque $\vec{m}_{q,q-1}$, known external forces and moments $\vec{f}_\tau^*$ and $\vec{m}_\tau^*$ and by inertia forces and moments of inertia forces $\vec{f}_\tau^{in}$ and $\vec{m}_\tau^{in}$, can be written as follows [6]:

$$v_{q,q-1}^v f_{q,q-1} + \omega_{q,q-1}^v m_{q,q-1} = \vec{u}_3^T \sum_{\tau=1}^{n} \{ v_{\tau,\tau-1}\vec{F}_\tau + \omega_{\tau,\tau-1}\vec{M}_\tau \}\,, \tag{8}$$

where

$$\vec{F}_\tau = \vec{F}_{\tau 0} + a_{\tau+1,\tau}^T \vec{F}_{\tau+1}\,,\ \vec{M}_\tau = \vec{M}_{\tau 0} + a_{\tau+1,\tau}^T \vec{M}_{\tau+1} + \tilde{r}_{\tau+1,\tau} a_{\tau+1,\tau}^T \vec{F}_{\tau+1} \tag{9}$$

$$\vec{F}_{\tau 0} = -\vec{f}_\tau^* - \vec{f}_{\tau 0}^{in}\,,\ \vec{M}_{\tau 0} = -\vec{m}_\tau^* - \vec{m}_{\tau 0}^{in}\,.$$

The dynamics model expressed by the recursive matrix equations (8) and (9) represents the explicit dynamics equations of a multi-bodies constrained system. The relations can be also applied to calculate any joint force or joint torque by cutting successively each joint O_k and writing the formulae as follows

$$f_{k,k-1}^x = \vec{u}_1^T \vec{F}_k\,,\ f_{k,k-1}^y = \vec{u}_2^T \vec{F}_k\,,\ f_{k,k-1}^z = \vec{u}_3^T \vec{F}_k$$

$$m_{k,k-1}^x = \vec{u}_1^T \vec{M}_k\,, m_{k,k-1}^y = \vec{u}_2^T \vec{M}_k\,, m_{k,k-1}^z = \vec{u}_3^T \vec{M}_k\,. \tag{10}$$

4 Example: Compass robot arm

The six degrees-of-freedom system of a spatial Compass robot arm is considered as example. We consider that the compass is initially located in the vertical plane $y_0 z_0$ of fixed frame $Ox_0 y_0 z_0$.

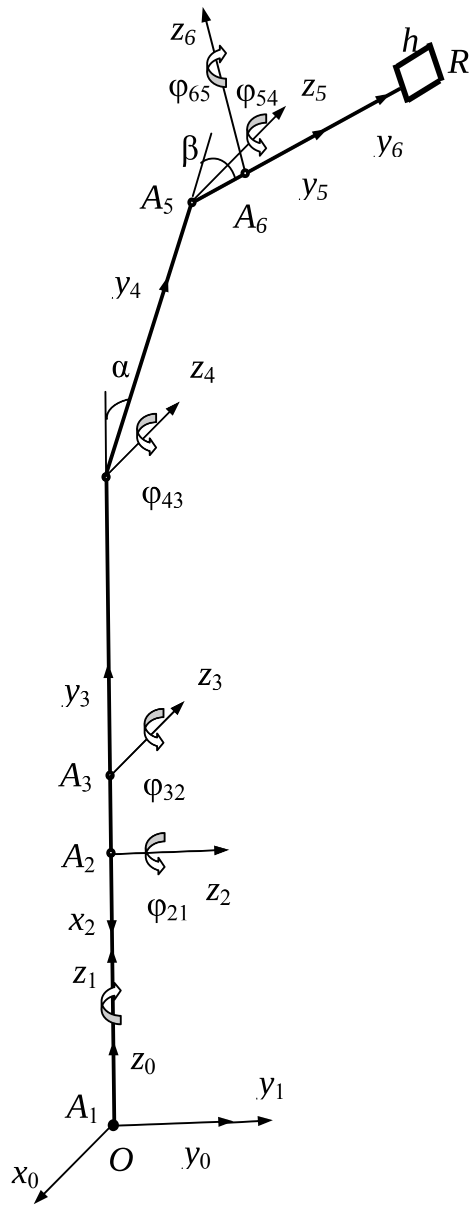

Fig. 2 Compass robot arm

The first link consists of a moving crank A_1A_2 of length l_1, mass m_1 and tensor of inertia $\hat{J}_1$ with respect $A_1x_1y_1z_1$ frame, which has rotation about z_1 axis with the angle φ_{10}, the angular velocity $\omega_{10}=\dot{\varphi}_{10}$ and the angular acceleration $\varepsilon_{10}=\ddot{\varphi}_{10}$. A second element of the leg is a rigid device jointed in A_2 and linked at the $A_2x_2y_2z_2$ frame, having a relative rotation with the angle φ_{21}, angular velocity $\omega_{21}=\dot{\varphi}_{21}$ and angular acceleration $\varepsilon_{21}=\ddot{\varphi}_{21}$. It has the length l_2, mass m_2 and tensor of inertia $\hat{J}_2$. Pursuing the serial robot we remark the presence of other four links, having known masses, inertia tensors and independent angles of rotation, angular velocities and angular accelerations (Fig. 2).

Starting from the fixed origin O and pursuing the kinematical chain we obtain following transformation matrices

$$a_{10}=a_{10}^{\varphi},a_{21}=a_{21}^{\varphi}\theta_1\theta_2,a_{32}=a_{32}^{\varphi}\theta_1\theta_2,a_{43}=a_{43}^{\varphi}a_{\alpha},a_{54}=a_{54}^{\varphi}a_{\beta},a_{65}=a_{65}^{\varphi}\theta_1, \quad (11)$$

where

$$a_{k,k-1}^{\varphi}=\begin{bmatrix}\cos\varphi_{k,k-1} & \sin\varphi_{k,k-1} & 0\\ -\sin\varphi_{k,k-1} & \cos\varphi_{k,k-1} & 0\\ 0 & 0 & 1\end{bmatrix},\ a_{k0}=\prod_{s=1}^{k}a_{k-s+1,k-s}\quad (k=1,2,...,6) \quad (12)$$

$$\theta_1=\begin{bmatrix}0&0&-1\\0&1&0\\1&0&0\end{bmatrix},\theta_2=\begin{bmatrix}0&1&0\\-1&0&0\\0&0&1\end{bmatrix},a_{\alpha}=\begin{bmatrix}\cos\alpha&\sin\alpha&0\\-\sin\alpha&\cos\alpha&0\\0&0&1\end{bmatrix},a_{\beta}=\begin{bmatrix}\cos\beta&\sin\beta&0\\-\sin\beta&\cos\beta&0\\0&0&1\end{bmatrix}.$$

Concerning the kinematics of this system we note following position vectors, velocities and accelerations

$$\begin{gathered}\vec{r}_{10}=\vec{0},\vec{r}_{21}=l_1\vec{u}_3,\vec{r}_{32}=-l_2\vec{u}_1,\vec{r}_{43}=l_3\vec{u}_2,\vec{r}_{54}=l_4\vec{u}_2,\vec{r}_{65}=l_5\vec{u}_2\\ \vec{r}_1^{\,C}=0.5\vec{r}_{21},\vec{r}_2^{\,C}=0.5\vec{r}_{32},\vec{r}_3^{\,C}=0.5\vec{r}_{43},\vec{r}_4^{\,C}=0.5\vec{r}_{54},\vec{r}_5^{\,C}=0.5\vec{r}_{65} \quad (13)\\ \vec{v}_{k,k-1}=\vec{0},\vec{\gamma}_{k,k-1}=\vec{0},\vec{\omega}_{k,k-1}=\dot{\varphi}_{k,k-1}\vec{u}_3,\vec{\varepsilon}_{k,k-1}=\ddot{\varphi}_{k,k-1}\vec{u}_3,\ (k=1,2,...,6).\end{gathered}$$

Starting from the matrix relations (6), (7) we determine the wrench for the inertia forces and the weights of six rods with respect the joints A_k. Replacing successively in the formulae (9), we obtain the vectors $\vec{F}_{\tau}$ and $\vec{M}_{\tau}$. Using the explicit dynamics equations of the multi-bodies constrained systems (8), the input torques and some joint forces or torques in internal joints A_k are quickly calculated

$$\begin{gathered}m_{10}=\vec{u}_3^{\,T}\vec{M}_1,\ f_{10}^{x}=\vec{u}_1^{\,T}\vec{F}_1,m_{21}=\vec{u}_3^{\,T}\vec{M}_2,\ f_{21}^{y}=\vec{u}_2^{\,T}\vec{F}_2\\ m_{32}=\vec{u}_3^{\,T}\vec{M}_3,\ f_{32}^{z}=\vec{u}_3^{\,T}\vec{F}_3,m_{43}=\vec{u}_3^{\,T}\vec{M}_4,m_{43}^{x}=\vec{u}_1^{\,T}\vec{M}_4 \quad (14)\\ m_{54}=\vec{u}_3^{\,T}\vec{M}_5,m_{54}^{y}=\vec{u}_2^{\,T}\vec{M}_5,m_{65}=\vec{u}_3^{\,T}\vec{M}_6,\ f_{65}^{z}=\vec{u}_3^{\,T}\vec{F}_6.\end{gathered}$$

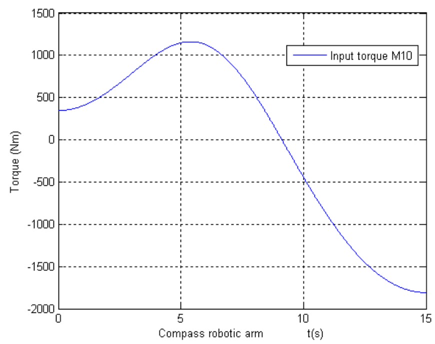

Fig. 3 Input torque m_{10}

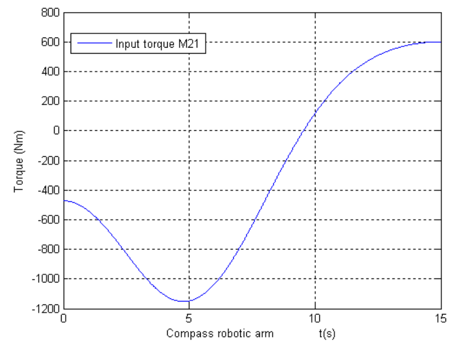

Fig. 4 Input torque m_{21}

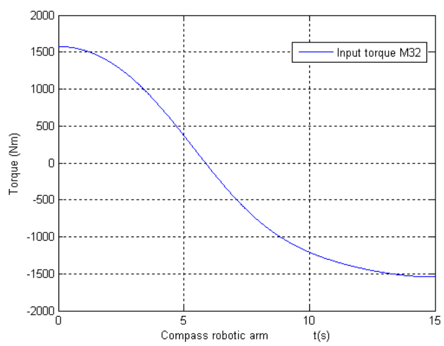

Fig. 5 Input torque m_{32}

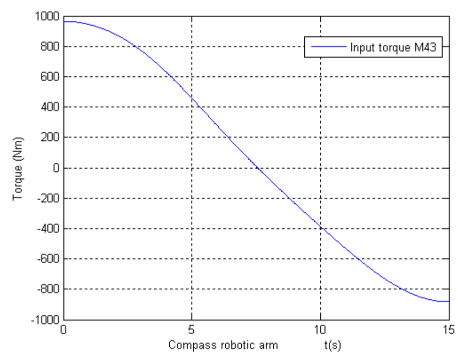

Fig. 6 Input torque m_{43}

In the inverse dynamics, we suppose that the time-history of the rotation motions of six links are known by following functions

$$\varphi_{k,k-1}(t) = \varphi^{*}_{k,k-11}[1-\cos(\frac{\pi}{15}t)],\ (k=1,2,...,6)\,. \tag{15}$$

For simulation purposes let us consider a robot arm which has the following mechanical and architectural characteristics:

$$\begin{aligned}
&\varphi^{*}_{10} = \frac{\pi}{6},\ \varphi^{*}_{21} = \frac{\pi}{12}, \varphi^{*}_{32} = \frac{\pi}{9}, \varphi^{*}_{43} = \frac{\pi}{36}, \varphi^{*}_{54} = \frac{\pi}{18}, \varphi^{*}_{65} = \frac{5\pi}{36} \\
&m_1 = 77.79kg, m_2 = 10kg, m_3 = 236kg, m_4 = 236kg \\
&m_5 = 10kg, m_{61} = 77.79kg, m_{62} = 1000, m_6 = m_{61} + m_{62} \\
&l_1 = 1.398m, l_2 = 0.34m, l_3 = 4.242m, l_4 = 4.242m \\
&l_5 = 0.34m, l_6 = 1.398m, h = 1m, R = 0.5m
\end{aligned} \tag{16}$$

$$\vec{r}_6^{\,C} = \{[0.5m_{61}l_6 + m_{62}(l_6 + 0.5h)]/m_6\}\vec{u}_2,$$
$$\alpha = \frac{\pi}{4},\ \beta = \frac{\pi}{4},\ \Delta t = 15s\ .$$

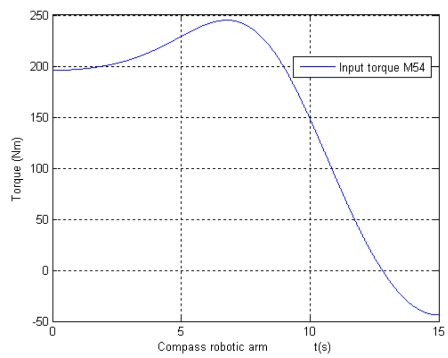

Fig. 7 Input torque m_{54}

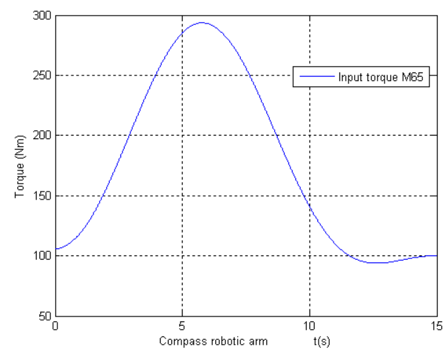

Fig. 8 Input torque m_{65}

Using the MATLAB software, a computer program was developed to solve the inverse dynamics of the Compass robot arm. To illustrate the algorithm, it is assumed that for a period $\Delta t = 15s$ the rods start at rest from initial position and rotate about its revolute joints. The active torques m_{10} (Fig. 3), m_{21} (Fig. 4), m_{32} (Fig. 5), m_{43} (Fig. 6), m_{54} (Fig. 7), m_{65} (Fig. 8), the internal joint forces f_{10}^{x} (Fig. 9), f_{21}^{y} (Fig. 10), f_{32}^{z} (Fig. 11), f_{65}^{z} (Fig. 12) and the internal joint torques m_{43}^{x} (Fig. 13), m_{54}^{y} (Fig. 14) are calculated by the program and plotted versus time.

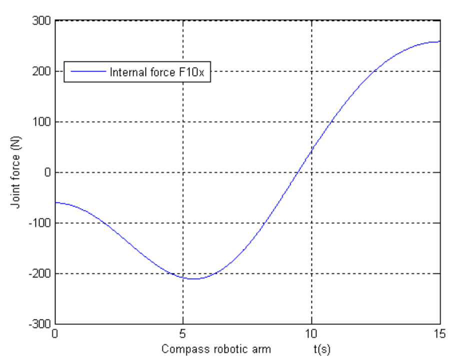

Fig. 9 Internal force f_{10}^{x}

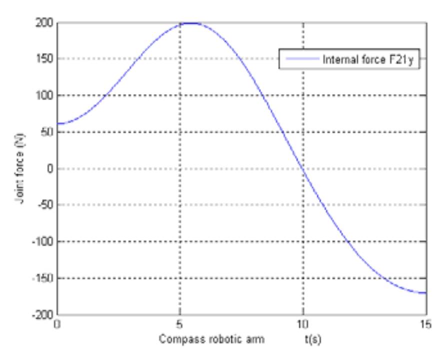

Fig. 10 Internal force f_{21}^{y}

5 Conclusions

In the kinematics analysis some exact relations that give in real-time the position, velocity and acceleration of each element of a multi-bodies system with constraints have been established in the present paper. The dynamics model takes into consideration the masses and forces of inertia introduced by all links of the mechanism. Based on the principle of virtual work, the approach establishes a direct determination of the time-history evolution for all input torques and reaction forces or torques in external and internal joints.

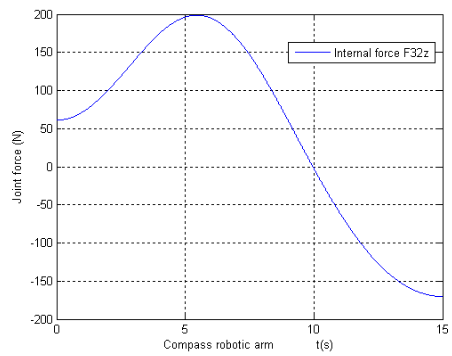

Fig. 11 Internal force f_{32}^{z}

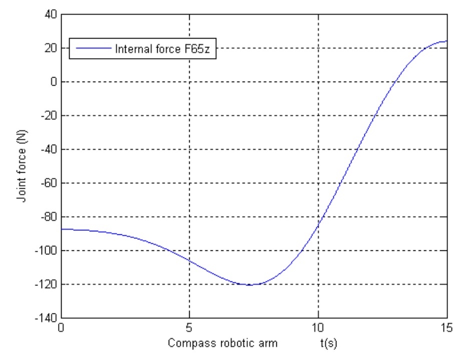

Fig. 12 Internal force f_{65}^{z}

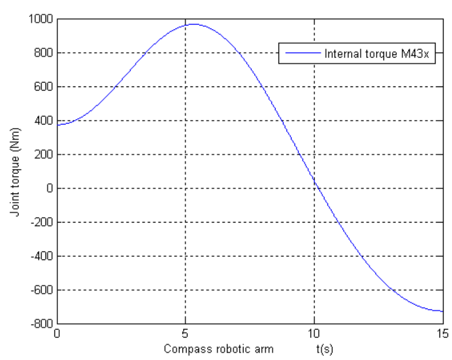

Fig. 13 Internal torque m_{43}^{x}

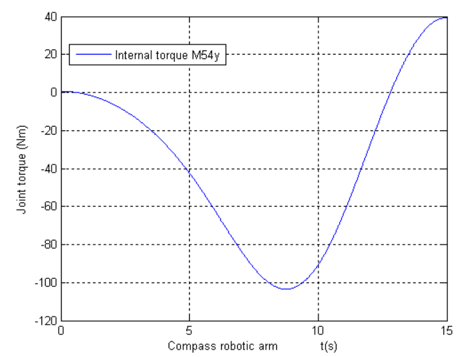

Fig. 14 Internal torque m_{54}^{y}

R E F E R E N C E S

[1] *Tsai, L-W.*, Robot analysis: the mechanics of serial and parallel manipulator, Wiley, 1999
[2] *Angeles, J.*, Fundamentals of Robotic Mechanical Systems: Theory, Methods and Algorithms, Springer, 2002
[3] *Merlet, J-P.*, Parallel robots, Kluwer Academic, 2000

[4] *Stroe, I., Staicu, S.,* Calculus of joint forces using Lagrange equations and principle of virtual work, Proceedings of the Romanian Academy, Series A, 11, 3, pp. 253-260, 2010
[5] *Staicu, S.*, Méthodes matricielles en cinématique des mécanismes, UPB Scientific Bulletin, Series D: Mechanical Engineering, 62, 1, pp. 3-10, 2000
[6] *Staicu, S.*, Modèle dynamique en robotique, UPB Scientific Bulletin, Series D: Mechanical Engineering, 61, 3-4, pp. 5-19, 1999