USING A MARKOV CHAIN FOR PRODUCT QUALITY IMPROVEMENT SIMULATION

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Improvements to product quality can be achieved only by controlling all factors impacting product quality (FIPQ). In a real business system there are a great number of FIPQ, while the processes that are going on are stochastic. For this reason a mathematical model of the system gives us the possibility of simulating different combinations of FIPQ, reducing the level of uncertainty in decision-making. A Markov chain will be used to model the stochastic processes of a system of quality management and selection of the optimum set of FIPQ.

Keywords: customer requirements; product quality; quality improvement; Markov chain; simulation.

1. Introduction

Meeting customer requirements is a complex process that entails a large number of impact factors. In order to identify the FIPQ the organisation needs to be viewed as a complex system of interrelated processes and subsystems. A large number of complex business processes are carried out in an organisation, using various resources, the goal of which is to satisfy customer requirements for products of appropriate quality in an adequate period of time. Crucial in meeting customer requirements is to accurately determine what they are as well as to identify and eliminate any factors that might cause a failure in meeting these requirements. Identification of the FIPQ means good knowledge of the way business processes are conducted, especially those affecting quality. This is further complicated by the fact that the execution of business processes involves a large number of internal customers (owners, employees, management, etc.) who also have their own requirements to be satisfied.

Mathematical models have an indispensable place and role in the process of decision-making, that is, in business decision-making in terms of the selection of the “best” solution (the best combination of factors which will ensure the

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fulfilment of customer requirements to the greatest extent). Models are used in order to investigate alternative FIPQ before practical action is taken. The probability of product conformity through application of a particular strategy where determined by mathematical means is an objective probability, as compared to subjective probability, which is of less value than the objective. The application of mathematical models and simulations has the goal of providing as much objective information as possible in decision-making conditions of uncertainty, that is in stochastic systems such as quality management systems.

Generally speaking, whenever there is a need to model and analyse a contingency in a system, the adequate tool for it proves to be a simulation. [1] Simulation has proved to be a reliable method and tool to support decision-making, one that can be helpful in the process of ongoing improvement, via the analysis and assessment of a “what-if” scenario. [2] A great number of researchers claim that simulation is a major tool in the process of reengineering and improvement of business effectiveness and performance. [3] At the same time, others maintain that simulation has great potential in aiding continual enhancement of quality improvement management systems themselves. [4],[5].

Anderson, Sweeney and Williams state that one method of study of processes in which behaviour is unpredictable and prone to change via repeated testing is simulation by Markov chain. [6] Markov chains are widely used in modelling various phenomena. Recently, Markov chains have also been implemented in the field of quality management. [7]-[24]

Reviewing the literature in this field, the authors have concluded that Markov chains have wide application in different fields. However, the application in quality management has not been identified yet, in part referring to managing factors that impact the quality of a product. The unique model for identifying the factors that impact the quality of a product and the choice of optimum set of factors by application of Markov chains are described in the paper. The defined model enables the simulation of the state of output conformance depending on the changes of factors that impact the quality. Reviewing the literature, this kind of simulation of stochastic system was not noted. The results in the paper add value to the field of applying the mathematical methods in quality management and have the possibility of practical application in all types of organizations dealing with product quality improvement. In the rest of this paper, a model will be presented for indentifying and evaluating FIPQ, as well as a way of simulating the impact of varying combinations of factors on the probability of delivery of conformant products.

2. Determining and evaluating the requirements of customers

Quality starts with the establishment of the requirements that the product has to meet. To provide customer-oriented products, listening to the voice of the customer is critical. [25] Establishing a culture of internal customers goes hand-in-hand with establishing a culture of respect for customers as a whole. Fig. 1. presents the relatedness between external and the internal customers as to their requirements.

Table 1 presents, for example, customers and their requirements against the example of internal and external customers shown in Fig. 1.

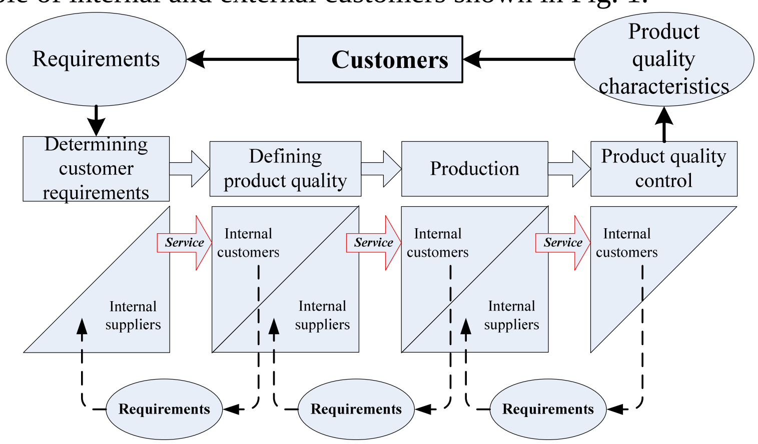

Fig. 1. Relatedness between internal and external customers

Table 1

Customers and their requirements

Name of activity	Department carrying out the activity	Customers	Description of customer requirements
Determining customer requirements	Sales	Design and development	Clear and precise specification of customer requirements Error-free recording of requirements
		External customer	Determining requirements in the shortest possible time Error-free recording of requirements
Defining product quality	Design and development	Production	Clear specification of product quality Defining quality within a time span allowing fulfilment of contractual obligations
		External customer	Defined quality is fully in accordance with requirements Defining quality within a time span allowing fulfilment of contractual obligations
Production	Production	Quality control	Production of products which are in accordance with specified requirements and quality
		External customer	Production of products which are in accordance with specified requirements and quality Production within a time span allowing fulfilment of contractual obligations
Quality control	Quality control	External customer	Adequate identification of nonconformities Product quality control within defined time spans Adequate accompanying documentation (quality control records, etc.)

Upon defining customer requirements, the importance of the requirements for each customer need to be defined (Table 2). Assessment of customer requirements involves all customers and it is desirable that the evaluation process include experts in quality management too. The identified relative importance of customer requirements serves as the basis for identifying the factors affecting the meeting of these requirements.

Table 2

Defining the importance of customer requirements

Activity	Requirement	Customers	Average importance
		Cus_j	
Act_l	Req_i	r_{ij}	$\bar{r}_{il}$

Let us suppose that $N \geq 1$ activities are given. Each activity can meet $n_l \geq 1,\ 1 \leq l \leq N$ requirements, the importance of which is evaluated by $m > 1$ users and experts. Let us indicate with r_{ij} the importance of the ith requirement for the jth user, $r_{ij} \in \{0,1,2 \ldots\}, 1 \leq i \leq n_l, 1 \leq j \leq m$. Let us also define the average value of the importance of each requirement for the user $\bar{r}_{il} = \frac{1}{m}\sum_{j=1}^{m} r_{ij}$, $i \ \in \{1, \ldots, n_l\}, \forall\ l \ \in \{1, \ldots, N\}$.

Notation:

Act_l activities $l \ \in \{1, \ldots, N\},\ N \geq 1$

Req_i requirements per activity $1 \leq i \leq n_l$

r_{ij} importance of the ith requirement of the lth activity for the jth user

$\bar{r}_{il}$ average value of the importance of each requirement for the user

3. Identifying the factors that impact the fulfilment of requirements

The identified requirements of customers and established significance of each requirement form the basis for identifying the factors impacting the fulfilment of those requirements. For the purposes of this paper, the “4 Ms” – Man, Material, Method, and Machine model of FIPQ provision will be used. [26] Using the “4 Ms” model, a different “4 Ms” set with accompanying characteristics can be defined for each activity that creates an output of a particular quality. It is those specific characteristics which impact the quality of outputs (of products and internal services), that is, the fulfilment of the requirements of internal and external customers.

4. Evaluating the impact of the factors on the fulfilment of requirements

For each activity, the impact of the listed factors on the fulfilment of requirements needs to be determined (Table 3).

Table 3

Determining the impact of the factors on the fulfilment of requirements

Act_l		
Factors *Requirement*	Fac_j	Average importance
Req_i	b_{ij}	$\bar{r}_{il}$
Absolute impact	$\boldsymbol{f_{jl}}$	
Relative influence	$\boldsymbol{\delta_{jl}}$	

We wish to determine the impact of $K \geq 1$ factors on the quality of output of each of N activities. Since each activity can fulfil n_l, $1 \leq l \leq N$ requirements which can be impacted by the factors, we define the type of impact of factors on the fulfilment of requirements $b_{ij} \in \{0,1,2\ldots\}, 1 \leq i \leq n_l, 1 \leq j \leq K$. The impact of the jth factor on the lth activity is the value $f_{jl} = \sum_{i=1}^{n_l} b_{ij}\ \bar{r}_{il}$, $1 \leq j \leq K$, $1 \leq l \leq N$. We indicate the relative value of the impact of factors on quality with $\delta_{jl} = f_{jl}/\sum_{j=1}^{k} f_{jl}$, $1 \leq j \leq K, 1 \leq l \leq N$.

Notation:

Fac_j factors $1 \leq j \leq K, K \geq 1$

b_{ij} impact of the jth factor on the fulfilment of the ith requirement

f_{jl} the absolute impact of the jth factor on the lth activity

δ_{jl} the relative impact of the jth factor on the lth activity

The calculated values for relative influence of the impact of factors on quality (δ_{jl}) are shown in Fig. 2, using the example with four activities and four factors influencing the quality of each activity output.

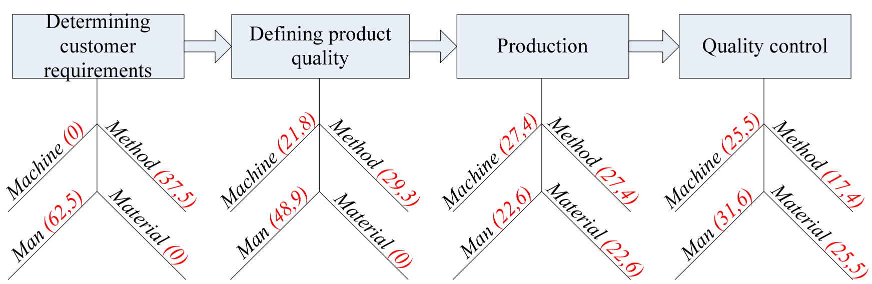

Fig. 2 – Example of calculated impact of factors on fulfilment of requirements

5. Defining a strategy for improvement of the factors

The identified critical customer requirements form the basis for identifying the factors that impact the fulfilment of those requirements. From the perspective of efficient and effective management, it is vital to determine those measures that will improve the positive factors or eliminate those negative ones that could present problems for the fulfilment of the critical requirements, before nonconformity occurs. These factors can be improved (e.g., training, process control, etc.) or eliminated (e.g., variations in processes, stoppages, waste, etc.) in order that customer requirements be fully met. Identification of the FIPQ is the basis for defining an optimum strategy of managing FIPQ.

A great many measures can be taken to eliminate the causes of nonconformity, i.e. improve the factors that affect the meeting of customer requirements. Some solutions that affect these factors are suggested in [27],[28],[29],[30],[31]. The question is how to select the right measure that will contribute most to the improvement of FIPQ, and, on the other hand, secure an opportunity for improvement at the lowest possible cost. According to Schiffauerova and Thomson, any serious attempt to improve quality has to take into account the costs of quality improvement. [32]

The identified FIPQ form the basis for defining an optimum strategy for the management of FIPQ, taking into account the criteria: 1) costs of improvement and 2) impact of improvement on product quality. The optimum strategy is that strategy which, for those factors which can be improved, has the greatest improvement impact with the least (or equal to the budgeted) costs of improvement. The question arises, how to select the right measure, which will most contribute to the improvement of FIPQ. The greatest problem is in determining how great the impact of improvement of the factor is on the quality of the product. Will, for example, staff training have a greater effect, or will improvement to preventive maintenance? Will, for example, buying new machines have a greater effect, or will improving the product planning and development process?

For each factor which impacts the output quality of an activity, the management can define varying improvement actions. We can define many improvement actions for each factor. Let us suppose that there is a set $A = \{a_1, \dots, a_n\}$ comprised of $n > 1$ different actions for the improvement of each of K FIPQ. Then the possible number of scenarios for one activity represents a variation with repetition of the Kth class of set A of n elements, $\overline{V_n^K} = n^K$. Since we have N activities in the model, the possible number of ways of improving the factors is $\overline{V_n^{KN}} = n^{KN}$.

Let us indicate with C_{ij}^l , $1 \le i \le n, 1 \le j \le K, 1 \le l \le N$, the cost of the ith action for the improvement of the jth factor in activity l. Each of the $\overline{V_n^{KN}}$ combinations of factor improvement actions has certain costs which are calculated as the sum of the costs of each individual action C_{ij}^l which belongs to the given combination, $C_s = \sum_{l=1}^{N} \sum_{j=1}^{K} C_{ij}^l$.

We can define, for example three improvement actions for each factor:

1. *Action 1*/No improvement
2. *Action 2*/Incremental improvement
3. *Action 3*/Radical improvement.

Since there are four FIPQ and three improvement scenarios for each factor, the possible number of combinations for one activity is $3^4 = 81$. Since our example has four activities, the possible number of combinations of factor improvement is $81^4 = 43.046.721$. Each of the 43.046.721 combinations of factor improvement actions has certain costs which are calculated as the sum of the costs of each individual action.

The series of *Actions 1* indicates that no factor impacting quality has been improved, while the series of *Actions 3* indicates that all factors have been radically improved. The costs of the first series of actions is zero, while the costs of the second series are equal to the sum of the costs of all the actions.

Each series of actions undertaken has a differing impact on the fulfilment of the requirements of internal and external customers, and therefore on product quality. The impact of improvement of FIPQ can be determined using a simulation of the effects of improvement. The simulation is conducted on the basis of the defined quality system model, whose comprising elements include all identified FIPQ. In order to define an adequate model for any system, especially for simulation purposes, all system elements and possible scenarios must be studied and defined in detail (Table 4).

Table 4

Costs and the impact of factor improvement on quality

Act_l, Fac_j				
	Costs	Impact of improvement	Impact of factor on quality	Overall impact on quality improvement
Acs_i	C_{ij}^l	U_{ij}^l	δ_{jl}	λ_{ij}^l

Let us indicate with U_{ij}^l, $1 \le i \le n, 1 \le j \le K, 1 \le l \le N$, the impact of improving the jth factor of activity l when undertaking the ith action, determined with the help of experts. The total impact of each of n actions on the improvement to quality is defined as $\lambda_{ij}^l = U_{ij}^l \delta_{jl}$, $\forall\, j \in \{1, \dots, K\}$, $\forall\, l \in \{1, \dots, N\}$.

Notation:

Acs_i factor improvement actions $1 \le i \le n, n > 1$

C_{ij}^l cost of the ith action for improving the jth factor in activity l

U_{ij}^l impact of the improvement of the jth factor of activity l when the ith action is undertaken

λ_{ij}^l overall impact of each action on the improvement of quality

C_s cost per the scenario

6. Selection of optimum strategy using Markov chain

For the effects of FIPQ to be improved a simulation of the effects of improvement needs to be implemented. The basis for conducting such a simulation is the definition of a model of a stochastic quality management system, the constituent parts of which are also all the identified FIPQ. Markov chains are an important tool for modelling stochastic processes. [33],[34],[35] A Markov chain with state space $S = \{S_1, S_2, \ldots\}$ and transition matrix P is a sequence of random variables $\{X_n, n \in N\}$ satisfying:

$$P\{X_{n+1} = S_{n+1} | X_1 = S_1, X_2 = S_2, \ldots, X_n = S_n\} = P\{X_{n+1} = S_{n+1} | X_n = S_n\} \quad (1)$$

Intuitively, a random process is called a Markov chain when, conditional on the current state of the process, its future is independent of its past. The process changes from state S_i to another state S_j, at time epochs $n = 1,2, \ldots$ with probability:

$$p_{ij}^{n,n+1} = P\{X_{n+1} = S_j | X_n = S_i\} \quad (2)$$

The chain is homogeneous if this probability is independent of time epoch n. In that case, the following notation is used:

$$p_{ij} = P\{X_{n+1} = S_j | X_n = S_i\} \quad (3)$$

Probability p_{ij} is called a one-step transition probability from state S_i to state S_j, while matrix $P = [p_{ij}]_{ij}$ is called a one step transition probability matrix, or simply transition matrix. The n-step transition probability of a Markov chain is the probability that it goes from state S_i to state S_j in n transitions:

$$[\![p(n)]\!]_{\downarrow(ij)} = P(X_{\downarrow(n+k)} = S_{\downarrow j} | X_{\downarrow k} = S_{\downarrow i}) \; \forall k, \quad (4)$$

and the associated transition matrix in n-steps is $P(n) = [p_{ij}(n)]_{ij}$. (5)

Using Chapman-Kolmogorov equations, calculating these probabilities in n steps is possible:

$$p(n+m)_{ij} = \sum_{k=1}^{\infty} p(n)_{ik} p(m)_{kj}, \quad (6)$$

i.e. in matrix notation:

$$P(n+m) = P(n)P(m). \quad (7)$$

Then, it follows that

$$P(n) = P^n. \quad (8)$$

If we wish to start the chain according to some initial distribution $p(0)$, then the state probabilities in matrix notation are given with:

$$p(n) = p(o)P^n. \quad (9)$$

Reaching a solution to the Markov model involves three steps: 1) setting the model, 2) working out the equation and 3) using Laplace transformations in solving the state equations. [36] Since deriving equations from the state diagram is a drawn-out process subject to errors, various computer programs are used as support for the simulation. Using Matlab, a complex computing procedure can be substantially shortened, simplified, and understood and recorded in a more accessible manner.

Below, an example of the implementation of the Markov processes for the example of the most important customer requirements given in will be presented, using the Matlab program. Let us say that we want to model the probability of delivery of conformant products dependent on changes to FIPQ, which directly impacts customer satisfaction. The following table lists the possible states of fulfilment of all the critical requirements identified.

Table 5

Possible states of fulfillment of customer requirements

Activity	**Description of customer requirements**	**State of fulfilment of requirement**	**Symbol**
		1) Initial state: customer comes with requirements	***S1***
Determining customer requirements	Error-free recording of requirements	1) Correctly determined customer requirement 2) Wrongly determined customer requirement	***S2*** ***S3***
Defining product quality	Quality is fully in accordance with requirements	1) Quality meets requirements 2) Quality does not meet requirements	***S4*** ***S5***
Production	Production of products which are in accordance with specified requirements	1) Conformant product produced 2) Nonconformant product produced	***S6*** ***S7***
Quality control	Adequate identification of nonconformities	1) Product quality control will identify nonconformities 2) Product quality control will not identify nonconformities	***S8*** ***S9***
		1) Conformant product delivered 2) Nonconformant product delivered	***S10*** ***S11***

On the basis of historical data from monitoring the performance of processes it is possible to determine the probabilities of transition from one state to another (Figure 3). The probability of the system being in states S_{10} or S_{11} is affected by errors in determining customer requirements, defining quality, production and quality control, i.e. a combination of the factors that impact the output quality of these activities.

Since $\lambda_{311} = \lambda_{511} = \lambda_{911} = \lambda_{610} = \lambda_{810} = 1; \lambda_{13} + \lambda_{12} = 1; \lambda_{24} + \lambda_{25} = 1; \lambda_{46} + \lambda_{47} = 1; \lambda_{78} + \lambda_{79} = 1$ we can conclude that the probability of the system being in state S_{10} or S_{11} is affected only by the probability of transitions $\lambda_{13}, \lambda_{25}, \lambda_{47}$ and λ_{79}. By reducing these probabilities, we increase the probability that a conformant product has been delivered. The improvement of each factor for an activity has a certain overall impact on quality (shown in Table 4) by which it reduces the aforementioned probability of nonconformity ($\lambda_{13}, \lambda_{25}, \lambda_{47}$ and λ_{79}).

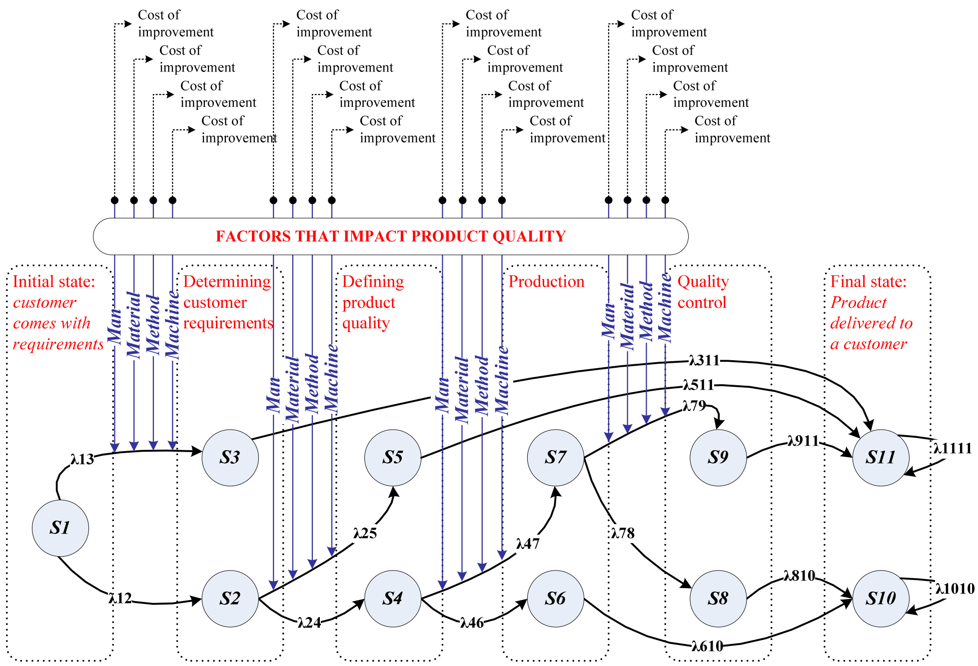

Fig. 3 State transition diagram

Let us say that a set of probabilities of the non-conformance of the product is given for each activity $q_{0l}, 1 \leq l \leq N$. If we want to improve the quality of the product for a specific value P we select a scenario s of improvement of factors impacting the quality of the results of activities. For each factor, on each activity, one improvement action is selected from the set A. The total costs of improvement to quality are equal to C_s, while the improvement itself is measured by using the change in the probability of the non-conformance of the output of the given activity $q_{sl} = q_{0l} - P\sum_{j=1}^{K} \lambda_{ij}^{l}, \quad \forall\, l \in \{1, \dots, N\}$, or the probability of the conformance of the output of the given activity $p_{sl} = 1 - q_{sl}$.

Based on the estimated values of improvement costs of individual FIPQ and the impact of improvement on quality (example of estimated values is shown in Table 6), we may define the following possible scenarios:

Scenario 1: *status quo*; In this case Action 1 and sum of costs of improvement applied for all FIPQs are equal to 0,

Scenario 2: Implement incremental improvements for all FIPQ; in this case the Action 2 and the sum of costs of improvement are applied for all FIPQ and come to 138,900

Scenario 3: Radically improve the "Man" factor for all activities; in this case for the "Man" factor the Action 3 and sum of costs of improvement are applied for all four activities and come to 78,000.

Scenario 4: Implement radical improvements for all FIPQ; in this case the Action 3 and sum of costs of improvement are applied for all FIPQ and come to 352,000.

Actions applied for FIPQ determine implementation costs, and also determine values for "Total impact on quality" which impact the transition probabilities. For instance, if the existing state is $\lambda_{13} = 0.15$ and we implement incremental improvements (Action 2) to all factors impacting quality for the activity "Determining customer requirements", the new state of λ'_{13} will be $0.15 – 0.15(0.1875+0.1875) = 0.095$ (for example, values of total impact on quality of the "Man" factor of 0.1875 and the "Method" factor of 0.1875 are defined in Table 6).

Table 6

Example of costs and the impact of factor improvement on quality

Activity	**Factor**	**Action**	**Costs**	**Impact of improvement**	**Impact of factor on quality**	**Total impact on quality**
1	2	3	4	5	6	7=5x6
Determining customer requirements	Man	Action 1	*0*	*0*	*62.5*	*0*
		Action 2	*1,000*	*30%*	*62.5*	*18.75*
		Action 3	*15,000*	*95%*	*62.5*	*59.375*
	Material	Action 1	*0*	*0*	*0*	*0*
		Action 2	*0*	*45%*	*0*	*0*
		Action 3	*0*	*100%*	*0*	*0*
	Method	Action 1	*0*	*0*	*37.5*	*0*
		Action 2	*10,000*	*50%*	*37.5*	*18.75*
		Action 3	*35,000*	*99%*	*37.5*	*37.125*
	Machine	Action 1	*0*	*0*	*0*	*0*
		Action 2	*0*	*60%*	*0*	*0*
		Action 3	*0*	*99%*	*0*	*0*
Defining product quality	Man	Action 1	*0*	*0*	*48.9*	*0*
		Action 2	*1,900*	*40%*	*48.9*	*19.56*
		Action 3	*10,000*	*80%*	*48.9*	*39.12*
	Material	Action 1	*0*	*0*	*0*	*0*
		Action 2	*0*	*30%*	*0*	*0*
		Action 3	*0*	*70%*	*0*	*0*
	Method	Action 1	*0*	*0*	*29.3*	*0*
		Action 2	*4,000*	*30%*	*29.3*	*8.79*
		Action 3	*12,000*	*75%*	*29.3*	*21.975*
	Machine	Action 1	*0*	*0*	*21.8*	*0*
		Action 2	*11,000*	*35%*	*21.8*	*7.63*
		Action 3	*20,000*	*70%*	*21.8*	*15.26*

Production	Man	Action 1	*0*	*0*	*22.6*	*0*
		Action 2	*15,000*	*50%*	*22.6*	*11.3*
		Action 3	*35,000*	*90%*	*22.6*	*20.34*
	Material	Action 1	*0*	*0*	*22.6*	*0*
		Action 2	*2,000*	*40%*	*22.6*	*9.04*
		Action 3	*8,000*	*90%*	*22.6*	*20.34*
	Method	Action 1	*0*	*0*	*27.4*	*0*
		Action 2	*7,000*	*45%*	*27.4*	*12.33*
		Action 3	*24,000*	*95%*	*27.4*	*26.03*
	Machine	Action 1	*0*	*0*	*27.4*	*0*
		Action 2	*50,000*	*50%*	*27.4*	*13.7*
		Action 3	*100,000*	*95%*	*27.4*	*26.03*
Quality control	Man	Action 1	*0*	*0*	*31.6*	*0*
		Action 2	*5,000*	*35%*	*31.6*	*11.06*
		Action 3	*18,000*	*97%*	*31.6*	*30.65*
	Material	Action 1	*0*	*0*	*25.5*	*0*
		Action 2	*2,000*	*40%*	*25.5*	*10.2*
		Action 3	*5,000*	*90%*	*25.5*	*22.95*
	Method	Action 1	*0*	*0*	*17.4*	*0*
		Action 2	*8,000*	*40%*	*17.4*	*6.96*
		Action 3	*25,000*	*80%*	*17.4*	*13.92*
	Machine	Action 1	*0*	*0*	*25.5*	*0*
		Action 2	*22,000*	*45%*	*25.5*	*11.47*
		Action 3	*45,000*	*97%*	*25.5*	*24.73*

Using the attached Matlab algorithm, the probability of delivery of a conformant product can be simulated for different variant scenarios. The simulation was conducted on a sample of 100 customers (cycles).

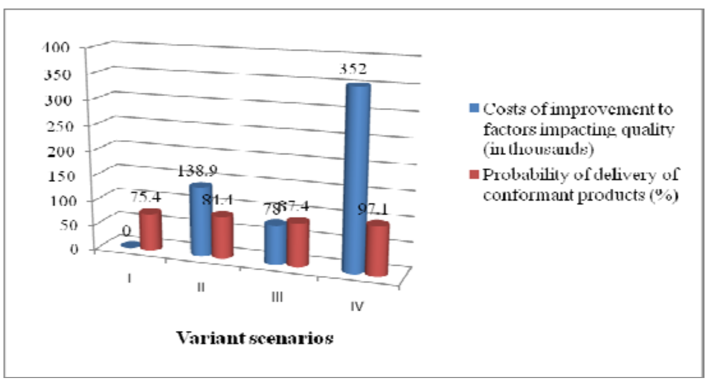

Fig. 4 – Costs of factor improvement and probability of delivery of conformant products

By simulating the application of differing variants of FIPQ we can conclude that the scenario of radical improvement to the “Man” factor for all activities will secure a greater degree of delivery of conformant products than

Scenario 2, together with significantly lower costs of improvement (Fig. 4). We can also conclude that by implementing Scenario 4 we can secure a probability of delivery of conformant products of 97.1% at most. Without a model defined in this way and the conduct of a simulation it would not be possible to select the best combination of FIPQ within certain limitations (in this case, limited improvement budget).

7. Conclusion

The study presents a hypothetical example of the improvement of FIPQ, taking into account the requirements of all customers, both external and internal. The basis for defining the requirements was obtained by identifying all the customers and their requirements, which was done on the basis of a study of the processes affecting product quality.

The methodology was presented on the example of comparison of the effects of four different strategies for the improvement of FIPQ on the probability of delivery of conformant products. The same methodology can be used to compare a larger number of alternatives and even include a larger number of activities and factors. The methodology thus conceived makes it possible to identify the areas in which improvement will be most effective, i.e., affect product quality, by observing essential performances of the process and simulating the effects of improvement. The simulation of a business process helps in the understanding, analysis and design of processes. It can be implemented when (re)designed processes have to be assessed and compared. Simulation offers qualitative assessments of the effects product design will probably have upon the functioning of the process, so that the best design can be selected with quantitative support**.**

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Appendix

Matlab algorithm

```
function [chain,state] = simulatemarkov(x,P,pi0,T);
% notation is folowing
%  x = vector of state values
%  P = one step transition matrix, P=[p(i,j)] i,j=1,...n
% pi0 = initial probability distribution
% T = number of time periods
% chain = sequence of realizations from the chain simulation
n = length(x);  % size of the state vector
E = rand(T,1); % random vector of dimension T necessary for iteration the chain i.e. for realization
of
cumsumP = P*triu(ones(size(P))); % creates a matrix whose rows are the cumulative sums of the
rows of P
% Initial state using initial probabilities pi0
E0 = rand(1,1);
ppi0 = [0,cumsum(pi0)];
s0 = ((E0<=ppi0(2:n+1)).*(E0>ppi0(1:n)))'; %
s = s0;
% Iterating on the chain
for t=1:T,
 state(:,t) = s;
```

```
 ppi = [0,s'*cumsumP];
 s = ((E(t)<=ppi(2:n+1)).*(E(t)>ppi(1:n)))'; % if E(t) ≥ p_{i,1}, chain stays in the same state in next time period t + 1, otherwise it moves in some other different state.
end
chain = x'*state;
```

Transition probability matrix

Variant I	**Variant II**	**Variant III**	**Variant IV**
P=[0,0.85,0.15,0,0,0,0,0,0,0,0;	P=[0,0.906,0.094,0,0,0,0,0,0,0,0	P=[0,0.939,0.061,0,0,0,0,0,0,0,0	P=[0,0.995,0.005,0,0,0,0,0,0,0,0
0,0,0,0.9,0.1,0,0,0,0,0,0;	,0;	,0;	,0;
0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0.936,0.064,0,0,0,0,0,0;	0,0,0,0.939,0.061,0,0,0,0,0,0;	0,0,0,0.976,0.024,0,0,0,0,0,0;
0,0,0,0,0,0.85,0.15,0,0,0,0;	0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0,0,0.92,0.08,0,0,0,0;	0,0,0,0,0,0.881,0.119,0,0,0,0;	0,0,0,0,0,0.989,0.011,0,0,0,0;
0,0,0,0,0,0,0,0,0,1,0;	0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0.9,0.1,0,0;	0,0,0,0,0,0,0,0,0,1,0;	0,0,0,0,0,0,0,0,0,1,0;	0,0,0,0,0,0,0,0,0,1,0;
0,0,0,0,0,0,0,0,0,1,0;	0,0,0,0,0,0,0,0.94,0.06,0,0;	0,0,0,0,0,0,0,0.931,0.069,0,0;	0,0,0,0,0,0,0,0.992,0.008,0,0;
0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0,0,0,0,0,0,1,0;	0,0,0,0,0,0,0,0,0,1,0;	0,0,0,0,0,0,0,0,0,1,0;
0,0,0,0,0,0,0,0,0,1,0	0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0,0,0,0,0,0,0,1;	0,0,0,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,1]	0,0,0,0,0,0,0,0,0,1,0	0,0,0,0,0,0,0,0,0,1,0	0,0,0,0,0,0,0,0,0,1,0
	0,0,0,0,0,0,0,0,0,0,1]	0,0,0,0,0,0,0,0,0,0,1]	0,0,0,0,0,0,0,0,0,0,1]

```
pi0 = [1,0,0,0,0,0,0,0,0,0,0];  % initial probability distribution
x = [1;2;3;4;5;6;7;8;9;10;11];  % state vector
T = 100; % simulation lenght
pi=pi0*P^T % vector of state probabilities in time T
[chain,states] = simulatemarkov(x,P,pi0,T);
```

Remark: In numerical implementation, general code for simulating Markov models was used. Code consists of two m files. The first file was used to provide simulation, i.e. the code was used to generate Markov chain with general number of states. Second m file uses data described in the paper (for the given 11 state vectors, transition matrix and initial probabilities) and calculates transition matrix in T steps and generates chain. Markov chain will then give sequence of realizations $\{x_t\}$ of Markov process $\{X_t\}$. For the last time period, the value $T = 100$ is chosen as an example for calculation of probability of the output conformance after 100 deliveries to the clients. The same values would be given for other values, as well, for T, because the system quickly comes to the stationary state.

Random variable E is necessary in order to iterate over the chain. E is designed to be random vector with length T. So if the current state is i and$E(t) \geq p_{i,1}$, the chain stays in the same state in next time period $t + 1$, otherwise it moves in some other different state. Unfortunately, there is an error. It should be power of T i.e. 100. Fortunately, it doesn't change results and probabilities, which remain almost the same. Additionally, while reviewing the algorithm I have noticed that it would be more appropriate to stay consistent with the theory mentioned above in the paper. For that reason, in this updated version of the code, the matrix is calculated by multiplying initial probability and transition matrix on the power of T, as explained in the paper. The same results (the same values of probabilities) are obtained as in the previous case. For this paper, the most important part of the code is the calculation of the probability of being in states S10 or S11 for the given number of time periods. Code for generation of states of the chain is mainly used for better understanding.