KINEMATICS MODELLING OF A PLANAR PARALLEL ROBOT WITH PRISMATIC ACTUATORS

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Lucrarea prezentă stabileşte relaţii matriceale recurente pentru cinematica robotului plan paralel cunoscut 3-RPR cu acţionare pneumatică sau hidraulică. Cele trei picioare identice ale robotului, care sunt legate la platforma mobilă, sunt localizate în acelaşi plan. Cunoscând mişcarea platformei, problema de cinematică inversă oferă expresii matriceale şi grafice pentru deplasările, vitezele şi acceleraţiile celor trei sisteme active.

Recursive matrix relations for kinematics of the commonly known 3-RPR planar parallel robot with pneumatic or hydraulic actuators are established in this paper. The three identical legs of the robot, connecting to the moving platform, are located in the same plane. Knowing the motion of the platform, the inverse kinematical problem offers matrix expressions and graphs for the displacements, velocities and accelerations of the three active systems.

Key-words: kinematics, parallel manipulator, platform

1. Introduction

Parallel manipulators are closed-loop mechanisms that consist of separate serial chains connecting the fixed base to the moving platform. Compared with serial manipulators, the followings are the potential advantages of parallel architectures: higher kinematical precision, lighter weight and better stiffness, greater load bearing, stabile capacity and suitable position of arrangement of actuators. But, from application point of view, a limited workspace and complicated singularities are two major drawbacks of parallel mechanisms.

Parallel manipulators can be equipped with revolute or prismatic actuators. They have a robust construction and can move bodies of large dimensions with high velocities and accelerations. This is the reason why the devices, which produce translations or spherical motion to a platform, technologically are based on the concept of parallel manipulators [1].

Over the past decades, parallel manipulators have received more and more attention from researchers and industries. Important companies such as Giddings & Lewis, Ingersoll, Hexel and others have developed them as high precision

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machine tools. Accuracy and precision in the direction of the tasks are essential since the robot is intended to operate on fragile objects, where positioning errors of the tool could end in costly damages.

Considerable efforts have been devoted to the kinematics and dynamic analysis of fully parallel manipulators. Among these, the class of manipulators known as Stewart-Gough platform focused great attention (Stewart [2], Merlet [3], Parenti-Castelli and Di Gregorio [4]). They are used in flight simulators and more recently for Parallel Kinematics Machines. The prototype of Delta parallel robot (Clavel [5], Tsai and Stamper [6], Staicu and Carp-Ciocardia [7]) developed by Clavel at the Federal Polytechnic Institute of Lausanne and by Tsai and Stamper at the University of Maryland as well as the Star parallel manipulator (Hervé and Sparacino [8]) are equipped with three motors, which train on the mobile platform in a three-degrees of freedom general translation motion. Angeles, Gosselin, Gagné and Wang [9], [10], [11] analysed the kinematics, dynamics and singularities loci of Agile Wrist spherical robot with three actuators.

Planar parallel robots are useful for manipulating an object on a plane. A mechanism is said to be a *planar robot* if all the moving links of the mechanism perform planar motions that are situated in parallel planes. For a planar mechanism, the loci of all points in all links can be drawn conveniently on a plane. In a planar linkage, the axes of all revolute joints must be normal to the plane of motion, while the direction of translation of a prismatic joint must be parallel to the plane of motion.

Aradyfio and Qiao [12] examined the inverse kinematics solution for the three different 3-DOF planar parallel robots. Gosselin and Angeles [13] and Pennock and Kassner [14] each presents a kinematical study of a 3-$R\underline{P}R$ planar parallel robot where a moving platform is connected to a fixed base by three links, each leg consisting of two binary links and three parallel revolute joints. Sefrioui and Gosselin [15] give an interesting numerical solution in the inverse and direct kinematics of this kind of planar robot.

Recently, more general approaches have been presented. Daniali et al. [16] present a study of velocity relationships and singular conditions for general planar parallel robots. Merlet [17] solved the forward pose kinematics problem for a broad class of planar parallel manipulators. Williams et al. [18] analysed the dynamics and the control of a planar three-degrees-of-freedom parallel manipulator at Ohio University while Yang et al. [19] concentrate on the singularity analysis of a class of 3-$\underline{R}RR$ planar parallel robots developed in its laboratory. Bonev, Zlatanov and Gosselin [20] describe several types of singular configurations by studying the direct kinematics model of a 3-$\underline{R}PR$ planar parallel robot with actuated base joints.

A recursive method is introduced in the present paper, to reduce significantly the number of equations and computation operations by using a set of matrices for the kinematics model of the 3-*RPR* planar parallel robots.

2. Kinematics analysis

Having a closed-loop structure, the planar parallel robot 3-*RPR* is a special symmetrical mechanism composed of three planar kinematical chains of variable length with identical topology, all connecting the fixed base to the moving platform (Fig. 1).

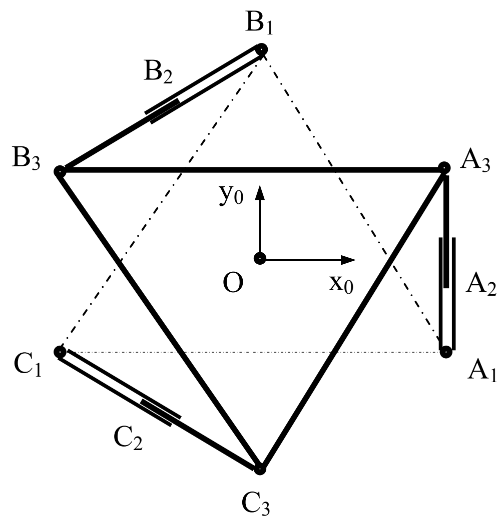

Fig. 1. The 3-*RPR* planar parallel robot

Its movable platform and base form two congruent equilateral triangles. The centres A_1, B_1, C_1 of three fixed pivots define the geometry of a fixed base while the three moving revolute joints A_3, B_3, C_3 define the geometry of the moving platform. Each leg or modulus consists of two links with two revolute joints and one prismatic joint in-between. Together, the mechanism consists of seven moving links, six revolute joints and three prismatic joints. Grübler mobility equation predicts that the device has certainly three degrees of freedom.

In the present kind of robot (*RPR*) we consider the moving platform as the output link while the pistons A_2A_3, B_2B_3, C_2C_3 as the input links. In order to analyse this robot, we attach to the fixed base a Cartesian frame $x_0y_0z_0(T_0)$ having the origin located at the triangle centre O, the axis z_0 perpendicular to the base and the axis x_0 pointing along the direction C_1A_1. Another mobile reference frame $x_G y_G z_G$ is attached to the moving platform. The origin of this coordinate central system is located just at the centre G of the moving triangle (Fig. 2).

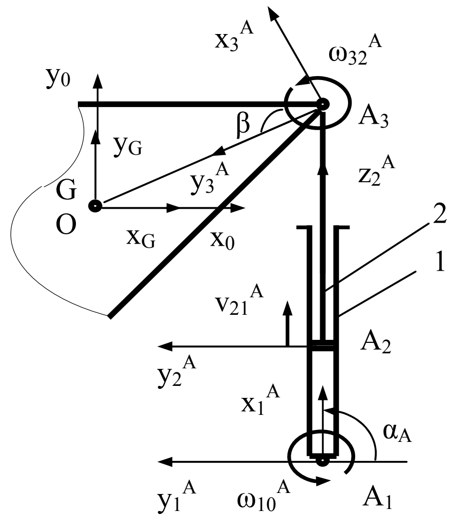

Fig. 2. Kinematical scheme of the first leg *A* of the mechanism

To simplify the graphical image of the kinematical scheme of the mechanism, in the followings we will represent the intermediate reference systems by only two axes, so as one proceeds in most of robotics papers [1], [3], [9]. The axis z_k is represented for each component element T_k. It is noted that a relative rotation by the angle $\varphi_{k,k-1}$ or a relative translation of the body T_k by $\lambda_{k,k-1}$ must be always pointing about or along the direction of the axis z_k.

In what follows we consider that the moving platform is initially located at a *central configuration* where the platform is not rotated with respect to the fixed base while the mass centre G is at the origin O of the fixed frame.

One of the three active legs (for example leg A) consists of a fixed revolute joint and a moving cylinder **1,** of length l_1, which has a rotation about the axis z_1^A with the angle φ_{10}^A, the angular velocity $\omega_{10}^A = \dot{\varphi}_{10}^A$ and the angular acceleration $\varepsilon_{10}^A = \ddot{\varphi}_{10}^A$. A prismatic joint is as well as a piston **2** of length l_2, linked to the frame $x_2^A y_2^A z_2^A$, which has a relative motion with the displacement λ_{21}^A, the velocity $v_{21}^A = \dot{\lambda}_{21}^A$ and the acceleration $\gamma_{21}^A = \ddot{\lambda}_{21}^A$. Finally, a revolute joint is introduced at a planar moving platform, which is schematised as an equilateral triangle having the edge $l = r\sqrt{3}$.

At the central configuration we consider also that all the legs are initially extended at equal lengths l_0 while the angles of orientation of the fixed pivots are given by

$$\alpha_A = \frac{\pi}{2}, \quad \alpha_B = -\frac{5\pi}{6}, \quad \alpha_C = -\frac{\pi}{6}$$
$$\beta_A = \beta_B = \beta_C = \beta = \frac{\pi}{6}. \tag{1}$$

Pursuing the first leg A in the $OA_1A_2A_3A_4$ way, we obtain the following matrices of transformation [21]:

$$a_{10} = a_{10}^{\varphi}\, a_{\alpha}^{A}, \quad a_{21} = \theta, \quad a_{32} = a_{32}^{\varphi}\, a_{\beta}\, \theta^T, \tag{2}$$

where

$$a_{\alpha}^{A} = \begin{bmatrix} \cos\alpha_A & \sin\alpha_A & 0 \\ -\sin\alpha_A & \cos\alpha_A & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad a_{\beta} = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$a_{k,k-1}^{\varphi} = \begin{bmatrix} \cos\varphi_{k,k-1}^{A} & \sin\varphi_{k,k-1}^{A} & 0 \\ -\sin\varphi_{k,k-1}^{A} & \cos\varphi_{k,k-1}^{A} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \theta = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$a_{k0} = \prod_{j=1}^{k} a_{k-j+1,\,k-j}, \qquad (k = 1, 2, 3). \tag{3}$$

Analogous relations can be written for the other two legs of the mechanism. Three relative displacements $\lambda_{21}^{A}, \lambda_{21}^{B}, \lambda_{21}^{C}$ of the active links are the joint variables that give the input vector $\vec{\lambda}_{21} = [\lambda_{21}^{A} \;\; \lambda_{21}^{B} \;\; \lambda_{21}^{C}]^T$ of the instantaneous position of the mechanism in the first study configuration. But, in the inverse geometric problem, we can consider that the position of the mechanism is completely given by the coordinates x_0^G, y_0^G of the mass centre G of the moving platform and by the orientation angle ϕ of the movable frame $x_G y_G z_G$. The orthogonal rotation matrix of the moving platform, from the reference system $x_0 y_0 z_0$ to $x_G y_G z_G$, is

$$R = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{4}$$

Further, we suppose that the position vector of the centre *G,* respectively $\vec{r}_0^{\,G} = [x_0^G \;\; y_0^G \;\; 0]^T$ and the orientation angle ϕ, which are expressed by the following analytical functions

$$x_0^G = x_0^{G*}(1-\cos\frac{\pi}{3}t)$$
$$y_0^G = y_0^{G*}(1-\cos\frac{\pi}{3}t) \qquad (5)$$
$$\phi = \phi^{*}(1-\cos\frac{\pi}{3}t)$$

can describe the general absolute motion of the moving platform.
From the rotation conditions of the moving platform

$$a_{30}^{\circ T} a_{30} = b_{30}^{\circ T} b_{30} = c_{30}^{\circ T} c_{30} = R\,, \qquad (6)$$

taking, for example,

$$a_{30}^{\circ} = \begin{bmatrix} \cos(\alpha_A+\beta) & \sin(\alpha_A+\beta) & 0 \\ -\sin(\alpha_A+\beta) & \cos(\alpha_a+\beta) & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad (7)$$

we obtain the following relations between angles:

$$\varphi_{10}^A + \varphi_{32}^A = \varphi_{10}^B + \varphi_{32}^B = \varphi_{10}^C + \varphi_{32}^C = \phi\,. \qquad (8)$$

The six variables $\varphi_{10}^A, \lambda_{21}^A,\; \varphi_{10}^B, \lambda_{21}^B,\; \varphi_{10}^C, \lambda_{21}^C$ will be determined by several vector-loop equations, as follows

$$\vec{r}_{10}^{\,A} + \sum_{k=1}^{2} a_{k0}^T\, \vec{r}_{k+1,k}^{\,A} + a_{30}^T\, \vec{r}_3^{\,GA} =$$
$$= \vec{r}_{10}^{\,B} + \sum_{k=1}^{2} b_{k0}^T\, \vec{r}_{k+1,k}^{\,B} + b_{30}^T\, \vec{r}_3^{\,GB} = \qquad (9)$$
$$= \vec{r}_{10}^{\,C} + \sum_{k=1}^{2} c_{k0}^T\, \vec{r}_{k+1,k}^{\,C} + c_{30}^T\, \vec{r}_3^{\,GC} = \vec{r}_0^{\,G}\;,$$

where one denoted

$$\vec{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\vec{r}_{10}^{\,A} = 0.5r[\sqrt{3} \quad -1 \quad 0]^T$$
$$\vec{r}_{10}^{\,B} = r[0 \quad 1 \quad 0]^T$$
$$\vec{r}_{10}^{\,C} = 0.5r[-\sqrt{3} \quad -1 \quad 0]^T \tag{10}$$
$$\vec{r}_{21}^{\,i} = (l_0 + \lambda_{21}^i)\vec{u}_1$$
$$\vec{r}_{32}^{\,i} = l_2\,\vec{u}_3, \quad \vec{r}_3^{\,Gi} = r\,\vec{u}_2 \qquad (i = A, B, C).$$

Actually, these vector equations means that there is only one inverse geometrical solution for the manipulator, namely:

$$(l_0 + l_2 + \lambda_{21}^i)\sin(\varphi_{10}^i + \alpha_i) = y_0^G - y_{10}^i - r\sin(\phi + \alpha_i + \beta)$$
$$(l_0 + l_2 + \lambda_{21}^i)\cos(\varphi_{10}^i + \alpha_i) = x_0^G - x_{10}^i + r\cos(\phi + \alpha_i + \beta)$$
$$(i = A,\ B,\ C)\,. \tag{11}$$

We will develop the inverse kinematics problem and determine the velocities and accelerations of the manipulator, supposing that the planar motion of the moving platform is known. Firstly, we compute the linear and angular velocities of each leg in terms of the angular velocity $\omega_0^G = \dot{\phi}\,\vec{u}_3$ and of the centre's velocity $\vec{v}_0^{\,G} = \dot{\vec{r}}_0^{\,G}$ of the moving platform.

The motions of the component elements of each leg (for example the leg A) is characterized by the following skew symmetrical matrices:

$$\tilde{\omega}_{k0}^A = a_{k,k-1}\,\tilde{\omega}_{k-1,0}^A\,a_{k,k-1}^T + \omega_{k,k-1}^A\,\tilde{u}_3\,, \qquad \omega_{k,k-1}^A = \dot{\varphi}_{k,k-1}^A \tag{12}$$

which are *associated* to the absolute angular velocities given by the following recursive relations

$$\vec{\omega}_{k0}^A = a_{k,k-1}\,\vec{\omega}_{k-1,0}^A + \omega_{k,k-1}^A\,\vec{u}_3\,, \quad (k = 1,2,3), \quad \omega_{21}^A = 0\,. \tag{13}$$

The following relations give the velocities $\vec{v}_{k0}^{\,A}$ of the joints A_k .

$$\vec{v}_{k0}^{\,A} = a_{k,k-1}\,\vec{v}_{k-1,0}^{\,A} + a_{k,k-1}\,\tilde{\omega}_{k-1,0}^A\,\vec{r}_{k,k-1}^{\,A} + v_{k,k-1}^A\,\vec{u}_3$$
$$v_{\sigma,\sigma-1}^A = 0 \quad (\sigma = 1, 3)\,. \tag{14}$$

The equations of geometrical constraints (8) and (9) can be derived with respect to the time to obtain the following *matrix conditions of connectivity* [22}

$$\omega_{10}^A\,\vec{u}_i^{\,T}\,a_{10}^T\,\tilde{u}_3\{\vec{r}_{21}^{\,A} + a_{21}^T\,\vec{r}_{32}^{\,A} + a_{21}^T\,a_{32}^T\,\vec{r}_3^{\,GA}\} + v_{21}^A\,\vec{u}_i^{\,T}\,a_{10}^T\,\vec{u}_1 +$$
$$+ \omega_{32}^A\,\vec{u}_i^{\,T}\,a_{30}^T\,\tilde{u}_3\,\vec{r}_3^{\,GA} = \vec{u}_i^{\,T}\,\dot{\vec{r}}_0^{\,G}\,, \quad (i = 1,2)$$

$$\omega_{10}^{A} + \omega_{32}^{A} = \dot{\phi} \ , \tag{15}$$

where $\tilde{u}_3$ is a skew-symmetric matrix associated to the unit vector $\vec{u}_3$ pointing in the positive sense of the axis *z*. We obtain, from these equations, the relative velocities $\omega_{10}^{A}, v_{21}^{A}, \omega_{32}^{A}$ as functions of the angular velocity of the platform and of the velocity of the mass centre *G*. But, the conditions (15) give the *complete* Jacobian matrix of the manipulator. This matrix is a fundamental element for the analysis of the robot workspace and for the particular configurations of the singularities where the manipulator becomes uncontrollable.

Since φ_{10}^{i} is a passive variable in the kind of the planar robot with prismatic actuators, it should be eliminated from the equations (11). So, summing the squares of these equations there results:

$$\begin{aligned} &(l_0 + l_2 + \lambda_{21}^{i})^2 = [x_0^G - x_{10}^{i} + r\sin(\phi + \alpha_i + \beta)]^2 = \\ &= [y_0^G - y_{10}^{i} - r\cos(\phi + \alpha_i + \beta)]^2 \end{aligned} \tag{16}$$

where the "zero" position $x_0^{0G} = 0,\ y_0^{0G} = 0,\ \phi^0 = 0$ corresponds to the joints variables $\vec{\lambda}_{10}^{0} = [0 \quad 0 \quad 0]^T$.

A new matrix relation is obtained by calculating the derivative of equation (15) with respect to the time, namely

$$J_{1p}\dot{\vec{\lambda}}_{21} = J_{2p}[\dot{x}_0^G \quad \dot{y}_0^G \quad \dot{\phi}]^T , \tag{17}$$

where

$$\vec{\lambda}_{21} = [\lambda_{21}^{A} \quad \lambda_{21}^{B} \quad \lambda_{21}^{C}]^T$$

$$J_{1p} = diag\{\delta_{Ap} \quad \delta_{Bp} \quad \delta_{Cp}\}$$

$$J_{2p} = \begin{bmatrix} \beta_{1p}^{A} & \beta_{2p}^{A} & \beta_{3p}^{A} \\ \beta_{1p}^{B} & \beta_{2p}^{B} & \beta_{3p}^{B} \\ \beta_{1p}^{C} & \beta_{2p}^{C} & \beta_{3p}^{C} \end{bmatrix} \tag{18}$$

$$\delta_{ip} = l_0 + l_2 + \lambda_{21}^{i},\ (i = A,\ B,\ C)$$

$$\beta_{1p}^{i} = x_0^G - x_{10}^{i} + r\sin(\phi + \alpha_i + \beta)$$

$$\beta_{2p}^{i} = y_0^G - y_{10}^{i} + r\cos(\phi + \alpha_i + \beta)$$

$$\beta_{3p}^{i} = r[(x_0^G - x_{10}^{i})\cos(\phi + \alpha_i + \beta) + (y_0^G - y_{10}^{i})\sin(\phi + \alpha_i + \beta)]. \tag{18}$$

As for the relative accelerations $\varepsilon_{10}^{A}, \gamma_{21}^{A}, \varepsilon_{32}^{A}$ of the robot, the derivatives with respect to the time of the equations (15) give the following conditions of connectivity [25]

$$\begin{aligned}
&\varepsilon_{10}^{A}\vec{u}_i^{\,T} a_{10}^{T}\tilde{u}_3\left\{\vec{r}_{21}^{\,A}+a_{21}^{T}\vec{r}_{32}^{\,A}+a_{21}^{T}a_{32}^{T}\vec{r}_3^{\,GA}\right\}+\\
&+\gamma_{21}^{A}\vec{u}_i^{\,T} a_{10}^{T}\vec{u}_1+\varepsilon_{32}^{A}\vec{u}_i^{\,T} a_{30}^{T}\tilde{u}_3\vec{r}_3^{\,GA}=\vec{u}_i^{\,T}\ddot{\vec{r}}_0^{\,G}-\\
&-\omega_{10}^{A}\omega_{10}^{A}\vec{u}_i^{\,T} a_{10}^{T}\tilde{u}_3\tilde{u}_3 a_{21}^{T}\left\{\vec{r}_{21}^{\,A}+a_{21}^{T}\vec{r}_{32}^{\,A}+a_{21}^{T}a_{32}^{T}\vec{r}_3^{\,GA}\right\}-\\
&-\omega_{32}^{A}\omega_{32}^{A}\vec{u}_i^{\,T} a_{30}^{T}\tilde{u}_3\tilde{u}_3\vec{r}_3^{\,GA}-2\omega_{10}^{A}v_{21}^{A}\vec{u}_i^{\,T} a_{10}^{T}\tilde{u}_3\vec{u}_1-\\
&-2\omega_{10}^{A}\omega_{32}^{A}\vec{u}_i^{\,T} a_{10}^{T}\tilde{u}_3 a_{21}^{T}a_{32}^{T}\tilde{u}_3\vec{r}_3^{\,GA}\,,\quad (i=1,2)\\
&\varepsilon_{10}^{A}+\varepsilon_{32}^{A}=\ddot{\phi}\,.
\end{aligned}\tag{19}$$

If the other two kinematical chains of the robot are pursued, analogous relations can be easily obtained.

The relationships (15) and (19) represent the *inverse kinematics model* of the planar parallel robot.

The following recursive relations give the angular accelerations $\vec{\varepsilon}_{k0}^{\,A}$ and the accelerations $\vec{\gamma}_{k0}^{\,A}$ of the joints A_k.

$$\begin{aligned}
&\vec{\varepsilon}_{k0}^{\,A}=a_{k,k-1}\vec{\varepsilon}_{k-1,0}^{\,A}+\varepsilon_{k,k-1}^{A}\vec{u}_3+\omega_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\vec{u}_3\\
&\tilde{\omega}_{k0}^{A}\tilde{\omega}_{k0}^{A}+\tilde{\varepsilon}_{k0}^{A}=a_{k,k-1}\left(\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A}+\tilde{\varepsilon}_{k-1,0}^{A}\right)a_{k,k-1}^{T}+\\
&\qquad+\omega_{k,k-1}^{A}\omega_{k,k-1}^{A}\tilde{u}_3\tilde{u}_3+\varepsilon_{k,k-1}^{A}\tilde{u}_3+2\omega_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\tilde{u}_3\\
&\vec{\gamma}_{k0}^{\,A}=a_{k,k-1}\left[\vec{\gamma}_{k-1,0}^{\,A}+a_{k,k-1}\left(\tilde{\omega}_{k-1,0}^{A}\tilde{\omega}_{k-1,0}^{A}+\tilde{\varepsilon}_{k-1,0}^{A}\right)\vec{r}_{k,k-1}^{\,A}\right]+\\
&\qquad+2v_{k,k-1}^{A}a_{k,k-1}\tilde{\omega}_{k-1,0}^{A}a_{k,k-1}^{T}\vec{u}_3+\gamma_{k,k-1}^{A}\vec{u}_3\,,\qquad (k=1,2,3)\\
&\varepsilon_{21}^{A}=0\,,\quad \gamma_{\sigma,\sigma-1}^{A}=0\quad (\sigma=1,3)\,.
\end{aligned}\tag{20}$$

As application let us consider a manipulator which has the following characteristics:

$$x_0^{G*}=-0.016\,m,\quad y_0^{G*}=0.017\,m,\quad \phi^*=\frac{\pi}{30},\quad r=0.3\,m,$$
$$l=r\sqrt{3},\ l_0=0.1\,m,\ l_1=l_2=0.2\,m,\ \Delta t=3s. \tag{21}$$

Using the MATLAB software, a computer program was developed to solve the inverse kinematics of the robot. Finally, the displacements (Fig. 3), the velocities (Fig. 4) and the accelerations (Fig. 5) of the three prismatic actuators were plotted versus time, using this program.

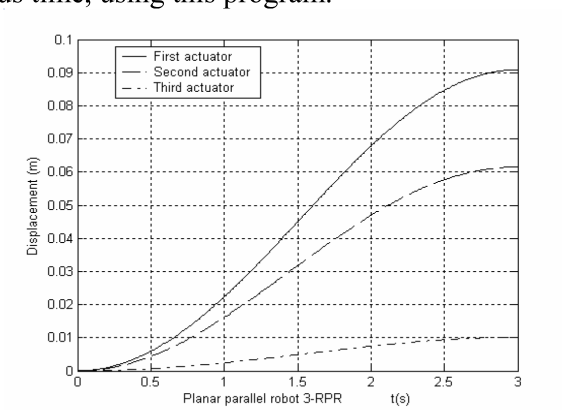

Fig. 3. Displacements of the three actuators

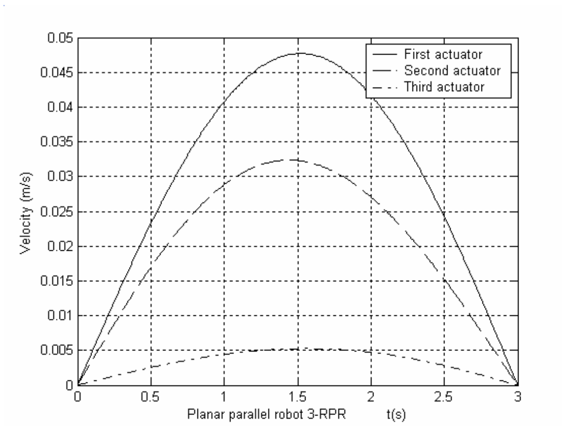

Fig. 4. Velocities of the three actuators

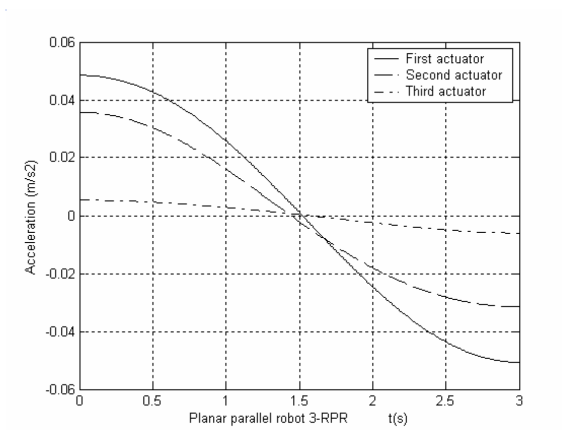

Fig. 5. Accelerations of the three actuators

We remark that the displacements of the platform are small with respect to the mechanism's sizes. The discontinuities of the accelerations of the component elements lead to the generation of the chocks in the joints.

3. Conclusions

Within the inverse kinematical analysis, some exact relations that give the time-history evolution of the displacements, velocities and accelerations of each element of the parallel robot have been established in the present paper.

The simulation by the presented program certifies that one of the major advantages of the current matrix recursive formulation is a reduced number of additions or multiplications and consequently a smaller processing time of numerical computation. Also, the proposed method can be applied to various types of complex robots, when the number of the components of the mechanism is increased.

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