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DETERMINATION OF OPERATING CHARACTERISTICS OF TWO-PHASED INDUCTION MOTORS
I. THE MODEL OF CIRCUIT ON PHASE

O. MAGDUN, M. COVRIG*

Caracteristicile de funcționare ale motoarelor asincrone bifazate se pot determina cu o mai bună acuratețe dacă se foloseşte un model de circuit al motorului bifazat şi parametrii de circuit calculați luând in considerare saturația miezului magnetic.

În lucrarea de față se propune un model de circuit original pentru motorul asincron bifazat cu înfăşurări neortogonale, dezvoltat pe baza analizei undelor magnetice în întrefier. În partea a doua a lucrării se prezintă rezultatele calculate cu acest model în comparație cu rezultatele experimentale. Se indică şi o metodă simplă bazată pe soluția problemei de câmp electromagnetic pentru determinarea parametrilor de circuit ai modelului.

The operating characteristics of the two-phased induction motors can be determined accurately if it is used a circuit model of the two-phased induction motor and circuit parameters computed by considering saturation of magnetic core.

In this part of paper, an original circuit model for the two-phased induction motor with non-orthogonal windings based on the magnetic waves analysis in air gap is proposed. In the second part of paper the computed results with this model is shown in comparison with experimental results. Also, a simple method based on the solution of the electromagnetic field problem to determine the circuit parameters of the model is proposed.

Keywords: two-phased induction motor, circuit model

Introduction

In the specialty literature, generally, the induction motor operating is analyzed using models of equivalent circuits with concentrated parameters ([1], [2]), in the hypothesis of unsaturated magnetic core. The saturation effects are considered by the correction of circuit parameters.

For two-phased induction motors were determined circuit models using the symmetrical components theory ([1], [2], [3]), bat these models are valid only for motors with orthogonal windings and only for the fundamental harmonic. Some trying on determination of operating characteristics of two-phased induction

* Lecturer, Dept. of Electrical Engineering, "Valahia" University of Targoviste; Prof., Dept. of Electrical Machines, Electrical Drives, and Materials, University "Politehnica" of Bucharest, ROMANIA

motor with non-orthogonal windings have been made with models of dynamical regime [4], [5], but the results are affected by errors, especially in the starting area and in the area of critical slip.

This part of paper deals with the equivalent circuit on phase of the two-phased induction motor with non-orthogonal asymmetrical windings.

1. Magnetic waves in the air gap of the two-phased induction motor

Consider a two-phased induction motor with windings situated on a cylindrical armature. The windings are represented by two concentric coils placed on their magnetic axes. The magnetic axis of a phase, denoted main phase (subscript a) is superposed on the stator spatial axis, and the magnetic axis of the second phase, denoted auxiliary phase (subscript b) is displaced with the geometrical angle β in the movement rotor direction (Fig. 1).

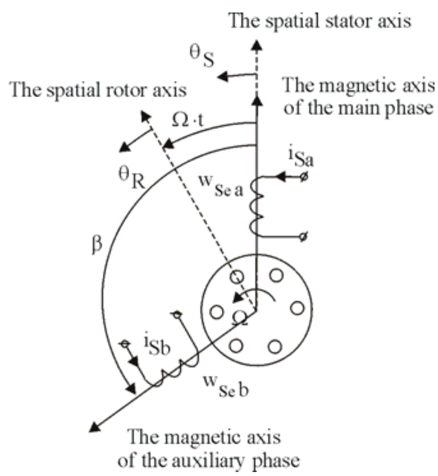


Fig. 1. Explanatory to displacement of motor windings

The motor has armature equals to ideal length l_i, the air gap is constant and equal with equivalent air gap δ_e, and the pole pitch is denoted τ. The rotor of the machine is in squirrel cage and it is considered that is moved with angular speed Ω corresponding with the slip s.

The spatial coordinate of the stator after movement direction is θ_S, and θ_R is the spatial coordinate of the rotor. It is admit that the magnetic circuit is made of an ideal ferromagnetic material with the propriety $\mu_{Fe} \to \infty$. Thus, all magnetic energy is located in the air gap, and the ferromagnetic core contributes only to guide the magnetic field lines. The magnetic saturation, the hysteresis phenomena and iron loses are neglected.

The air gap mmf wave obtained by superposing of wave produced by each winding, it can be decomposed in spatial harmonics [8]:

$$f_S(\theta_S,t) = \sum_{\nu} f_{Sa_\nu}(\theta_S,t) + \sum_{\nu} f_{Sb_\nu}(\theta_S,t) \tag{1}$$

where:

- the mmf wave harmonic ν produced by the current i_{Sa} flowing through the main winding has expression:
-

$$f_{Sa_\nu}(\theta_S,t) = \frac{2}{\pi}\cdot\frac{w_{Sa_\nu}}{p}\cdot I_{Sa}\sqrt{2}\cdot sin(\omega_S t-\varphi)\cdot cos(\nu\cdot p\theta_S) \tag{2}$$

$$i_{Sa} = I_{Sa}\sqrt{2}\cdot sin(\omega_S t-\varphi), \qquad w_{Sa_\nu} = w_{Sa}\cdot k_{wSa_\nu}\cdot\frac{1}{\nu}$$

- the mmf wave of harmonic ν produced by the current i_{Sb} flowing through the auxiliary winding has expression:
-

$$f_{Sb_\nu}(\theta_S,t) = \frac{2}{\pi}\cdot\frac{w_{Sb_\nu}}{\nu\cdot p}\cdot I_{Sb}\sqrt{2}\cdot sin(\omega_S t-\gamma)\cdot cos[\nu\cdot p\cdot(\theta_S-\beta)] \tag{3}$$

$$i_{Sb} = I_{Sb}\sqrt{2}\cdot sin(\omega_S t-\varphi), \qquad w_{Sb_\nu} = w_{Sb}\cdot k_{wSb_\nu}\cdot\frac{1}{\nu}$$

where w_{Sa_ν} and w_{Sb_ν} are the numbers of equivalent turns for spatial harmonic ν, which depend by both, numbers of turns of the main and auxiliary windings: w_{Sa}, w_{Sb} and the winding factors for spatial harmonic ν: k_{wSa_ν}, k_{wSb_ν}. [6]

If the auxiliary winding is related to the main winding, from the conserving condition of the maximum solenation, it is obtain the related expression of the auxiliary current flowing through by the auxiliary phase, and respectively from relation (3), the expression of the mmf wave:

$$f_{Sb_\nu}(\theta_S,t) = \frac{2}{\pi}\cdot\frac{w_{Sa_\nu}}{\nu\cdot p}\cdot I_{Sb_\nu}'\sqrt{2}\cdot sin(\omega_S t-\gamma)\cdot cos[\nu\cdot p\cdot(\theta_S-\beta)] \tag{4}$$

$$i_{Sb_\nu}' = I_{Sb_\nu}'\sqrt{2}\cdot sin(\omega_S t-\gamma)$$

$$I_{Sb_\nu}{}' = \frac{w_{Sb_\nu}}{w_{Sb_\nu}} I_{Sb}$$

2. Static structure of equivalent machine determination

The harmonic serial development of the air gap mmf wave suggests that two-phased machine correspond with *n* two-phased equivalent machines of harmonic (Fig. 2). Two-phased machine of harmonic ν has two identical windings having $w_{Sa\nu}$ equivalent turns and νp pair poles.

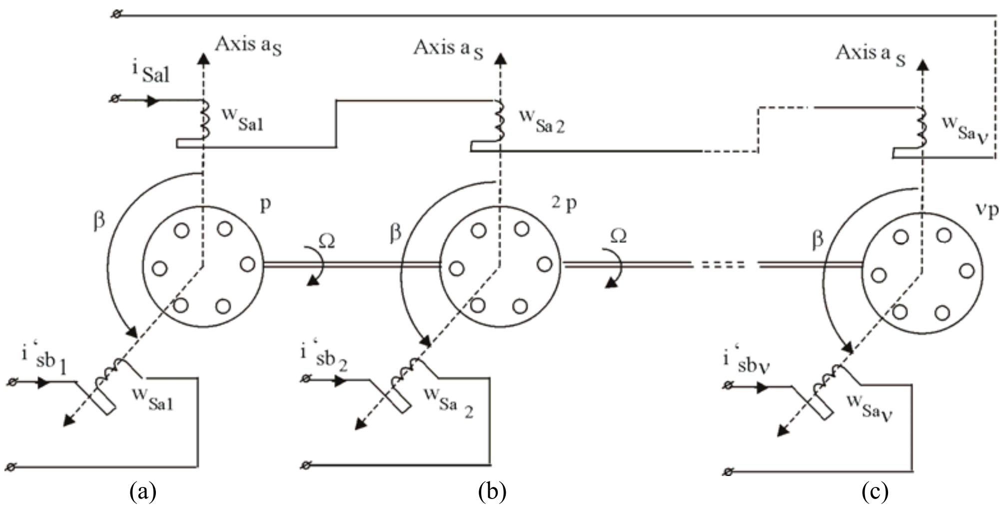


Fig.2. Two-phased machine represented by two-phased equivalent machines of harmonic ν
a) Two-phased machine of harmonic 1
b) Two-phased machine of harmonic 2
c) Two-phased machine of harmonic ν

The windings are made by filiform conductors placed sinusoidal on the exterior surface of armature and each determine in the air gap, only mmf wave of harmonic ν. Due to the constant air gap, the magnetic waves produced by the stator windings of the machine of harmonic ν have the expressions:

$$b_{Sa_\nu}(\theta_S,t) = \frac{\mu_0}{\delta_e} f_{Sa_\nu}(\theta_S,t) \tag{5}$$

$$b_{Sb_\nu}(\theta_S,t) = \frac{\mu_0}{\delta_e} f_{Sb_\nu}(\theta_S,t)$$

Particular case

If consider only the spatial harmonic 1 and 3, in the hypothesis of using of the model shown in Fig. 2, the total magnetic flux of main winding determined only by stator windings is:

$$\phi_{aS} = L_{a1} \cdot i_{Sa} + L_{a1} \cdot cos(p\beta) \cdot i_{Sb1}' + L_{a3} \cdot i_{Sa} + L_{a3} \cdot cos(3p\beta) \cdot i_{Sb3}' \qquad (6)$$

$$L_{a1} = \left(\frac{2}{\pi}\right)^2 \cdot \mu_0 \cdot \frac{w_{Sa1}^2}{p \cdot \delta_e} \cdot \tau \cdot l_i$$

$$L_{a3} = \left(\frac{2}{\pi}\right)^2 \cdot \mu_0 \cdot \frac{w_{Sa3}^2}{p \cdot \delta_e} \cdot \tau \cdot l_i$$

where L_{a1} and L_{a3} are the self inductances of the main windings from equivalent machines of fundamental harmonic and harmonic 3.

The air gap magnetic waves of the two-phased machine of harmonic ν produced by the stator windings (5), can be decomposed in sum of progressive waves:

$$b_{Sa_\nu}(\theta_S, t) = b_{Sad_\nu}(\theta_S, t) + b_{Sai_\nu}(\theta_S, t)$$

$$b_{Sad_\nu}(\theta_S, t) = \frac{1}{2} \cdot \frac{2}{\pi} \cdot \mu_0 \cdot \frac{w_{Sa_\nu}}{p \cdot \delta} \cdot I_{Sa}\sqrt{2} \cdot sin(\omega_S t - \varphi - \nu \cdot p\theta_S)$$

$$b_{Sai_\nu}(\theta_S, t) = \frac{1}{2} \cdot \frac{2}{\pi} \cdot \mu_0 \cdot \frac{w_{Sa_\nu}}{p \cdot \delta} \cdot I_{Sa}\sqrt{2} \cdot sin(\omega_S t - \varphi + \nu \cdot p\theta_S)$$

$$b_{Sb_\nu}(\theta_S, t) = b_{Sbd_\nu}(\theta_S, t) + b_{Sbi_\nu}(\theta_S, t)$$

$$b_{Sbd_\nu}(\theta_S, t) = \frac{1}{2} \cdot \frac{2}{\pi} \cdot \mu_0 \cdot \frac{w_{Sa_\nu}}{p \cdot \delta} \cdot I_{Sb_\nu}'\sqrt{2} \cdot sin[\omega_S t - \gamma - \nu \cdot p(\theta_S - \beta)]$$

$$b_{Sbi_\nu}(\theta_S, t) = \frac{1}{2} \cdot \frac{2}{\pi} \cdot \mu_0 \cdot \frac{w_{Sa_\nu}}{p \cdot \delta} \cdot I_{Sb_\nu}'\sqrt{2} \cdot sin[\omega_S t - \gamma + \nu \cdot p(\theta_S - \beta)]$$

resulting that in the machine air gap it actuates the forward traveling magnetic wave:

$$b_{Sd\nu}(\theta_S,t)=b_{Sad_\nu}(\theta_S,t)+b_{Sbd_\nu}(\theta_S,t)=$$

$$=\frac{1}{\pi}\cdot\mu_0\cdot\frac{w_{Sa\nu}}{p\cdot\delta}\cdot I_{Sd\nu}\sqrt{2}\cdot\sin\left[\omega_S t-\nu\cdot p\theta_S-\xi_{d\nu}\right] \quad (7)$$

$$I_{Sd\nu}=\sqrt{I_{Sd1_\nu}{}^2+I_{Sd2_\nu}{}^2}\,, \qquad \xi_{d\nu}=arctg\left(\frac{I_{Sd2_\nu}}{I_{Sd1_\nu}}\right)$$

$$I_{Sd1_\nu}=I_{Sa}\cdot\cos\varphi+I_{Sb_\nu}{}'\cdot\cos(\gamma-\nu\cdot p\beta)$$

$$I_{Sd2_\nu}=I_{Sa}\cdot\sin\varphi+I_{Sb_\nu}{}'\cdot\sin(\gamma-\nu\cdot p\beta)$$

respectively, the backward traveling magnetic wave:

$$b_{Si\nu}(\theta_S,t)=b_{Sai_\nu}(\theta_S,t)+b_{Sbi_\nu}(\theta_S,t)=$$

$$=\frac{1}{\pi}\cdot\mu_0\cdot\frac{w_{Sa\nu}}{p\cdot\delta}\cdot I_{Si\nu}\sqrt{2}\cdot\sin\left[\omega_S t+\nu\cdot p\theta_S-\xi_{i\nu}\right] \quad (8)$$

$$I_{Si\nu}=\sqrt{I_{Si1_\nu}{}^2+I_{Si2_\nu}{}^2}\,, \qquad \xi_{i\nu}=arctg\left(\frac{I_{Si2_\nu}}{I_{Si1_\nu}}\right)$$

$$I_{Si1_\nu}=I_{Sa}\cdot\cos\varphi+I_{Sb_\nu}{}'\cdot\cos(\gamma+\nu\cdot p\beta)$$

$$I_{Si2_\nu}=I_{Sa}\cdot\sin\varphi+I_{Sb_\nu}{}'\cdot\sin(\gamma+\nu\cdot p\beta)$$

In the rotor phases in squirrel cage, the stator magnetic fields of forward and backward sequences produce motion-induced voltage determining polyphased systems of currents $i_{rd\nu}$ şi $i_{ri\nu}$ of direct and reverse sequences. The currents through the reference phase of the rotor have the expressions:

$$i_{r\,d_\nu} = I_{r\,d_\nu}\sqrt{2}\cdot sin\left(s_{d_\nu}\cdot\omega_S t - \zeta_{d_\nu} - \xi_{d_\nu}\right)$$

$$i_{r\,i_\nu} = I_{r\,i_\nu}\sqrt{2}\cdot sin\left(s_{i_\nu}\cdot\omega_S t - \zeta_{i_\nu} - \xi_{i_\nu}\right)$$

where:

- s_{d_ν} is the forward wave slip of harmonic ν:

$$s_{d_\nu} = (1-\nu) + \nu\cdot s$$

- s_{i_ν} is the backward wave slip of harmonic ν:

$$s_{i_\nu} = (1+\nu) - \nu\cdot s$$

The two systems of currents determine forward magnetic progressive wave b_{rd_ν} and backward b_{ri_ν} respectively, that can be written mathematically in terms of stator coordinates [6]:

$$b_{rd_\nu}(\theta_S,t) = \frac{3}{2}\frac{1}{\pi}\cdot\mu_0\cdot\frac{w_{e_\nu}}{p\cdot\delta}\cdot I_{rd_\nu}'\sqrt{2}\cdot sin\left[\omega_S t - \nu\cdot p\theta_S - \zeta_{d_\nu} - \xi_{d_\nu}\right] \quad (9)$$

$$b_{ri_\nu}(\theta_S,t) = \frac{3}{2}\frac{1}{\pi}\cdot\mu_0\cdot\frac{w_{e_\nu}}{p\cdot\delta}\cdot I_{ri_\nu}'\sqrt{2}\cdot sin\left[\omega_S t + \nu\cdot p\theta_S - \zeta_{i_\nu} - \xi_{i_\nu}\right]$$

Due to the fact that the progressive magnetic waves are determined by systems of symmetrical and balanced currents flowing through symmetrical windings polyphased, the rotor of the two-phased machine can be assimilate to two three-phased windings fixed on stator. For the easy expression, the static windings modeling the rotor windings have been related to the number of equivalent turns of the main stator winding. Practically, due to the magnitude of the superior harmonics can be considered adequately a model that to uses only the harmonics 1 and 3 (Fig. 3).

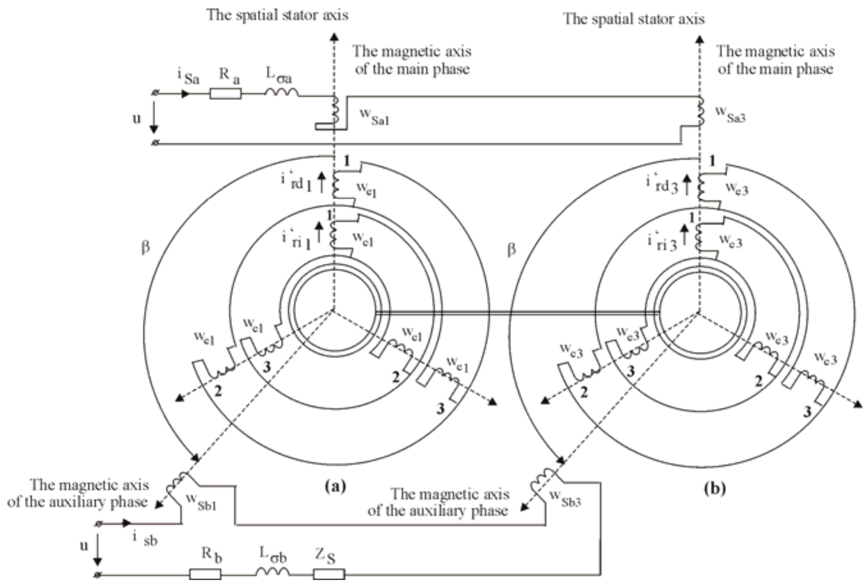


Fig.3. The static equivalent model of the two-phased machine with two-phased windings on stator
a) The equivalent machine of fundamental harmonic
b) The equivalent machine of harmonic 3

Particular case

In the hypothesis of using of the model shown in Fig. 3, the magnetic flux component from main winding determined only by rotor windings is:

$$\begin{aligned}\phi_{ar} &= L_{c\,ar\,1} \cdot i_{r\,d\,1}' + L_{c\,ar\,1} \cdot i_{r\,i\,1}' + L_{c\,ar\,3} \cdot i_{r\,d\,3}' + L_{c\,ar\,3} \cdot i_{r\,i\,3}' \\ &= \frac{1}{2} \cdot \left[L_{a\,1} \cdot \left(i_{r\,d\,1}' + i_{r\,i\,1}' \right) + L_{a\,3} \cdot \left(i_{r\,d\,3}' + i_{r\,i\,3}' \right) \right] \end{aligned} \tag{10}$$

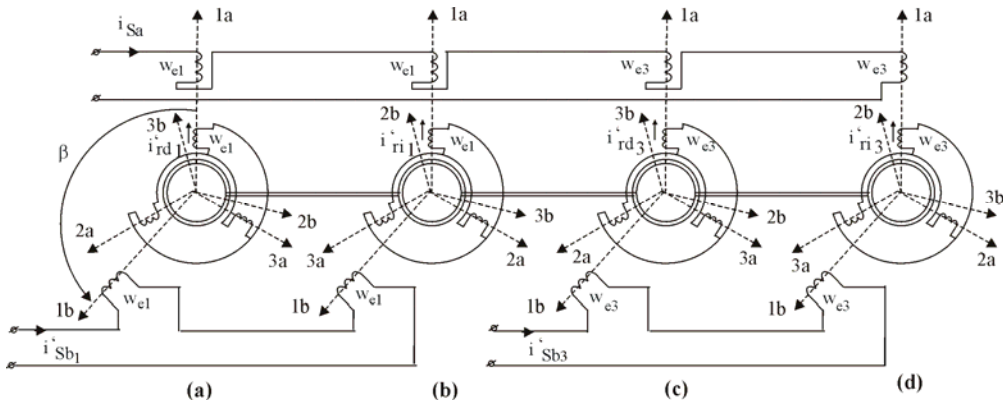
where the cyclic inductances $L_{c\,ar\,1}$ and $L_{c\,ar\,3}$ have the expressions [8]:

$$L_{c\,ar\,1} = \frac{3}{2} \cdot \left(\frac{2}{\pi} \right)^2 \cdot \mu_0 \cdot \frac{w_{Sa\,1} \cdot w_{e\,1}}{p \cdot \delta_e} \cdot \tau \cdot l_i = \frac{1}{2} \cdot \left(\frac{2}{\pi} \right)^2 \cdot \mu_0 \cdot \frac{w_{Sa\,1}^{\;2}}{p \cdot \delta_e} \cdot \tau \cdot l_i = \frac{1}{2} \cdot L_{a\,1}$$

$$L_{c\,ar\,3} = \frac{3}{2} \cdot \left(\frac{2}{\pi} \right)^2 \cdot \mu_0 \cdot \frac{w_{Sa\,3} \cdot w_{e\,3}}{3 \cdot p \cdot \delta_e} \cdot \tau \cdot l_i = \frac{1}{2} \cdot \left(\frac{2}{\pi} \right)^2 \cdot \mu_0 \cdot \frac{w_{Sa\,3}^{\;2}}{p \cdot \delta_e} \cdot \tau \cdot l_i = \frac{1}{2} \cdot L_{a\,3}$$

where $L_{a\,1}$ and $L_{a\,3}$ are the self inductances defined by equalities (6).

The stator single-phase windings from model shown in Fig. 3 can be assimilate to three phase, so that it is obtained the classic model of the two-phased induction machine, which for each harmonic ν is formed by two three-phased machines: a machine of direct sequence and a machine of reverse sequence (Fig. 4).



Observation. *They have been shown only the reference phases of the stator equivalent windings and integral rotor equivalent winding*

Fig.4. The static equivalent model of the two-phased machine using three-phased machines:

a) the direct machine of harmonic 1
b) the reverse machine of harmonic 1
c) the direct machine of harmonic 3
d) the reverse machine of harmonic 3

Particular case

If consider only the spatial harmonic 1 and 3, in the hypothesis of using of the model shown in Fig. 4, the total magnetic flux of reference phase determined only by stator three phase windings, is [8]:

$$\begin{aligned}
\phi_{as}' = {} & L_{c\,p\,1} \cdot i_{Sa} + L_{c\,p\,1} \cdot I_{Sb\,1}'\sqrt{2} \cdot sin(\omega_S t - \gamma + p\beta) + \\
& + L_{c\,p\,1} \cdot i_{Sa} + L_{c\,p\,1} \cdot I_{Sb\,1}'\sqrt{2} \cdot sin(\omega_S t - \gamma - p\beta) + \\
& + L_{c\,p\,3} \cdot i_{Sa} + L_{c\,p\,3} \cdot I_{Sb\,3}'\sqrt{2} \cdot sin(\omega_S t - \gamma + 3p\beta) + \\
& + L_{c\,p\,3} \cdot i_{Sa} + L_{c\,p\,3} \cdot I_{Sb\,3}'\sqrt{2} \cdot sin(\omega_S t - \gamma - 3p\beta) = \\
= {} & 2 \cdot L_{c\,p\,1} \cdot \left[i_{Sa} + i_{Sb\,1}' cos(p\beta) \right] + 2 \cdot L_{c\,p\,3} \cdot \left[i_{Sa} + i_{Sb\,3}' cos(3p\beta) \right] = \\
= {} & \frac{1}{3} \cdot \left[(L_{a\,1} + L_{a\,3}) \cdot i_{Sa} + L_{a\,1} cos(p\beta) \cdot i_{Sb\,1}' + L_{a\,3} cos(3p\beta) \cdot i_{Sb\,3}' \right]
\end{aligned} \qquad (11)$$

where $L_{c\,p\,1}$ and $L_{c\,p\,3}$ are the ciclic self inductances of harmonic [7]:

$$L_{c\,p\,1} = \frac{3}{2}\left(\frac{2}{\pi}\right)^2 \cdot \mu_0 \cdot \frac{{w_{e\,1}}^2}{p \cdot \delta_e} \cdot \tau \cdot l_i = \frac{1}{6}\left(\frac{2}{\pi}\right)^2 \cdot \mu_0 \cdot \frac{{w_{Sa\,1}}^2}{p \cdot \delta_e} \cdot \tau \cdot l_i = \frac{1}{6} \cdot L_{a\,1}$$

$$L_{c\,p\,3} = \frac{3}{2}\left(\frac{2}{\pi}\right)^2 \cdot \mu_0 \cdot \frac{{w_{e\,3}}^2}{9p \cdot \delta_e} \cdot \tau \cdot l_i = \frac{1}{6}\left(\frac{2}{\pi}\right)^2 \cdot \mu_0 \cdot \frac{{w_{Sa\,3}}^2}{9p \cdot \delta_e} \cdot \tau \cdot l_i = \frac{1}{6} \cdot L_{a\,3}$$

and rotor component has expression [6]:

$$\begin{aligned}\phi_{ar}' &= L_{c\,p\,1} \cdot i_{r\,d\,1}' + L_{c\,p\,1} \cdot i_{r\,i\,1}' + L_{c\,p\,3} \cdot i_{r\,d\,3}' + L_{c\,p\,3} \cdot i_{r\,i\,3}' = \\ &= \frac{1}{6} \cdot \left[L_{a\,1} \cdot \left(i_{r\,d\,1}' + i_{r\,i\,1}'\right) + L_{a\,3} \cdot \left(i_{r\,d\,3}' + i_{r\,i\,3}'\right)\right]\end{aligned} \tag{12}$$

The expression of the total flux is:

$$\phi_a' = \phi_{as}' + \phi_{ar}' \tag{13}$$

3. Voltage equations of stator windings

In order to determine the equations of the two-phased induction motor mathematic model and to determine the parameters of the model using the three-phased model, it is applied the Faraday's law for the main phase of the model shown in Fig. 3:

$$u = R_a \cdot i_{Sa} + L_{\sigma a} \cdot \frac{di_{Sa}}{dt} + \frac{d\phi_a}{dt} \tag{14}$$

$$\phi_a = \phi_{as} + \phi_{ar}$$

where:

- u is the supply voltage of the two stator windings: main and auxiliary, connected in parallel;
- R_a and $L_{\sigma a}$ are the resistance and the leakage inductance of the main winding, respectively;
- the magnetic flux ϕ_a has two components: ϕ_{as} determined by stator windings (6) and ϕ_{ar} determined by rotor windings (10).

Analyzing the expressions of the stator magnetic flux on phase (6) and (11) and analogous for rotor, from comparison of the expressions (10) and (12) of the magnetic fluxes on rotor phase results the dependence:

$$\phi_a' = \frac{\phi_a}{3} \tag{15}$$

Considering (15), in the voltage equation (14) it is obtained expression:

$$u = R_a \cdot i_{Sa} + L_{\sigma a} \cdot \frac{di_{Sa}}{dt} + 3 \cdot \frac{d\phi_a'}{dt}$$

respectively:

$$u_1 = R_{a3} \cdot i_{Sa} + L_{\sigma 3} \cdot \frac{di_{Sa}}{dt} + \frac{d\phi_a'}{dt} \quad (16)$$

where:

- the terminal voltage u_1 depends of supply voltage:

$$u_1 = \frac{u}{3}$$

- the resistance R_{a3} that corresponds with the phase resistance of the three-phased winding with w_{e1} turns on phase:

$$R_{a3} = \frac{R_a}{3}$$

- inductance that corresponds with same phase resistance of tree-phased winding with w_{e1} turns:

$$L_{\sigma a3} = \frac{L_{\sigma a}}{3}$$

The complete form of the voltage equation (16), for the main winding and stationary regime, written in complex is:

$$\underline{U}_1 = R_{a3}\underline{I}_{Sa} + j \cdot X_{\sigma a3}\underline{I}_{Sa} +$$
$$+ j \cdot X_{p1}\left(\underline{I}_{Sa} + \underline{I}_{Sb1}' \cdot e^{j \cdot p\beta} + \underline{I}_{rd1}'\right) + j \cdot X_{p1}\left(\underline{I}_{Sa} + \underline{I}_{Sb1}' \cdot e^{-j \cdot p\beta} + \underline{I}_{ri1}'\right) +$$
$$+ j \cdot X_{p3}\left(\underline{I}_{Sa} + \underline{I}_{Sb3}' \cdot e^{j \cdot 3p\beta} + \underline{I}_{rd3}'\right) + j \cdot X_{p3}\left(\underline{I}_{Sa} + \underline{I}_{Sb1}' \cdot e^{-j \cdot 3p\beta} + \underline{I}_{ri3}'\right) \quad (17)$$

The using of the magnetic flux expression ϕ_a' determined in the case of the equivalent model with three-phased machines (Fig. 4) has the crossing advantage of stator windings spatial displacement $\nu \cdot p\beta$ in the currents phase difference – see equation (17).

Equation (17) suggests a model of circuit for main winding (Fig. 5.a.), model where the auxiliary windings effects are represented by current sources that depend on both the current flowing through auxiliary winding and the spatial displacement of windings. For each direct harmonic or reverse, the rotor is modeled by resistances and the leakage reactances related corresponding of a phase.

Proceeding analogous for the auxiliary phase, it is obtained the mathematic model of the auxiliary winding. The equation of the auxiliary phase is:

$$\underline{U}_1' = R_{b3}' \underline{I}_{Sb1}' + j \cdot X_{\sigma b3}' \underline{I}_{Sb1}' + \underline{Z}_{S3}' \cdot \underline{I}_{Sb1}' +$$

$$+ j \cdot X_{p_1} \left(\underline{I}_{Sa} + \underline{I}_{Sb_1}' \cdot e^{j \cdot p\beta} + \underline{I}_{rd_1}' \right) \cdot e^{-j \cdot p\beta} +$$

$$+ j \cdot X_{p_1} \left(\underline{I}_{Sa} + \underline{I}_{Sb_1}' \cdot e^{-j \cdot p\beta} + \underline{I}_{ri_1}' \right) \cdot e^{j \cdot p\beta} +$$

$$+ j \cdot X_{p_3} \cdot k_w \cdot \left(\underline{I}_{Sa} + \underline{I}_{Sb_3}' \cdot e^{j \cdot 3p\beta} + \underline{I}_{rd_3}' \right) \cdot e^{-j \cdot 3p\beta} +$$

$$+ j \cdot X_{p_3} \cdot k_w \cdot \left(\underline{I}_{Sa} + \underline{I}_{Sb_1}' \cdot e^{-j \cdot 3p\beta} + \underline{I}_{ri_3}' \right) \cdot e^{j \cdot 3p\beta} \quad (18)$$

where:

$$\underline{U}_1' = \frac{w_{Sa1}}{w_{Sb1}} \cdot \underline{U}_1 ,$$

$$\underline{I}_{Sb_1}' = \frac{w_{Sb1}}{w_{Sa1}} \cdot \underline{I}_{Sb} ,$$

$$R_{b3}' = \frac{R_b'}{3} , \qquad R_b' = \left(\frac{w_{Sa1}}{w_{Sb1}} \right)^2 \cdot R_b ,$$

$$X_{\sigma b3}' = \frac{X_{\sigma b}'}{3} , \qquad X_{\sigma b}' = \left(\frac{w_{Sa1}}{w_{Sb1}} \right)^2 \cdot \omega_S L_{\sigma b} ,$$

$$\underline{Z}_{S3}' = \frac{\underline{Z}_S'}{3} , \qquad \underline{Z}_S' = \left(\frac{w_{Sa1}}{w_{Sb1}} \right)^2 \cdot \underline{Z}_S ,$$

$$k_w = \frac{w_{Sa\,1}}{w_{Sb\,1}} \cdot \frac{w_{Sb\,3}}{w_{Sa\,3}},$$

relations where R_b and $L_{\sigma b}$ represent the resistance and leakage inductance of the auxiliary winding and it has considered that the winding has connected in serial an additional consumer of impedance $\underline{Z}_S{}'$. The model of circuit corresponding with equation (18) is presented in Fig. 5.b where the effects of the main windings are represented by current sources.

By applying of the Kirchhoff's theorem for the two models, they are obtained n equations that in the knowledging hypothesis of both the effective value of the supply voltage and the circuit parameters for a certain value of slip, determine the currents flowing through the machine phase.

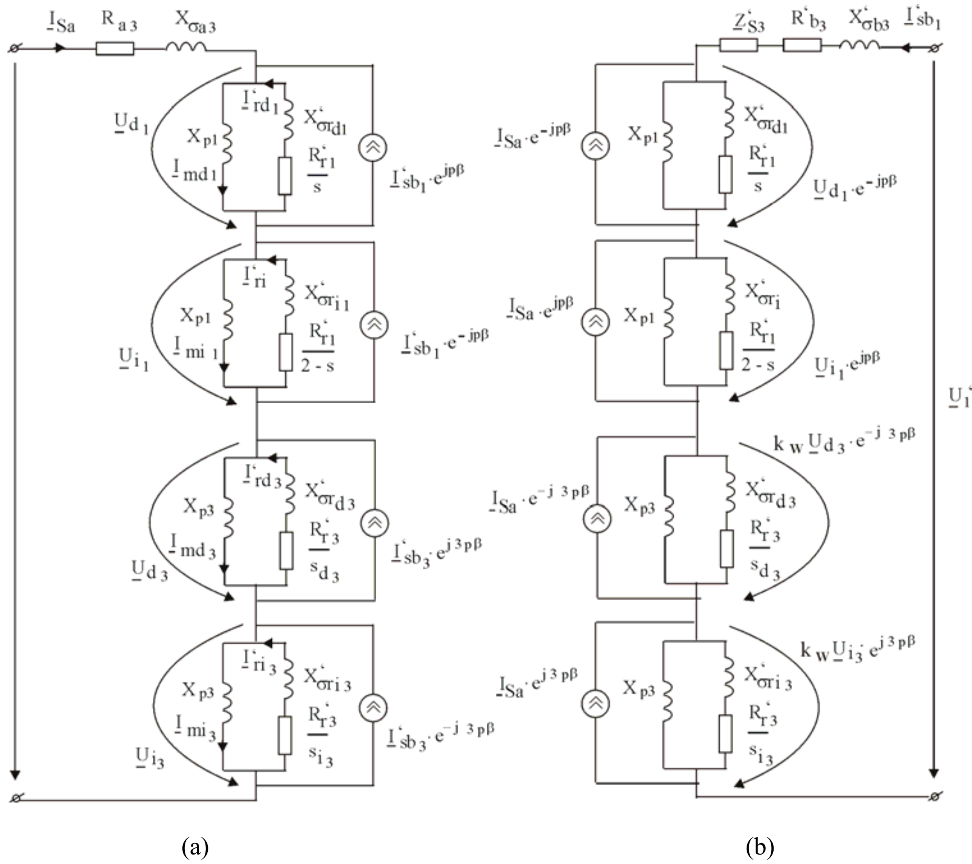


Fig. 5. The equivalent circuit model

a) for the main phase
b) for the auxiliary phase

Observation:

- In the case when the motor has a single phase, from the model of main phase (see Fig. 5.a.) by the annulment of current $\underline{I}_{Sb}$ it is obtained the classic model from literature [2], [7].
- In the case when the two phases are spatial displaced with angle $\beta = \frac{\pi}{2}$ it is obtained the model of two-phased induction motor with orthogonal winding [1], [2], [3].

Conclusions

In the paper is presented a new model of circuit for the two-phased induction motor with the different phases displaced non-orthogonal. The model has the advantage that uses theoretical computation methods of parameters analogue with the computation methods of the three-phased induction motors parameters and also, it considers the superior harmonics.

The model is simple and works directly with the parameters of phase and can be applied for all variety of two-phased induction machines.

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