ANALYSIS OF TRANSMISSION CHARACTERISTIC OF LIMACON GEAR

Dehua TAO*[1], Ning LI[2], Lijuan ZHENG[3], Xiao WANG[4]

The paper shows a non-circular gear named Limacon gear, which can achieve nonlinear transmission. The mathematical model of pitch curve and transmission characteristics of Limacon gear are established. Visual design platform for designing and analyzing Limacon gear is established. An example of Limacon gear mechanism is designed, and its transmission characteristics is analyzed, its 3D model is established and virtual prototype simulation test is carried out, the experimental curve is well fitted on the theoretical one, which verifies the correctness of the mathematical model of pitch curves and transmission characteristics of Limacon gear, and its feasibility in practical applications.

Keywords: Limacon gear, transmission characteristics, visual design platform, virtual prototype simulation test.

1. Introduction

Non-circular gear [1] is a mechanical mechanism, which can achieve nonlinear transmission. There are various types of non-circular gear widely used in variable speed instrument [2], such as Elliptical gear, Eccentric gear, Fourier non-circular gear, Sinusoidal gear and et al. Yazar, M[5] has used the polar equations and complex numbers of curves to design elliptical gears, the distribution of tooth stress and frequency analysis were preformed, the results prove that elliptical gears can be used in many industrial applications. Wang et al.[6] Proposed a new traverse mechanism which realizes this requirement that the thickness of the silk package formed by the traverse mechanism is uniform. XU et al.[7] Proposed the differential velocity vane pumps driven by the hybrid Higher-order Fourier non-circular gears, and the coupling dynamics is analyzed by means of fluid mechanics and solid mechanics simulation.

In this paper, a non-circular gear named Limacon gear is proposed. The mathematical model of pitch curves and transmission characteristics of Limacon gear are formulated by means of the geometric mathematics. A visual design platform for analyzing the transmission characteristics of Limacon gear is

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programmed by MATLAB. There are effect factors on Limacon gear analyzed by single factor analysis method. An example for verifying the correctness of the mathematical model of pitch curves and transmission characteristics of Limacon gear is chosen, and the 3D solid model is established and virtual prototype simulation test is tested by SolidWorks.

2. The principle of generating the Limacon curve

According to the principle of generating Limacon curve [8], the polar equation of the Limacon curve radii can be formulated as

$$R_1(\theta) = b\cos\theta + l \tag{1}$$

Where $R_1(\theta)$ is the Limacon curve radii, b, l, θ are, respectively, the diameter of pitch curve's generated circle, the fixed length, the angle between X axis and radius vector of arbitrary point on pitch curves. Fig.1 shows the pitch curve of Limacon gear with b is 5*mm* and l is 40*mm*.

The Limacon curves with different design parameters are shown in Fig.2 by taking 1.b=10*mm*,l=40*mm*(blank curve); 2. b=20*mm*, l=40*mm* (cyan curve); 3. b=30*mm*, l=40*mm* (red curve); 4. b=40*mm*,l=40*mm*(green curve);5. b=50*mm*, l=40*mm* (blue curve) respectively. It can be observed that, the shape of pitch curve is affected by the change of diameter of generated circle b and fixed length.

i) Where fixed length l is bigger than or equal to twice diameter of generated circle b, the Limacon curve is closed but without any concavity (blank curve and cyan curve);

ii) Where fixed length l is less than twice diameter of generated circle b, but bigger than diameter of generated circle, the Limacon curve is a concave curve without curling (red curve),;

iii) Where fixed length l is equal to diameter of generated circle b, the Limacon curve is heart-shaped (green curve), tooth profile on such Limacon curves is anisotropic, and its transmission is unstable;

iiii) Where fixed length l is less than diameter of generated circle b, the Limacon curve is a curled self-intersecting curve(blue curve), it's unable to generate tooth profile on such Limacon curves;

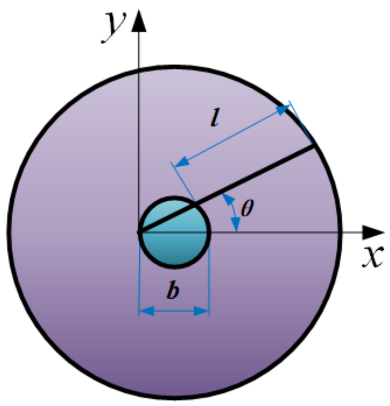
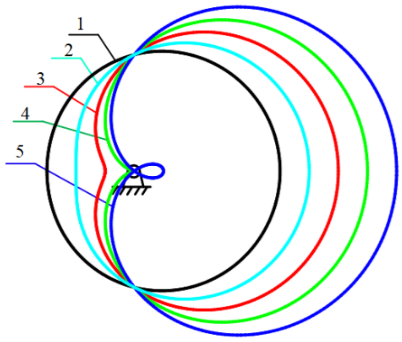


Fig.1 Limacon curve



1. b=10mm,l=40mm; 2.b=20mm,l=40mm;3. b=30mm, l=40mm; 4. b=40mm,l=40mm;5. b=50mm,l=40mm

Fig.2 Limacon curve with different designed parameters

3. Mathematical model of the Limacon gear and transmission characteristic

3.1 Mathematical model of the Limacon gear

Fig.3 shows the Limacon gear mechanism, the mechanism is schematically represented as consisting of two members: pitch line 1 is driving gear, pitch line 2 is driven gear. Driving gear rotates at a constant speed with an angular velocity ω, the driven gear rotates at a variable speed with an angular velocity of Ω, and the variable angular velocity of the driven gear depends on the design parameters of the Limacon gear.

In order to promise the condition for continuous movement [9], and the pitch curve of driven gear is closed, the relationship between rotating angle of driving gear with driven gear is

$$2\pi = \int_0^{2\pi} \frac{R_1(\theta)}{a - R_1(\theta)} \mathrm{d}\theta = \int_0^{2\pi} \frac{(b\cos\theta + l)}{a - (b\cos\theta + l)} d\theta \tag{2}$$

By solving the Eq.2, the distance of driving gear and driven gear can be deduced. By using the advance-retreat method, the interval for the distance a can be searched; and the precise value of distance a can be determined by using the golden section method.

By considering the distance a between driving gear and driven gear, the general mathematical model of pitch curve and rotating angle of driven gear are

$$\begin{cases} R_2(\delta) = a - R_1(\theta) \\ \delta(\theta) = \int_0^{\theta} \frac{1}{\tau(\theta)} d\theta = \int_0^{\theta} \frac{R_1(\theta)}{a - R_1(\theta)} d\theta = \int_0^{\theta} \frac{b\cos\theta + l}{a - (b\cos\theta + l)} d\theta \end{cases} \tag{3}$$

Where $\tau(\theta)$ is the gear ratio.

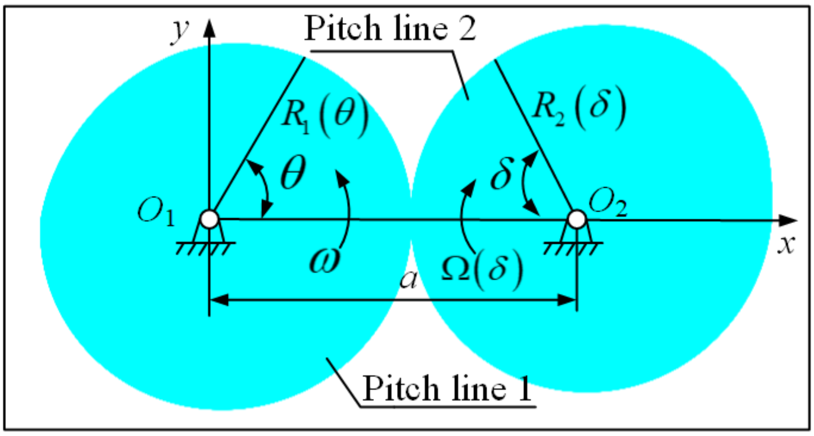


Fig.3. Schematic representation of Limacon gear mechanism

According to the principle of gear meshing, once ω and $\Omega(\delta(\theta))$ (or $R_1(\theta)$ and $R_2(\delta(\theta))$) are known, the variable gear ratio of Limacon gear can be evaluated as a function θ in the form

$$\tau(\theta)=\frac{\Omega(\delta(\theta))}{\omega}=\frac{R_1(\theta)}{R_2(\delta(\theta))} \tag{4}$$

3.2 Concavity and convexity

In order to machining the tooth profile successfully, and avoiding root cutting, the value of curvature of pitch curve of Limacon gear outweighs zero constantly, equation of pitch curve of Limacon gear can be formulated as

$$\rho=\frac{\left(R^2+\left(\frac{dR}{d\theta}\right)^2\right)^{3/2}}{R^2+2\left(\frac{dR}{d\theta}\right)^2-R\frac{d^2R}{d\theta^2}} \tag{5}$$

The following requirement must be observed: curvature radius should be greater than zero at every point.

Equation of pitch curve of driving and driven gear without concavity can be formulated as

$$\begin{cases}\rho_1=\dfrac{\left(l^2+b^2+2bl\cos\theta\right)^{3/2}}{l^2+3bl\cos\theta+2b^2}\geq 0\\ \rho_2=\dfrac{\left(a-R_1(\theta)\right)\left(R_1^{\,2}(\theta)+b^2\sin^2\theta\right)^{3/2}}{R_1^{\,3}(\theta)+\left(2b^2R_1(\theta)-ab^2\right)\sin^2\theta+\left(bR_1^{\,2}(\theta)-abR_1(\theta)\right)\cos\theta}\geq 0\end{cases} \tag{6}$$

After deduction, the condition that pitch curve of driving gear without concavity is: $b/l\leq 1/2$, satisfying case i) in Fig.2 in section 2.

According to the closure of the pitch curve and the uniformity of the gear teeth on the pitch curve, the arc length should meet the equation.

$$S=\int_0^{2\pi}\sqrt{b^2+l^2+2bl\cos\theta}\,d\theta=\pi mz \tag{7}$$

Where m is module of gear, and z is the number of gear teeth. Substituting equation (3) into equation (7) yields the required arc length of the driven gear's pitch curve.

3.3 Pressure angle

The conditions of force transmission by Limacon gear are varied in the process of motion due to variation of profile normal.

The pressure angle α_{12} is formed by $\boldsymbol{R}_n$ and the velocity v_{12} of the driven point, as shown in figure 4. It is given by

$$\alpha_{12}(\theta)=\mu(\theta)+\alpha_0-\frac{\pi}{2} \tag{8}$$

Wherein the driving profile is the left-side one, as shown in figure 4 (a).

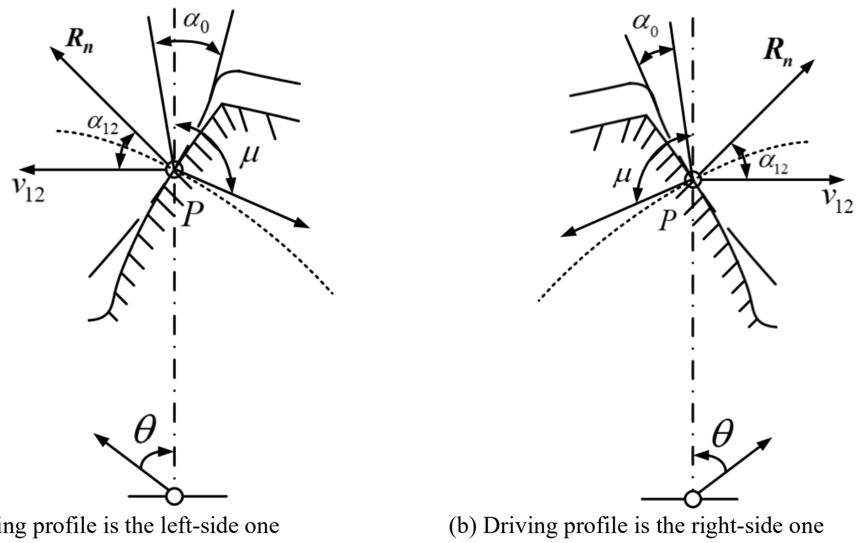
If the driving profile is the right-side one (figure 4 (b)), the formulation of the pressure angle is

$$\alpha_{12}(\theta)=\mu(\theta)-\alpha_0-\frac{\pi}{2} \tag{9}$$

Where α_0 is standard pressure angle of gear 20deg. $\mu(\theta)$ can be solved by the equation (10).

$$\tan\mu(\theta)=\frac{R(\theta)}{dR(\theta)/d\theta}=-\frac{\tau(\theta)+1}{\tau'(\theta)} \tag{10}$$

The following requirement must be observed: The pressure angle should not exceed the limiting value.



(a) Driving profile is the left-side one (b) Driving profile is the right-side one

Fig.4 Pressure angle

3.4 Contact ratio

Contact ratio [10] is one of the important indicators for measuring the performance of gear transmission, the value of contact ratio in the process of motion should be exceed the limiting value. Generally, the limiting value is 1.4.

The contact ratio of standard spur gears are constant ratio for all teeth of meshing, but it is not the case with the Limacon gears, the contact ratio of non-circular gear is shown in figure 5. By the definition of contact ratio, the formulation of the contact ratio is

$$\varepsilon=\frac{\sqrt{(\rho_1+h_{a1})^2-(\rho_1\cos\alpha_0)^2}-\rho_1\sin\alpha_0+\sqrt{(\rho_2+h_{a2})^2-(\rho_2\cos\alpha_0)^2}-\rho_2\sin\alpha_0}{\pi m\cos\alpha_0} \tag{11}$$

where ρ_1 , ρ_2 , h_{a1} , h_{a2} , α_0 and m are, respectively, curvature radius of driving gear and driven gear, addendum coefficient of driving gear and driven gear, standard pressure angle of gear is 20deg, module of gear.

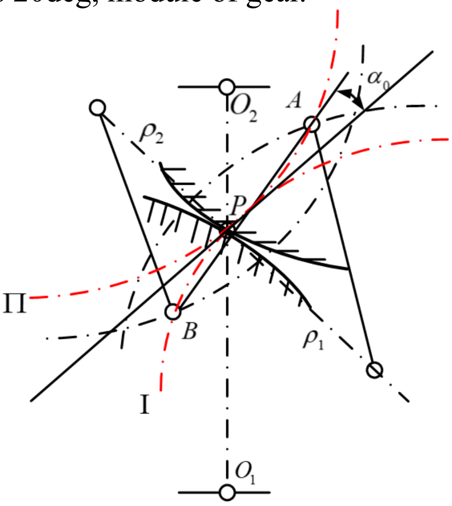


Fig.5 Contact ratio of non-circular gear

4. Visual design platform of the Limacon gear

For the sake of analyzing, a visual design platform of Limacon gear is established, as shown in Fig.6. The effect factor on transmission characteristics of Limacon gear can be analyzed by the software, and user can obtain the design parameters of Limacon gear for the design requirement.

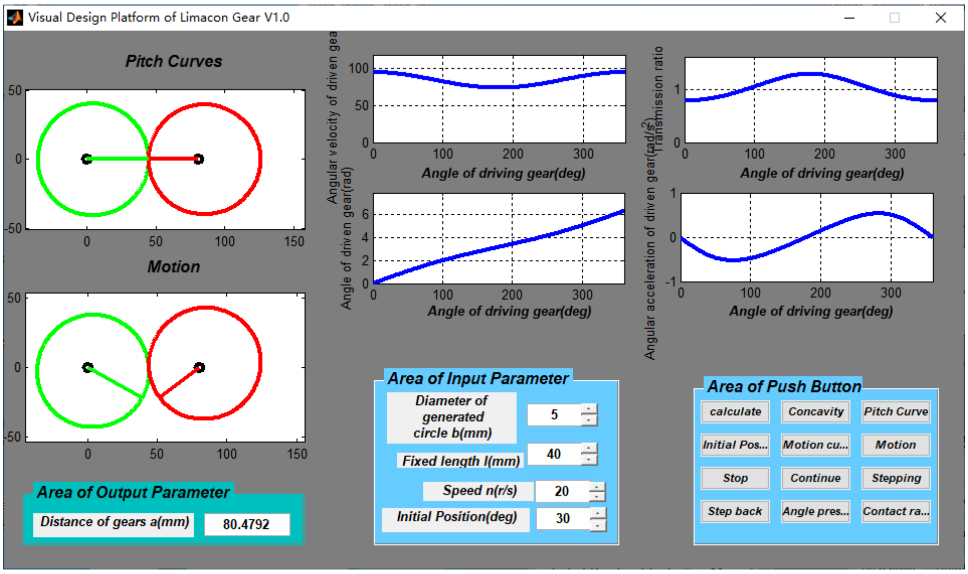


Fig.6. Visual design platform of Limacon gear

In this paper, generation of tooth profile of Limacon gear is based on the following equation:

$$\frac{dy}{d\theta} = \cos^2 \alpha_0 \left[R(\theta) - \frac{dR(\theta)}{d\theta} \tan \alpha_0 \right] \tag{12}$$

Where dy is the displacement of the contact point, as projected onto the normal direction to the Limacon gear crossing O_1 and O_2, $d\theta$ is the rotation of the Limacon gear.

In order to calculate coordinates of the tooth profile of Limacon gear, a software for generating tooth profile is developed by MATLAB, as shown in Fig.7.

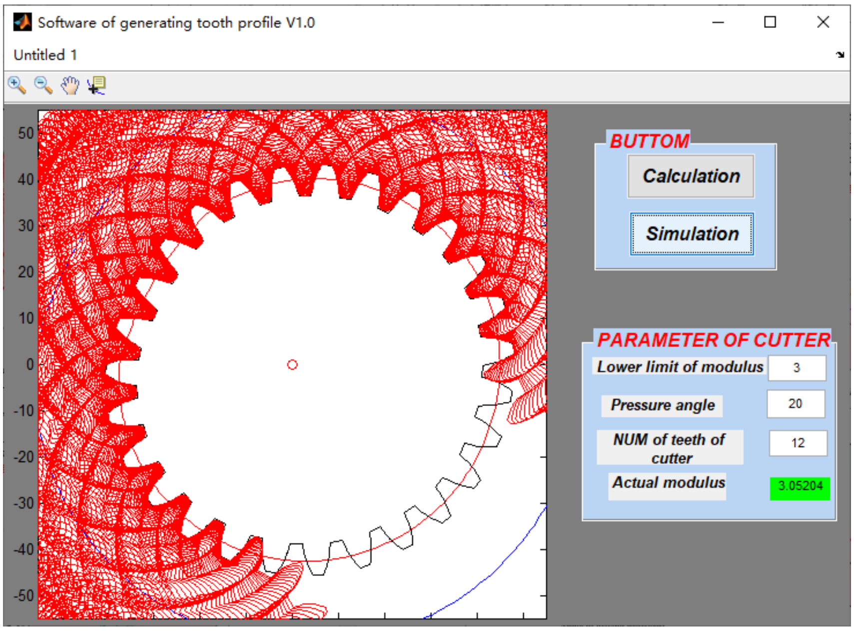
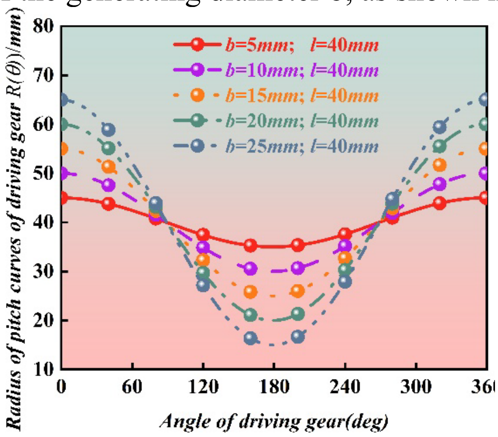
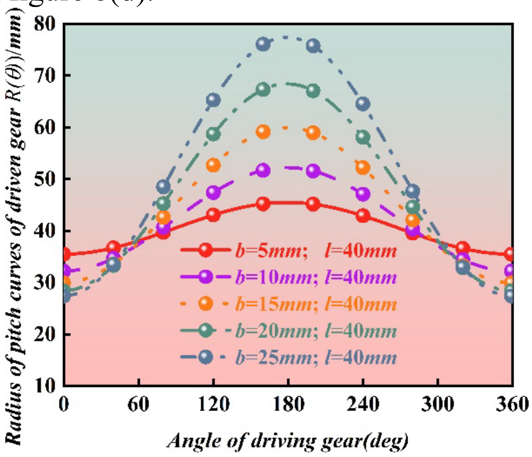


Fig.7 Software of generating tooth profile

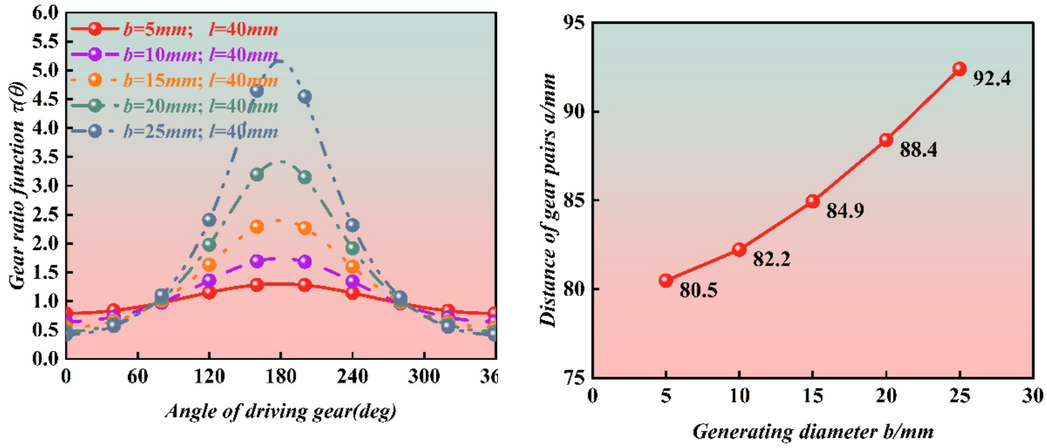
In order to analysis the effect of diameter of generated circle, the value of fixed length is maintained at 40mm, but the value of diameter of generated circle is changed. The radius of pitch curves of driving and driven gear, the gear ratio function with various values of parameter of generating diameter b are shown in Fig.8. The amplitude of the transmission ratio and radius of pitch curves of driving and driven gear increase along with the increase of the generating diameter b. In conclusion, the change of diameter of generated circle will influence the amplitude of gear ratio functions. The distance a of gear pair increases along with the increase of the generating diameter b, as shown in figure 8(d).



(a) Radius of pitch curves of driving gear



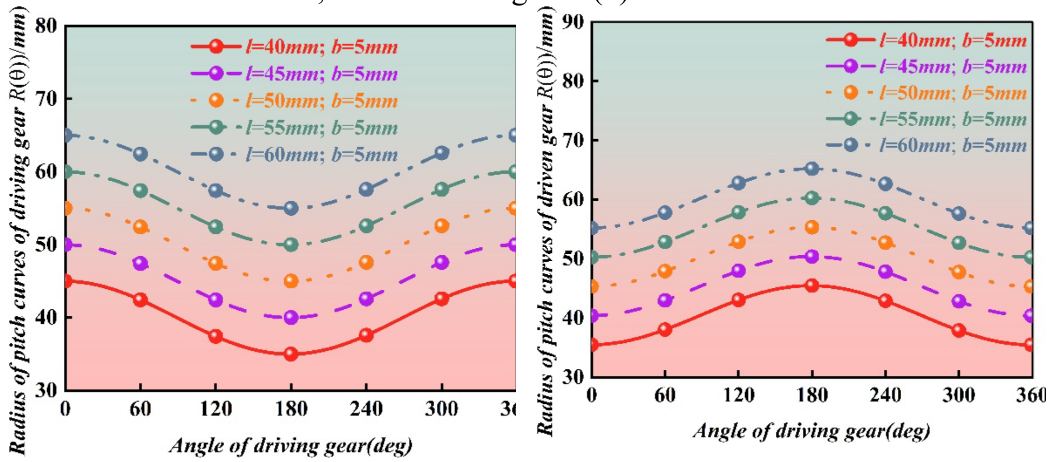
(b) Radius of pitch curves of driven gear



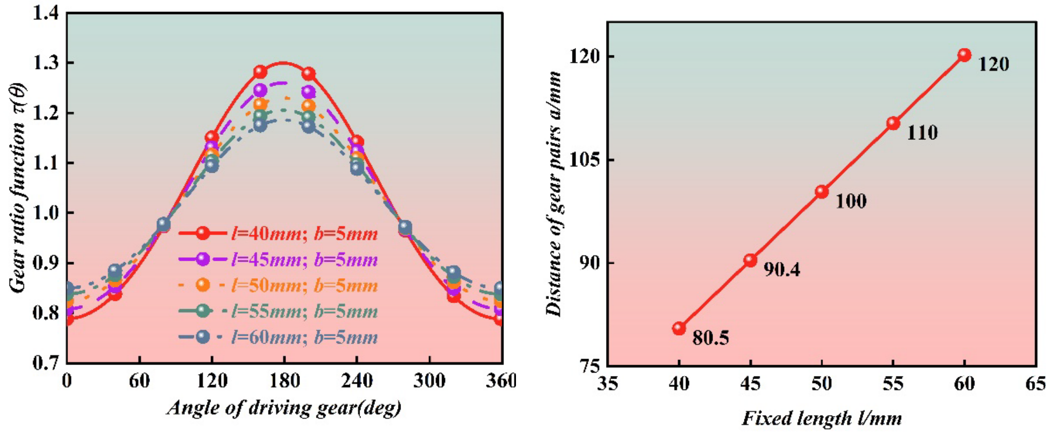
(c) Gear ratio function (d)Distance of gear pair

Fig.8 The impact of different diameter of generated circle

In order to analysis the effect of fixed length, the value of diameter of generated circle is maintained at 5mm, but the value of fixed length is changed. The radius of pitch curves of driving and driven gear, the gear ratio function with various values of fixed length l are shown in Fig.9. Radius of pitch curves of driving and driven gear increase with the fixed length, however, the change of fixed length has no impact on the amplitude of the radius of pitch curves of driving and driven gear. The amplitude of the gear ratio function decreases with the increase of fixed length, the influence of fixed length is more than the diameter of generated circle. The distance a of gear pair increases along with the increase of the fixed length l, and shows a linear correlation, as shown in figure 9(d).



(a)Radius of pitch curves of driving gear (b) Radius of pitch curves of driven gear



(c) Gear ratio function (d) Distance of gear pair

Fig.9 The impact of different fixed length

5. Examples, simulation and result analysis

In order to verify the correctness of the established mathematical model, an example with diameter of generated circle is 10*mm*, fixed length is 40*mm* and module is 4 is designed. The pitch curves and its tooth profile[11] of Limacon gear are generated as represented in Fig.10 by the visual design platform. Fig.11 shows the virtual prototype model of Limacon gear pair is established by SolidWorks.

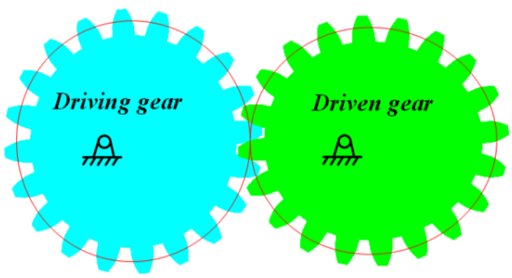


Fig.10 Pitch curve and its tooth profile of Limacon gear pairs

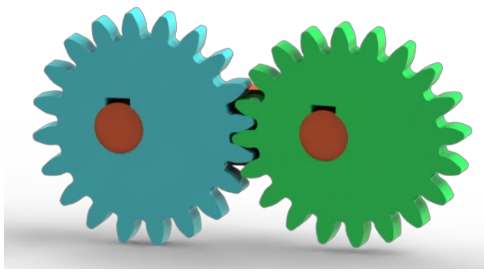


Fig.11 Virtual prototype model of Limacon gear pair

The range of transmission ratio variation of Limacon gear is [0.644478662, 1.74079777]; The range of pressure angle variation of Limacon gear is [5.52300048°, 34.47699645°], the maximum pressure angle is less than the maximum allowable pressure angle for self-locking of the gear [65°]; The range of contact ratio variation of Limacon gear is [1.557157814, 1.58298692], the minimum contact ratio is greater than the allowable contact ratio required for smooth gear transmission [1.4]; as shown in Fig.12. Proving that the Limacon gear mechanism has great transmission characteristics.

Fig.13 shows the relationship between theoretical and virtual simulation test gear ratio function of Limacon gear, the experimental curve is well fitted on the theoretical one, which verifies the correctness of the established mathematical model.

The cause analysis of not completely consistent:

i) The tooth profile calculation of Limacon gear is not accurate enough;

ii) The tooth profile modeling is established by using the built-in spline curve fitting in SolidWorks software, which may result in modeling errors.

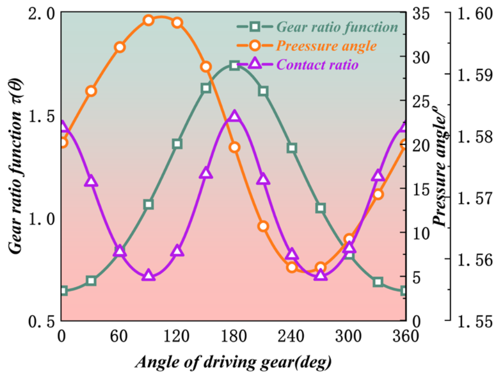


Fig.12 transmission characteristics of Limacon gear

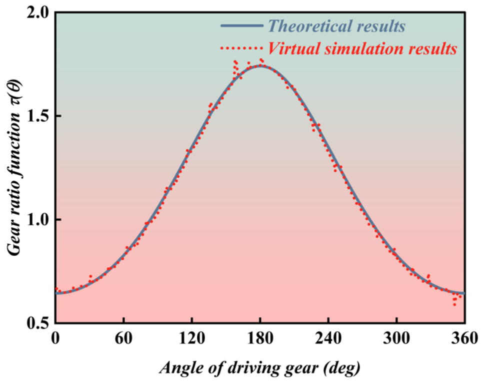


Fig.13 Comparison between theoretical results and virtual simulation test results

By looking at results in Fig.8-13 a comparison of characteristics can be outlined in term of the following observations:

- The pitch curve of driving gear and the transmission ratio curve is always symmetrical about a polar angle of 180 degrees.

- The amplitude of gear ratio functions increase along with the increase of the generated diameter, and decreases with the increase of fixed length.

- If fixed length *l* is bigger than or equal to twice diameter of generated circle b (curve 1 and 2 in Fig.2), the pitch curve of Limacon gear is closed without concavity, and the pressure angle curve and coincidence degree curve are smooth and have no abrupt changes, proving it has great transmission characteristics.

6. Conclusions

In this paper a non-circular gear, namely, Limacon gear mechanism has been designed to achieve nonlinear transmission. Mathematical model of pitch curves and transmission characteristics of Limacon gear have been established. A visual design platform for designing and analyzing Limacon gear has been established, and the effect factors on the pitch curves and transmission characteristics of Limacon gear have been analyzing, the change of diameter of generated circle and fixed length will influence the amplitude of gear ratio functions, the impact of diameter of generated circle is greater than that of fixed length. An example with Limacon gear has been designed, and its transmission characteristics has been analyzed, its 3D model has been established by SolidWorks, and virtual prototype simulation test have been carried out. Gear ratio function of virtual simulation test approximate closely the theoretical, with small deviations throughout the entire transmission process, which verifies the correctness of the mathematical model of pitch curves and transmission characteristics of Limacon gear, and its feasibility in practical applications.

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