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SOLID ROCKET MOTOR INTERNAL BALLISTICS SIMULATION USING DIFFERENT BURNING RATE MODELS

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The burning rate of solid rocket propellants is measured in Crawford bomb and is expressed by Vieille's law. Actual burning rate in solid motor rocket can be greater or less than the burning rate measured in Crawford bomb. Determination of different impacts on the actual burning rate is a complex task, which includes introduction of several assumptions in order to estimate influence of erosive burning or hump effect on the total burning rate.

The realized experimental studies involve determining the burning rate for the PRTF 111 double base propellant and static firings for the S-5K solid rocket motor in S.C. Electromecanica Ploieşti S.A. The developed 0-D internal ballistic module has the capability to provide a high degree of accuracy information on solid rocket motor performances.

Keywords: solid rocket motors, double base propellant, burning rate

1. Introduction

The combustion mechanisms for solid rocket propellants are quite complex and dependent on many local fluid, chemical, and thermal phenomena. Many solid rocket propellant burning-rate models are greatly simplified because of limited computational power and understanding of the combustion process.

There are several quasi-steady formulations to predict the burning rate of an energetic solid material. One of them is the Vieille's or Saint Robert's law, which is an empirical model [1].

Erosive burning refers to increase in the propellant burning rate caused by the high-velocity flow of combustion gases over the burning propellant surface. In this regard, Lenoir and Robillard [2] developed an expression for the rate of steady burning produced by erosive heating. In 1968, Lawrence proposed a modified Lenoir and Robillard (L-R) erosive burning model that was more accurate for large-scale motors [3].

A further improvement to the L-R model is presented by the authors of the solid propellant rocket motor performance computer program (SPP) [4] using the work from R. A. Beddini [5]. These improvements retain the heat-transfer theory

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of the original L-R model, but also improve the ability of the model to predict the erosive burning contributions for large-scale motors.

In order to use within a 0-D internal ballistic module, the L-R erosive burning model raises a number of adaptations; for this reason, in this paper, the author submits the replacement of the mass flux term, which is specific for dimensional flow analysis.

Influence of the solid propellant grain processing, which is called hump effect, consider a radial variation of burning rate as a function of the web burned. The greatest influence of the hump effect appears at the half of the web thickness, and calculations have shown the burning rates were increased in 3 to 7% [6], compared to the basic burning rate.

2. Theoretical aspects

The mathematical model which describes rocket motor behavior is based on the continuity equation of mass in zero-dimensional form. Basic assumptions for this model are:

- the working substance (chemical reaction products) is homogeneous;
- all the species of the working fluid are gaseous; any condensed phases (liquid or solid) add a negligible amount to the total mass;
- the working substance obeys the perfect gas law;
- there is no heat transfer across the rocket walls; therefore, the flow is adiabatic;
- chemical equilibrium is established within the rocket chamber and the gas composition does not change in the nozzle (frozen flow).

Calculation of pressure inside a rocket motor (combustion chamber) as a function of time is based on the continuity equation – mass of gas generated by combustion of propellant charge is equal to the sum of the mass of combustion products accumulated in a rocket motor and mass of combustion products through nozzle [7]:

$$\frac{dp}{dt}=\frac{RT}{V_c}\left[r\,A_b\left(\rho_p-\frac{p}{RT}\right)-\sqrt{\frac{\gamma}{RT}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}\,A_{tr}\,p\right]. \quad (1)$$

The burning rate of solid propellant is r, p is the chamber pressure, R is the specific gas constant, T is the absolute chamber temperature, ρ_p is the solid propellant density, γ is the specific heat ratio of the combustion gases, V_c is the chamber gas cavity volume, A_b is the solid propellant burning area, and A_{tr} is the nozzle throat area.

For many double-base propellants, also referred to as smokeless powders, it was established that the burning rates obey a relationship known as Vieille's or Saint Robert's empirical law, represented by Eq. (2), in which the burning rate is proportional to the pressure raised to a power n, known as the pressure exponent of burning rate [1]:

$$r_0 = a\, p^n . \tag{2}$$

The exponent n needs to be less than one for stable burning (slow deflagration, versus rapid explosive rate). For common double-base propellants, n can range from 0.2 to 0.5. Coefficient a is a function of the propellant's initial temperature T_p:

$$a = a_0\, e^{\sigma_p \left(T_p - T_0\right)} , \tag{3}$$

where a_0 and T_0 are defined for reference conditions and σ_p is the temperature sensitivity of burning rate. Values for σ_p typically range between 0.001 and 0.009 per degree Kelvin.

The positive erosive burning augmentation on the base burning rate (r_0) has been observed experimentally over the years to be more evident at lower port-to-throat area ratios (A_p / A_{tr}). Influence on burning rate in rocket motor chamber is considered using a relatively model for erosive burning, based on heat transfer, first developed in 1956 by Lenoir and Robillard [2] and modified by author. In this model, total burning rate contains a component of burning rate without erosion, r_0, and r_e, the increase in burn rate due to erosion effects:

$$r_b = r_0 + r_e . \tag{4}$$

The modified Lenoir and Robillard model defines the erosive burning contribution as:

$$r_e = \alpha \left(\rho_p\, r_b \frac{A_b}{A_p} \right)^{0.8} L^{-0.2}\, e^{-\beta A_p / A_b} , \tag{5}$$

where L is the characteristic length of solid propellant grain and β is empirically constant with a value of about 270 for a for a nitropolymer propellant [8].

The expression of α was determined from heat transfer considerations to be:

$$\alpha = \frac{0.0288\, c_p\, \mu^{0.2}\, \mathrm{Pr}^{-2/3}}{\rho_p\, c_s} \frac{T_p - T_s}{T_s - T_0} . \tag{6}$$

Here c_p is specific heat of the combustion gases, μ is the absolute viscosity of the gas and Pr is the effective Prandtl number ($\mu c_p / k$) of the gas, where k is the thermal conductivity of the gas, c_s is the heat capacity of solid propellant, and T_s is the solid propellant surface temperature.

Variation of the burning rate, caused by hump effect, is defined as a ratio of actual burning rate (within a real rocket motor) to the basic burning rate (measured by Crawford bomb) under same chamber pressures [9]:

$$\delta_{hump} = \frac{1}{A_b\left(\rho_p - \frac{p}{RT}\right)}\left(\frac{V_c}{RT}\frac{dp}{dt} + \sqrt{\frac{\gamma}{RT}\left(\frac{2}{\gamma+1}\right)^{(\gamma+1)/(\gamma-1)}}A_{tr}\,p\right)\cdot\frac{1}{r_0(p)}. \quad (7)$$

3. Experimental research

Basic composition of double-base rocket propellant PRTF 111 is consisted of 56% nitrocellulose (NC) with 11.8% nitrogen (N) as a binder, 26.7% nitroglycerin (NG) as plasticizer, 10.5% dinitrotoluene (DNT) as plasticizer, 3% ethyl centralite (EC) as stabilizer, 1.7% calcium carbonate as stabilizer, 1.2% fuel oil or wax as plasticizer and 0.9% lead oxide as burning rate catalyst.

The combustion chamber used at S.C. UPS Făgăraş S.A. for measurements of burning rate is called a "strand burner" (Crawford bomb). The burning rate is obtained as a function of pressure and of initial temperature, from which pressure exponent of burning rate and temperature sensitivity of burning rate are deduced.

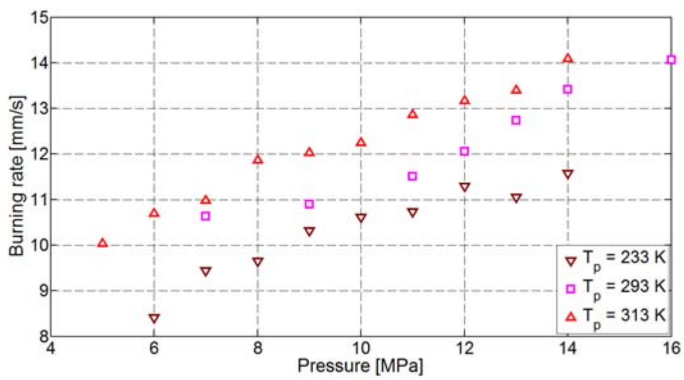


Fig. 1. Burning rate laws of the PRTF 111 propellant measured in the Crawford bomb

Static testing of the S-5K rocket motor, for conditioned temperatures of 223 K, 293 K and 323 K, to obtain the pressure-time characteristics are realized by author at S.C. Electromecanica Ploieşti S.A.

Acquiring accurate chamber pressure readings over the duration of the motor burn is an important aspect of static testing. From pressure data, it is possible to derive key performance parameters such as thrust, total impulse, specific impulse and characteristic velocity. It is also possible to determine how well a motor performed comparatively to design prediction, and such data can provide indication of anomalous behavior such as erosive burning. Data acquisition system consist of one HBM P3MB pressure transducer and for this static testing recording rate is 1000 samples per second.

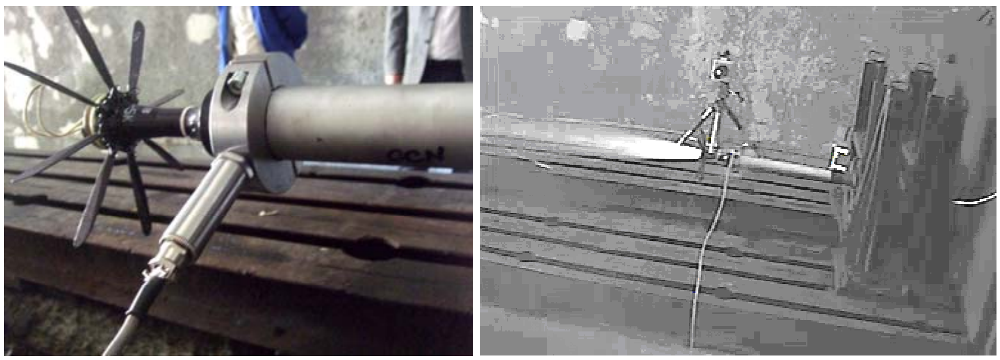


Fig. 2. HBM P3MB pressure transducer mounted on S-5K rocket motor

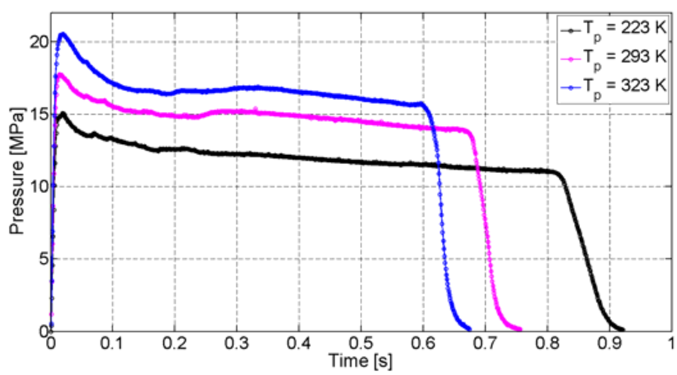


Fig. 3. Pressure-time curves for the S-5K rocket motor, experimental results obtained by the author

4. Results

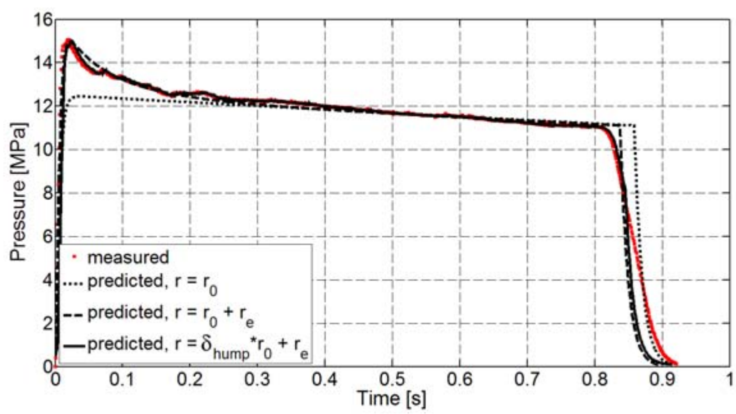
The elements balance equations and the second law of thermodynamics are used to determine the equilibrium composition of the combustion products. This model is based on minimization of the Gibbs free energy, and the resulted constrained optimization problem uses the Lagrange multiplier method [10], [11]. The program calculates chemical equilibrium product concentrations and determines thermodynamic and transport properties for the gaseous combustion products mixture at constant-pressure condition.

For example, we present some obtained results at PRTF 111 solid propellant combustion, for chamber pressure at 15 Mpa, needful for solid rocket motor internal ballistics simulation: T_p = 2077.542 K, R = 376.175 J/(kg·K), γ = 1.255, c_p = 1851.247 J/(kg·K), μ = 1.251·10^{-4} N·s/m^2, and Pr = 0.797.

The parameters of the burning law are evaluated from measured data in Crawford bomb by the help of linear regression using the least squares method. The result of computing is the burning law in form:

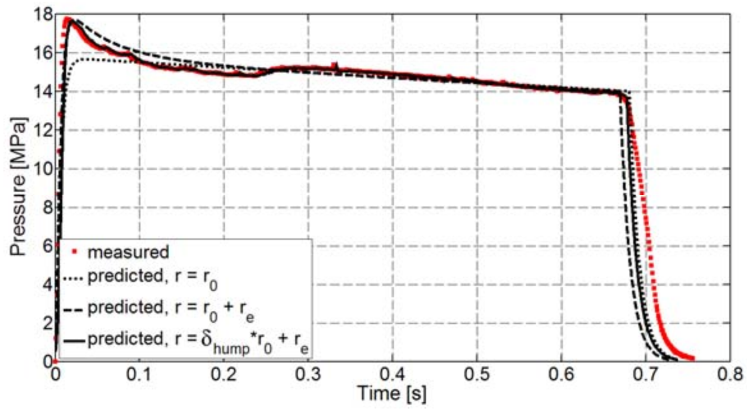
$$r = 5.332 \cdot 10^{-5} e^{2.218 \cdot 10^{-3}\left(T_p - T_0\right)} p^{0.336} . \tag{8}$$

The chamber pressure is solved using Eq. (1) in a fourth-order Runge-Kutta time scheme. Within Fig. 4 to 6 there are presented in a comparative manner results of chamber pressure vs. time for prediction models and experimental test. In simulation, basic burning rate is corrected with erosive burning influence. Also, influence of hump effect was analyzed. The prediction has shown good agreement with test results.



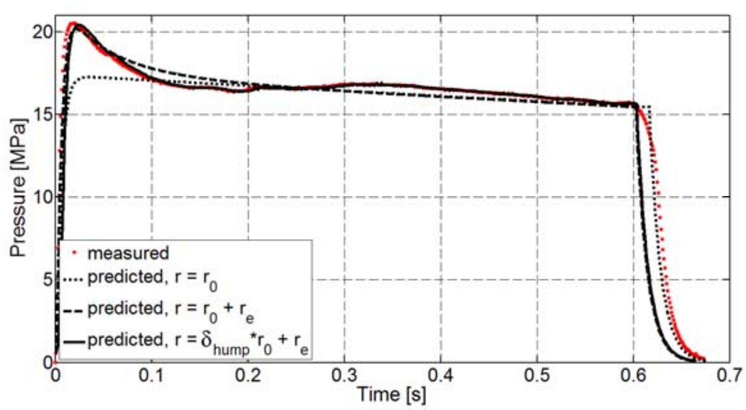
	measured	predicted, $r = r_0$	predicted, $r = r_0 + r_e$	predicted, $r = \delta_{hump}\, r_0 + r_e$
Burn time, s	0.819	0.857	0.836	0.844
Peak pressure, MPa	15.069	12.452	14.959	15.016
Medium pressure, MPa	12.059	11.778	12.025	11.996
Total pressure integral, MPa·s	10.319	10.189	10.182	10.184

Fig. 4. Chamber pressure vs. time for the S-5K rocket motor, T_p = 223 K



	measured	predicted, $r = r_0$	predicted, $r = r_0 + r_e$	predicted, $r = \delta_{hump}\, r_0 + r_e$
Burn time, s	0.675	0.679	0.668	0.676
Peak pressure, MPa	17.739	15.653	17.677	17.637
Medium pressure, MPa	14.935	14.824	15.026	14.918
Total pressure integral, MPa·s	10.417	10.182	10.180	10.192

Fig. 5. Chamber pressure vs. time for the S-5K rocket motor, $T_p = 293$ K



	measured	predicted, $r = r_0$	predicted, $r = r_0 + r_e$	predicted, $r = \delta_{hump}\, r_0 + r_e$
Burn time, s	0.610	0.615	0.601	0.602
Peak pressure, MPa	20.530	17.259	20.186	20.441
Medium pressure, MPa	16.695	16.353	16.669	16.724
Total pressure integral, MPa·s	10.492	10.185	10.183	10.188

Fig. 6. Chamber pressure vs. time for the S-5K rocket motor, $T_p = 323$ K

5. Conclusions

The 0-D internal ballistics model provides the capability to approximate first-order rocket performance. Two main observations deserve to be highlighted:

- For the safety and suitability for service (S3) of solid rocket motor it is mandatory taking into consideration the influence of erosive burning when determining the peak pressure. In the study, the usage of Vieille's burning rate law provides considerable differences between numerical prediction and experimental results, ranging from 12 to 18%.
- For evaluations of key performance parameters such as total impulse or specific impulse a basic model can be used. The differences between numerical prediction and experimental results are within 3% for total pressure integral, and also for the time integral of the thrust.

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