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EVALUATING THE PERFORMANCE AND CLASSIFYING THE INTERVAL DATA IN DATA ENVELOPMENT ANALYSIS

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Standard data envelopment analysis (DEA) supposes that measure status from point of view input or output is known. Nevertheless, in some situations, determining the status of a performance measure is not easy. Measures with unknown status of input /output are called flexible measures. Moreover, traditional DEA models do not deal with imprecise data and assume that all input and output are exactly known. This paper proposes methods to evaluate efficiency and to classify data where the inputs and outputs of decision making units (DMUs) are known to lie within bounded interval and where flexible measures exist. A sample of bank branches is used to illustrate the application of the proposed models.

Keywords: DEA, Interval data, Flexible measures, Efficiency

1. Introduction

Data envelopment Analysis (abbreviated as DEA and originated by Charnes et al. [1]) has been known as an effective tool for measuring the relative efficiency of peer decision making units (DMUs) with multiple inputs and outputs. We see that the conventional application of DEA assumes that each measure should be assigned an explicit designation specifying whether it is an input or output. Assessing the status of a performance measure is not clear in some situations, so Cook and Zhu [2] presented a mixed integer linear programming problem (MILP) to handle such flexible measures. Toloo [3] claimed that the Cook and Zhu's [2] model may produce incorrect efficiency scores due to a computational problem as a result of introducing a large positive number to the model. Amirteimoori et al. [4] mentioned that the revised model of Toloo is a special case of that of Cook and Zhu and that this revised model is infeasible in many real cases. Additionally, Amirteimoori et al. [5] proposed an alternative model to calculate the technical efficiency of DMUs with flexible measures. These all made Toloo [6] to consider alternative solutions for classifying inputs and outputs in data envelopment analysis.

Contrary to the claim of the original DEA models [1] that inputs and outputs are measured by exact values on a ratio scale we see that in some applications inputs and outputs are unknown decision variables such as bounded

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data, ordinal data, and ratio bounded data. Authors such as Despotis and Smirlis [7], Joe Zhu [8], and Wang et al. [9] provided papers on the theoretical development of DEA with interval data.

This paper considers a situation that variables are interval data while some measures can play either the role of input or output. Here in the study models with interval data are introduced that treat and identify the status of flexible measures. To illustrate the application of suggested models in real world let us reflect a study of evaluating bank's branches' efficiency to attract investments. In this case a factor such as 'deposits' can be assumed as a flexible and interval measure. It is clear that it can be assumed as an interval measure similar to Jahanshahloo et al. [10]; moreover, according to Cook and Zhu [2] it is a source of revenue for the branch thus it is regarded as an output. It should be considered here that, arguments have been made claiming that staff time expended in processing customers who are making deposits or opening deposit accounts, could be used as an advantage to sell more profitable products to the customers, this factor can be supposed as an input. Presentation of models with flexible and interval data is essential and beneficial because there are many situations in real world like the described one above.

This paper is organized as follows. Section 2 provides a method to evaluate efficiency and to determine the status of flexible measures in presence of interval data. Section 3 describes an alternative method for the same purpose of section 2. In the section 4, a numerical example is used for clarification. Conclusions are finally made in section 5.

2. A new method to determine the status of flexible measures in presence of interval data

Assume that there are n DMUs producing the same set of outputs by consuming the same set of inputs. Unit j is denoted by $\text{DMU}_\text{j}\ (j=1,2,...,n)$, whose *ith* input and *rth* output are denoted by $x_{ij}\ (i=1,...,m)$ and $y_{rj}\ (r=1,...,s)$, respectively. Contrary to the original DEA model, it is supposed that the levels of inputs and outputs are unknown; it is only known that the input-output values are in certain bounded intervals, i.e. $x_{ij}\in[x_{ij}^L, x_{ij}^U]$ and $y_{rj}\in[y_{rj}^L, y_{rj}^U]$, where upper and lower bounds of the intervals are given as fixed numbers and it is assumed that $x_{ij}^L>0$ and $y_{rj}^L>0$.

Let efficiency of DMU_j be equivalent to

$$\theta_j = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \quad j = 1, \ldots, n.$$

It is clear that θ_j should also be an interval number, let us assume

$$\theta_j = [\theta_j^L, \theta_j^U] = \left[\frac{\sum_{r=1}^{s} u_r y_{rj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U}, \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \right] \subseteq (0,1], \quad j = 1, \ldots, n.$$

The proposed models by Wang et al. [9] in order to measure the upper and lower bounds of the efficiency of DMU_0 are given by:

$$\begin{aligned} & \text{maximize} \quad \theta_{jo}^U = \frac{\sum_{r=1}^{s} u_r y_{rj_0}^U}{\sum_{i=1}^{m} v_i x_{ij_0}^L} \\ & \textit{subject to} \quad \theta_j^U = \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \le 1, \quad j = 1, \ldots, n, \\ & \qquad u_r, v_i \ge \varepsilon \quad \forall r, \forall i. \end{aligned} \tag{1}$$

and

$$\begin{aligned} & \text{maximize} \quad \theta_{jo}^L = \frac{\sum_{r=1}^{s} u_r y_{rj_0}^L}{\sum_{i=1}^{m} v_i x_{ij_0}^U} \\ & \textit{subject to} \quad \theta_j^U = \frac{\sum_{r=1}^{s} u_r y_{rj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L} \le 1, \quad j = 1, \ldots, n, \\ & \qquad u_r, v_i \ge \varepsilon \quad \forall r, \forall i. \end{aligned} \tag{2}$$

Now assume that there exist L flexible measures $w_{lj} (l = 1, \ldots, L)$, whose input/ output status is unidentified. For each measure l, like Cook and Zhu's [2] model, we introduce the binary variables $d_l \in \{0,1\}$, where $d_l = 1$ designates that measure l is an output, and $d_l = 0$ designates it as an input. Suppose γ_l be the weight for each measure l .

θ_j can be rewritten as:

$$\theta_j = \frac{\sum_{r=1}^{s} u_r [y_{rj}^L, y_{rj}^U] + \sum_{l=1}^{L} d_l \gamma_l [w_{lj}^L, w_{lj}^U]}{\sum_{i=1}^{m} v_i [x_{ij}^L, x_{ij}^U] + \sum_{l=1}^{L} (1-d_l)\gamma_l [w_{lj}^L, w_{lj}^U]} =$$

$$\frac{\left[\sum_{r=1}^{s} u_r y_{rj}^L, \sum_{r=1}^{s} u_r y_{rj}^U\right] + \left[\sum_{l=1}^{L} d_l \gamma_l w_{lj}^L, \sum_{l=1}^{L} d_l \gamma_l w_{lj}^U\right]}{\left[\sum_{i=1}^{m} v_i x_{ij}^L, \sum_{i=1}^{m} v_i x_{ij}^U\right] + \left[\sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^L, \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^U\right]} =$$

$$\frac{\left[\sum_{r=1}^{s} u_r y_{rj}^L + \sum_{l=1}^{L} d_l \gamma_l w_{lj}^L, \sum_{r=1}^{s} u_r y_{rj}^U + \sum_{l=1}^{L} d_l \gamma_l w_{lj}^U\right]}{\left[\sum_{i=1}^{m} v_i x_{ij}^L + \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^L, \sum_{i=1}^{m} v_i x_{ij}^U + \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^U\right]} =$$

$$\left[\frac{\sum_{r=1}^{s} u_r y_{rj}^L + \sum_{l=1}^{L} d_l \gamma_l w_{lj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U + \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^U}, \frac{\sum_{r=1}^{s} u_r y_{rj}^U + \sum_{l=1}^{L} d_l \gamma_l w_{lj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L + \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^L}\right], \quad j = 1, \ldots, n.$$

Thus

$$\theta_j = [\theta_j^L, \theta_j^U] = \left[\frac{\sum_{r=1}^{s} u_r y_{rj}^L + \sum_{l=1}^{L} d_l \gamma_l w_{lj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U + \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^U}, \frac{\sum_{r=1}^{s} u_r y_{rj}^U + \sum_{l=1}^{L} d_l \gamma_l w_{lj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L + \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^L}\right] \subseteq (0,1], \quad j = 1, \ldots, n.$$

$$\theta_j^U = \frac{\sum_{r=1}^{s} u_r y_{rj}^U + \sum_{l=1}^{L} d_l \gamma_l w_{lj}^U}{\sum_{i=1}^{m} v_i x_{ij}^L + \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^L} \le 1, \qquad j = 1, \ldots, n,$$

$$\theta_j^L = \frac{\sum_{r=1}^{s} u_r y_{rj}^L + \sum_{l=1}^{L} d_l \gamma_l w_{lj}^L}{\sum_{i=1}^{m} v_i x_{ij}^U + \sum_{l=1}^{L} (1-d_l)\gamma_l w_{lj}^U} > 0 \qquad j = 1, \ldots, n.$$

The following fractional programming models are presented to measure the upper and lower bounds of the efficiency of DMU_o :

$$\begin{aligned}&\text{maximize}\quad \theta_o^U=\frac{\sum_{r=1}^{s}u_r y_{ro}^U+\sum_{l=1}^{L}d_l\gamma_l w_{lo}^U}{\sum_{i=1}^{m}v_i x_{io}^L+\sum_{l=1}^{L}(1-d_l)\gamma_l w_{lo}^L}\\&\text{subject to}\quad \frac{\sum_{r=1}^{s}u_r y_{rj}^U+\sum_{l=1}^{L}d_l\gamma_l w_{lj}^U}{\sum_{i=1}^{m}v_i x_{ij}^L+\sum_{l=1}^{L}(1-d_l)\gamma_l w_{lj}^L}\le 1,\qquad j=1,\dots,n,\\&\qquad u_r,v_i,\gamma_l\ge\varepsilon\quad \forall r,\forall i,\forall l\qquad d_l\in\{0,1\}.\end{aligned}\tag{3}$$

$$\begin{aligned}&\text{maximize}\quad \theta_o^L=\frac{\sum_{r=1}^{s}u_r y_{ro}^L+\sum_{l=1}^{L}d_l\gamma_l w_{lo}^L}{\sum_{i=1}^{m}v_i x_{io}^U+\sum_{l=1}^{L}(1-d_l)\gamma_l w_{lo}^U}\\&\text{subject to}\quad \frac{\sum_{r=1}^{s}u_r y_{rj}^U+\sum_{l=1}^{L}d_l\gamma_l w_{lj}^U}{\sum_{i=1}^{m}v_i x_{ij}^L+\sum_{l=1}^{L}(1-d_l)\gamma_l w_{lj}^L}\le 1,\qquad j=1,\dots,n,\\&\qquad u_r,v_i,\gamma_l\ge\varepsilon\quad \forall r,\forall i,\forall l\qquad d_l\in\{0,1\}.\end{aligned}\tag{4}$$

These models can be transformed to linear programs by using the Charnes and Cooper transformation [11], substituting $\delta_l=d_l\gamma_l\,(l=1,2,...,L)$, and imposing the following constrains:

$$0\le\delta_l\le M d_l$$
$$\delta_l\le\gamma_l\le\delta_l+M\ (1-d_l)$$

M is a large positive number. Therefore, models (3) and (4) can be reformulated to the following mixed integer linear programs:

$$\begin{aligned}&\text{maximize}\quad \theta_o^U=\sum_{r=1}^{s}u_r y_{ro}^U+\sum_{l=1}^{L}\delta_l w_{lo}^U\\&\text{subject to}\ \sum_{i=1}^{m}v_i x_{io}^L+\sum_{l=1}^{L}\gamma_l w_{lo}^L-\sum_{l=1}^{L}\delta_l w_{lo}^L=1\\&\quad\sum_{r=1}^{s}u_r y_{rj}^U+\sum_{l=1}^{L}\delta_l w_{lj}^U-\sum_{i=1}^{m}v_i x_{ij}^L-\sum_{l=1}^{L}\gamma_l w_{lj}^L+\sum_{l=1}^{L}\delta_l w_{lo}^L\le 0,\qquad j=1,...,n,\\&\qquad 0\le\delta_l\le Md_l\\&\qquad \delta_l\le\gamma_l\le\delta_l+M\,(1-d_l)\\&\quad u_r,v_i,\gamma_l\ge\varepsilon\ \ \forall r,\forall i,\forall l\qquad d_l\in\{0,1\}.\end{aligned}\tag{5}$$

$$\begin{aligned}
&\text{maximize} \quad \theta_o^L = \sum_{r=1}^{s} u_r y_{ro}^L + \sum_{l=1}^{L} \delta_l w_{lo}^L \\
&\textit{subject to} \quad \sum_{i=1}^{m} v_i x_{io}^U + \sum_{l=1}^{L} \gamma_l w_{lo}^U - \sum_{l=1}^{L} \delta_l w_{lo}^U = 1 \\
&\sum_{r=1}^{s} u_r y_{rj}^U + \sum_{l=1}^{L} \delta_l w_{lj}^U - \sum_{i=1}^{m} v_i x_{ij}^L - \sum_{l=1}^{L} \gamma_l w_{lj}^L + \sum_{l=1}^{L} \delta_l w_{lo}^L \le 0, \qquad j = 1,\dots,n, \\
&0 \le \delta_l \le M d_l \\
&\delta_l \le \gamma_l \le \delta_l + M(1-d_l) \\
&\delta_l, u_r, v_i, \gamma_l \ge \varepsilon \quad \forall r, \forall i, \forall l \quad d_l \in \{0,1\}.
\end{aligned} \tag{6}$$

Where θ_o^U is the best relative efficiency when all the DMUs are in the most favorite conditions, while θ_o^L is the lower bound of the best relative efficiency. It is obvious that they compose the optimistic efficiency interval $[\theta_o^L, \theta_o^U]$. Afterwards, a simple majority decision rule is utilized for deciding the status of flexible variables in settings where such flexibility is present.

The relationship between θ_o^U and θ_o^L is stated in the form of following theorem:

Theorem 1 Assume θ_o^U and θ_o^L are the best relative efficiency according to models (5) and (6). Thus we have $\theta_o^{L^*} \le \theta_o^{U^*}$. Moreover, only when all the inputs and outputs for DMU_o are reduced from interval data to exact data, the equality happens.

Proof.Let $u_r^*(r = 1,\dots,s)$, $v_i^*(i = 1,\dots,m)$, $\gamma_l^*(l = 1,\dots,L)$, and δ_l^* $(l = 1,\dots,L)$ are the optimal solution to model(6). We define:

$$\psi_0 = \sum_{i=1}^{m} v_i^* x_{io}^L + \sum_{l=1}^{L} \gamma_l^* w_{lo}^L - \sum_{l=1}^{L} \delta_l^* w_{lo}^L,$$

$$\tilde{u}_r = \frac{u_r^*}{\psi_0},\ r = 1,\dots,s,\quad \tilde{v}_i = \frac{v_i^*}{\psi_0},\ i = 1,\dots,m,$$

$$\tilde{\gamma}_l = \frac{\gamma_l^*}{\psi_0},\ l = 1,\dots,L,\quad \tilde{\delta}_r = \frac{\delta_l^*}{\psi_0},\ l = 1,\dots,L,$$

Therefore we have

$$\psi_0 = \sum_{i=1}^{m} v_i^* x_{io}^L + \sum_{l=1}^{L} \gamma_l^* w_{lo}^L - \sum_{l=1}^{L} \delta_l^* w_{lo}^L \le \sum_{i=1}^{m} v_i^* x_{io}^U + \sum_{l=1}^{L} \gamma_l^* w_{lo}^U - \sum_{l=1}^{L} \delta_l^* w_{lo}^U = 1,$$

$$\sum_{i=1}^{m} \tilde{v}_i x_{io}^L + \sum_{l=1}^{L} \tilde{\gamma}_l w_{lo}^L - \sum_{l=1}^{L} \tilde{\delta}_l w_{lo}^L = \sum_{i=1}^{m} \frac{v_i^*}{\psi_0} x_{io}^L + \sum_{l=1}^{L} \frac{\gamma_l^*}{\psi_0} w_{lo}^L - \sum_{l=1}^{L} \frac{\delta_l^*}{\psi_0} w_{lo}^L =$$

$$\frac{1}{\psi_0}\left(\sum_{i=1}^{m} v_i^* x_{io}^L + \sum_{l=1}^{L} \gamma_l^* w_{lo}^L - \sum_{l=1}^{L} \delta_l^* w_{lo}^L\right) = 1$$

$$\sum_{r=1}^{s} \tilde{u}_r y_{rj}^U + \sum_{l=1}^{L} \tilde{\delta}_l w_{lj}^U - \sum_{i=1}^{m} \tilde{v}_i x_{ij}^L - \sum_{l=1}^{L} \tilde{\gamma}_l w_{lj}^L + \sum_{l=1}^{L} \tilde{\delta}_l w_{lo}^L \le$$

$$\frac{1}{\psi_0}\left(\sum_{r=1}^{s} u_r^* y_{rj}^U + \sum_{l=1}^{L} \delta_l^* w_{lj}^U - \sum_{i=1}^{m} v_i^* x_{ij}^L - \sum_{l=1}^{L} \gamma_l^* w_{lj}^L + \sum_{l=1}^{L} \delta_l^* w_{lo}^L\right) \le 0, \quad j = 1,\ldots,n,$$

$$\tilde{u}_r = \frac{u_r^*}{\psi_0} \ge \frac{\varepsilon}{\psi_0} \ge \varepsilon \quad r = 1,\ldots,s, \quad \tilde{v}_i = \frac{v_i^*}{\psi_0} \ge \frac{\varepsilon}{\psi_0} \ge \varepsilon, \; i = 1,\ldots,m,$$

$$\tilde{\gamma}_l = \frac{\gamma_l^*}{\psi_0} \ge \frac{\varepsilon}{\psi_0} \ge \varepsilon, \; l = 1,\ldots,L, \quad \tilde{\delta}_r = \frac{\delta_l^*}{\psi_0} \ge \frac{\varepsilon}{\psi_0} \ge \varepsilon, \; l = 1,\ldots,L,$$

Obviously, $\tilde{u}_r$, $\tilde{v}_i$, $\tilde{\gamma}_l$, and $\tilde{\delta}_r$ are feasible solutions for model(5). Thus

$$\sum_{r=1}^{s} \tilde{u}_r y_{ro}^U + \sum_{l=1}^{L} \tilde{\delta}_l w_{lo}^U \le \theta_o^{U^*}$$

$$\theta_o^{L^*} = \sum_{r=1}^{s} u_r^* y_{ro}^L + \sum_{l=1}^{L} \delta_l^* w_{lo}^L \le \sum_{r=1}^{s} u_r^* y_{ro}^U + \sum_{l=1}^{L} \delta_l^* w_{lo}^U = \sum_{r=1}^{s} (\psi_0 \tilde{u}_r) y_{ro}^U + \sum_{l=1}^{L} (\psi_0 \tilde{\delta}_l) w_{lo}^U$$

$$= \psi_0 \left(\sum_{r=1}^{s} \tilde{u}_r y_{ro}^U + \sum_{l=1}^{L} \tilde{\delta}_l w_{lo}^U\right) \le \psi_0 \theta_o^{U^*} \le \theta_o^{U^*}.$$

If $x_{io}^L = x_{io}^U, w_{lo}^L = w_{lo}^U$ and $y_{ro}^L = x_{ro}^U$, then equality occurs. □

3. An alternative method to determine the status of flexible measures in the presence of interval data

In this section we use the dual form of Wang et al. [9] models, which are known as Wang envelopment models and are given by:

$$\begin{aligned}
\text{minimize} \quad & \theta_o^U \\
\textit{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^L \le \theta_o^U x_{io}^L \quad \forall i, \\
& \sum_{j=1}^{n} \lambda_j y_{kj}^U \ge y_{ko}^U \qquad \forall k, \\
& \theta_o^U \;\; \textit{free}, \; \lambda_j \ge 0 \;\; \forall j.
\end{aligned} \tag{7}$$

and

$$\begin{aligned} \text{minimize} \quad & \theta_o^L \\ \text{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^L \le \theta_o^L x_{io}^U \quad \forall i, \\ & \sum_{j=1}^{n} \lambda_j y_{kj}^U \ge y_{ko}^L \quad \forall k, \\ & \theta_o^L \ \text{free}, \ \forall l \quad \lambda_j \ge 0 \ \forall j. \end{aligned} \tag{8}$$

Similar to the section 2, we assume that there exist L flexible measures $w_{lj}\,(l = 1,\dots,L)$.

Certainly, flexible measures are either inputs or outputs. Therefore, the following constraints (9) and (10) are added to models (7) and (8), respectively.

$$\begin{aligned} & \sum_{j=1}^{n} \lambda_j w_{lj}^L \le \theta_o^U w_{lo}^L + M\bar{d}_l \quad \forall l \\ & \sum_{j=1}^{n} \lambda_j w_{lj}^U \ge w_{lo}^U - M\,(1-\bar{d}_l) \quad \forall l \end{aligned} \tag{9}$$

and

$$\begin{aligned} & \sum_{j=1}^{n} \lambda_j w_{lj}^L \le \theta_o^L w_{lo}^U + M\bar{d}_l \quad \forall l \\ & \sum_{j=1}^{n} \lambda_j w_{lj}^U \ge w_{lo}^L - M\,(1-\bar{d}_l) \quad \forall l \end{aligned} \tag{10}$$

Indeed, a binary variable $\bar{d}_l$ is introduced to transform the either/or constraints to a form which only one constraint is held that $\bar{d}_l = 0$ expresses w_l is an input and it is an output if $\bar{d}_l = 1$. M is a large positive number. Hence, models (7) and (8) are revised for determining the status of flexible measures as follows:

$$\begin{aligned} \text{minimize} \quad & \theta_o^U \\ \text{subject to} \quad & \sum_{j=1}^{n} \lambda_j x_{ij}^L \le \theta_o^U x_{io}^L \quad \forall i \\ & \sum_{j=1}^{n} \lambda_j y_{kj}^U \ge y_{ko}^U \quad \forall k \\ & \sum_{j=1}^{n} \lambda_j w_{lj}^L \le \theta_o^U w_{lo}^L + M\bar{d}_l \quad \forall l \\ & \sum_{j=1}^{n} \lambda_j w_{lj}^U \ge w_{lo}^U - M\,(1-\bar{d}_l) \quad \forall l \\ & \bar{d}_l \in \{0,1\} \ \forall l \quad \lambda_j \ge 0 \ \forall j \end{aligned} \tag{11}$$

$$\begin{aligned} &\text{minimize} \quad \theta_o^L \\ &\textit{subject to} \quad \sum_{j=1}^{n} \lambda_j x_{ij}^L \le \theta_o^L x_{io}^U \quad \forall i \\ &\sum_{j=1}^{n} \lambda_j y_{kj}^U \ge y_{ko}^L \quad \forall k \\ &\sum_{j=1}^{n} \lambda_j w_{lj}^L \le \theta_o^L w_{lo}^U + M\bar{d}_l \quad \forall l \\ &\sum_{j=1}^{n} \lambda_j w_{lj}^U \ge w_{lo}^L - M(1-\bar{d}_l) \quad \forall l \\ &\bar{d}_l \in \{0,1\} \ \forall l \quad \lambda_j \ge 0 \ \forall j \end{aligned} \tag{12}$$

The above models result in the best upper and lower bound efficiency for each DMU and identify input/output status of flexible measures. In addition, like previous section we utilize the majority rule to establish the status of flexible measures. It is evident that models (5) and (6) do not have equal optimal objective values to models (11) and (12). Because models (5) and (6) assign a status for flexible measures that maximum amounts of objective functions can be obtained whilst, determining the status of flexible measures when objective functions take the minimum amounts is the purpose of models (11) and (12). Furthermore, if $\bar{d}_l^* = 0$, then $d_l^* = 1$ and $d_l^* = 0$ when $\bar{d}_l^* = 1$. This means whenever a flexible measure is identified as an input in models (11) and (12) then it is designed as an output in models (5) and (6) and vice versa.

Definition In each of methods, DMU_o is efficient if the best upper bound efficiency (i.e. models (5) in the first method and model (11) in the second method) be 1 ($\theta_o^{U^*} = 1$); otherwise, if $\theta_o^{U^*} < 1$, it is inefficient.

4. Examples

To illustrate the above ideas, consider the data set of bank branches in Iran. In this study personnel and computer terminals are assumed as input variables and loan is an output variable while deposit is supposed as a flexible measure. The data of 10 bank branches can be seen in Table 1.

The second column denotes optimal solutions when deposit is supposed as an input variable and the third column shows optimal values when it is considered as an output variable in Tables 2 and 3. In addition, the efficiency of models (5) and (6), when deposit is assumed as a flexible measure, are shown in the fourth column of Tables 2 and 3, respectively. Also optimal d is displayed in the fifth column of Tables 2 and 3.

Table 1

Data of the bank branches

#DMU	Personnel	Computer terminals	Deposits	Loans
1	[9,11]	[11.61,13.61]	[600,800]	[50,70]
2	[8,10]	[10.29,12.29]	[500,700]	[30,50]
3	[7,9]	[6.66,8.66]	[200,400]	[40,60]
4	[7.5,9.5]	[7.81,9.81]	[400,600]	[20,40]
5	[8.5,10.5]	[8.88,10.88]	[550,750]	[460,480]
6	[4,6]	[9.33,11.33]	[850,1050]	[850,870]
7	[6,8]	[14.10,16.10]	[750,950]	[840,860]
8	[6.5,8.5]	[8.74,10.74]	[600,800]	[400,420]
9	[7.5,9.5]	[10.31,12.31]	[350,550]	[810,830]
10	[8,10]	[11.28,13.28]	[800,1000]	[620,640]

The result of model (5) and (6) are showed in Table 2 and 3, respectively.

Table 2

Results of models (1) and (5)

#DMU	Input	Output	Flexible	*d*
1	0.071388	0.61228	0.61228	1
2	0.058006	0.60447	0.60447	1
3	0.126506	0.533677	0.533677	1
4	0.060716	0.682641	0.682641	1
5	0.62314	0.750483	0.750483	1
6	1	1	1	0 or 1
7	0.889547	0.659004	0.889547	0
8	0.544585	0.813338	0.813338	1
9	1	0.86334	1	0
10	0.639312	0.787741	0.787741	1

Table 3

Results of models (2) and (6)

#DMU	Input	Output	Flexible	*d*
1	0.042698	0.391729	0.391729	1
2	0.028505	0.361502	0.361502	1
3	0.055494	0.205213	0.205213	1
4	0.02355	0.362313	0.362313	1
5	0.47879	0.45341	0.47879	0
6	0.804547	0.804547	0.804547	0 or 1
7	0.668346	0.55952	0.668346	0
8	0.415934	0.496409	0.496409	1
9	0.793758	0.70565	0.793758	0
10	0.520336	0.535284	0.535284	1

As it can be seen in the fourth column of Table 2, deposit is identified as output in 7 DMUs while 2 DMUs consider it as input. Nevertheless, only 1 DMU designs deposit as input or output variable without any influence on the amount of efficiency.

Having a glance at Table 3 reveals that only 1 DMU treats deposit as either input or output while, 6 out of 9 remaining DMUs design deposits as output. By utilizing the majority choice among results of Tables 2 and 3, it is clear that the majority of DMUs identify deposit as an output.

Now we use the data set in Table 1 for evaluating of efficiency of models (11) and (12). The second and third columns of Tables 4 and 5 show optimal values whenever deposit is assumed as input and output, respectively. The fourth column of Tables 4 and 5 indicates the efficiency of models (11) and (12) respectively where deposit is supposed as a flexible measure. The optimal $\bar{d}$ is depicted in the fifth column of Tables 4 and 5.

Table 4

Table 4 Results of models (7) and (11)

#DMU	Input	Output	Flexible	$\bar{d}$
1	0.0714	0.6123	0.0714	0
2	0.058	0.6045	0.058	0
3	0.1265	0.5337	0.1265	0
4	0.0607	0.6826	0.0607	0
5	0.6231	0.7505	0.6231	0
6	1	1	1	0 or 1
7	0.8895	0.659	0.659	1
8	0.5446	0.8133	0.5446	0
9	1	0.8633	0.8633	1
10	0.6393	0.7877	0.6393	0

Table 5

Table 5 Results of models (8) and (12)

#DMU	Input	Output	Flexible	$\bar{d}$
1	0.0427	0.3917	0.0427	0
2	0.0285	0.3615	0.0285	0
3	0.0555	0.2052	0.0555	0
4	0.0235	0.3623	0.0235	0
5	0.4788	0.4534	0.4534	1
6	0.8045	0.8045	0.8045	0 or 1
7	0.6683	0.5595	0.5595	1
8	0.4159	0.4964	0.4159	0
9	0.7938	0.7057	0.7057	1
10	0.5203	0.5353	0.5203	0

As it can be seen DMUs that design deposits as input in Tables 2 and 3 treat deposits as output in Tables 4 and 5 and vice versa. In addition, DMU 6 treats the flexible measure as input or output in Tables 2, 3, 4, and 5. It is obvious that the most DMUs determine the flexible measure as input in Tables 4 and 5.

Thus according to the majority rule, the flexible measure is identified as input in models (11) and (12).

5. Conclusions

In this paper based on envelopment and multiplier forms, models have been proposed to evaluate the relative performance of DMUs in presence of interval data where there are measures with unknown status of input/ output. Moreover, a majority role has been used to determine the status of flexible measures. Through a numerical example, the applicability of the proposed models has been demonstrated. Nevertheless, changing efficiency scores by altering M is a drawback which is seen in the proposed models in the current paper, also all introduced models to identify the status of flexible measure confront it (i.e. Cook and Zhu, Toloo, and Amirteimoori et al. models). Therefore, selecting the best and the most suitable M is a significant issue.

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