ROBUST CONTROL FOR UNCERTAIN TIME DELAY PROCESSES

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Această lucrare tratează problema conducerii robuste a sistemelor liniare cu incertitudine cu argument întârziat ce conțin elemente de execuție cu saturație. Utilizând fundamente teoretice, cum ar fi teoria Razumikhin cu privire la stabilitatea sistemelor cu incertitudine cu argument întârziat, teoria Lyapunov a stabilității ecuațiilor diferențiale funcționale, tehnici de optimizare via inegalități liniare matriciale, această lucrare propune o procedură de determinare a unui criteriu pătratic compatibil cu incertitudinile unui sistem cu argument întârziat şi o procedură de sinteză a unei legi de conducere robuste pentru sistemul în cauză ce urmează a fi implementată folosind elemente de execuție cu saturație.

This paper is concerned with the problem of robust control of linear systems with delay dependence containing saturating actuators in the presence of non-linear parametric perturbation. Based on Razumikhin's approach to the stability of uncertain systems with delay dependence, on Lyapunov theory to the stability of functional differential equations and on optimization technical via matrices linear inequalities, this paper propose an procedure for determine a square criteria compatible with the uncertainness of the concerned system and for synthesize of robust control law for the concerned system.

Keywords: time delay system, stabilization robustness, non-linear parametric perturbation, saturating actuators

1. Introduction

In the last years three major thinks play an important role in the control theory, thus Yola showed that it is possible to parametrize all stabilizing controller for a particular system, Zames postulated that measuring performance in terms of the ∞-norm rather than the traditional 2-norm, might be much closer to the practical needs, Doyle argued that model uncertainty is often described very effectively in terms of norm bounded perturbations. For this perturbations and the

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H_∞ performance objective he developed a power full tool (the structured singular value) for testing *robust stability* (i.e. stability in the presence of model uncertainty).

In [1], Ioan Dumitrache et al approach one of the main problems of Smith predictor-based control of time delay process: the availability of an adequate model. In that paper, is presents a hierarchical control structure based on Smith predictor and a fuzzy decision system. Considering more operating regimes, the decision system selects the appropriate model parameters based on an on-line identification of the process parameters in order to preserve the performance robustness in presence of uncertainties.

In [2], Corneliu Popeea and Boris Jora presents a computational oriented overview on the order reduction problem for passive (positive real) linear systems. The numerical difficulties associated with the standard order reduction procedure, which involves the stabilizing solutions of two Riccati equations, are discussed. Some new approximate methods are proposed.

The traditional role of the gas turbine as fast response unit, ideal for improving primary control response of the power system, has been to a certain extent lost, due to relatively high constraints in ramping up and down the power output during the normal operation. This trend must be fully reflected in the modeling and simulation of the gas turbines in power system analysis programs. The [3] is a contribution to the systematic analysis of power-frequency control concepts for multi shaft high power gas turbines and the enhancement of their modeling in conventional power system analysis software.

Structures with inverse model represent one of the successful solutions for the real-time control of the nonlinear processes. The use of these structures imposes solving some specific problems, like determination of static characteristic of the process, construction of inverse model or robust control law design. In [4] a structure and the correspondent original methodology of designing and physically implementation based on inverse model command is proposed. The applicability of the structure is proved using a real-time structure with an RST control algorithm.

In [5] are presents the advantages offered by the mathematical theory of linear time invariant system control problem and the advantages offered by the microprocessors directly applied to digital process control. The main advantage of proposed method is the independence of the sampling period by the step of sampling. More exactly, has been derived a method based on the digital version of the exact model matching problem in conjunction with an adequate selection of the sampling period.

The [6] is a complete overview of the process control, prepares control design, implementation problems, problem formulations, analysis of posed control thesis of alternative control systems.

In [7] is presented a project about the system structure and the system design philosophy.

The uncertain time delay systems are of interest to control theorists and practitioners for various reasons. Control problems have been formulated and solved for such systems since the classical period and the mostly known result is that based on Smith predictor. The systems thus designed could be either non-robust or unstable. For this reason the more recent techniques based on state space, ensuring feedback stabilization and optimality of some quadratic criterion were applied [8].

In [9] are studies the Brunovsky forms and the connection between Hautus criteria and reaction equivalence.

In [10] is presented a feedback control systems designing procedure on the basis of the root locus. In order to design the regulator compensatory one may use: phase-lead compensatory, phase-lag compensatory or both types.

In [11] is used the thermal balance theoretical models in order to stay at a level imposed by the adjusted parameters.

The problems of stability analysis and stabilization of dynamic systems with time delay are of theoretical and practical importance and have attracted considerable attention. A keen interest has been taken in robust stabilization for uncertain time delay systems containing saturating actuators.

Based on Razumikhin's approach to the stability of uncertain systems with delay dependence, on Lyapunov theory to the stability of functional differential equations and on optimization technical via matrices linear inequalities, this paper propose an procedure for determine a square criteria compatible with the uncertainness of the concerned system and for synthesize of robust control law for the concerned system.

This paper is organized as follows: the section 1 is the introduction; the section 2 presents briefly the problem of robust control of the linear systems with delay dependence containing saturating actuators in the presence of non-linear parametric perturbation.; the section 3 gives the proposed robust control results for above problem; in the section 4 are presented the numerical examples; the section 5 gives the conclusion.

2. Preliminary

Consider the uncertain plant described by the following differential equation [12]:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h(t)) + (B+\Delta B(t))u(t) + f_0(x(t),t) + f_1(x(t-h(t)),t) \qquad (2.1)$$

for $\forall t > t_0 \geq 0$, where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control vector, A_0 and A_1 are $n \times n$ real matrices, B and $\Delta B(t)$ are $n \times m$ real matrices.

The time delay, $h(t)$, is a nonnegative bounded and continuous function, i.e. $0 < h(t) < h^*$ where h^* is a positive constant. Here, it is worth noticing that we do require for the time derivative of the time delay $h(t)$ to be less than one. The initial condition for (2.1) is given by [12]:

$$x(\theta) = \varphi(\theta) \quad t_0 - h^* \le \theta \le t_0 \tag{2.2}$$

where φ is a given continuous vector valued function on the interval established. Here $\Delta B(t)$ is the time-varying perturbation matrix of input matrix B.

The uncertainties functions $f_0(x(t),t)$ and $f_1(x(t-h(t)),t)$, which are smooth vector valued functions satisfying $f_0(0,t) = 0, \forall t$ and $f_1(0,t) = 0, \forall t$, are unknown and represent the system's nonlinear parametric perturbations with respect to the current state and the delayed state, respectively. In general, it is assumed that $\|\Delta B(t)\|, \|f_0(x(t),t)\|$ and $\|f_1(x(t-h(t)),t)\|$ are bounded, i.e.:

$$\|\Delta B(t)\| < \delta, \|f_0(x(t),t)\| < \beta_0, \|f_1(x(t-h(t)),t)\| < \beta_1 \tag{2.3}$$

where δ, β_0 and β_1 are given [12]. The control vector $u(t) \in \mathbb{R}^m$ is assumed to belong to a compact set: $\{u(t) \in \mathbb{R}^m : u_m \le u(t) \le u_M\}$.

Assumption 2.1:
The pair $(A_0 + A_1, B)$ can be stabilized and all the states of the system are available.

Note that Assumption 2.1, which is equivalent with stabilizing the system (2.1) without time delay and uncertainty is necessary for the existence of a stabilizing memory less state feedback control law for the system (2.1). By implementing a saturated controller [12]:

$$u = sat(Fx), F \in \mathbb{R}^{m \times n} \tag{2.4}$$

the system (2.1) becomes:

$$\dot{x}(t) = A_0 x(t) + A_1 x(t-h(t)) + (B + \Delta B(t)) sat(Fx(t)) + f_0(x(t),t) + f_1(x(t-h(t)),t) \tag{2.5}$$

for $\forall t > t_0 \ge 0$, where the saturation term is given by:

$$\begin{cases} sat:\mathbb{R}^m \to \mathbb{R} \\ sat(u) = (asat(u_i, i): i \in \{1,\dots,m\}) \\ asat(u_i, i) = \begin{cases} u_M^i, u_i \geq u_M^i \\ u_m^i, u_i \leq u_m^i \\ 0, u_i = 0 \end{cases} \\ w \leq \dfrac{asat(u_i, i)}{u_i} \leq 1 \qquad i \in \{1,\dots,m\} \end{cases} \tag{2.6}$$

The operation range of the nonlinear saturation is considered inside the sector $[w,1]$. The saturating actuator saturates at u_m and u_M, where [12]:

$$u_m = \begin{bmatrix} u_m^1 & \dots & u_m^m \end{bmatrix}^T \qquad u_M = \begin{bmatrix} u_M^1 & \dots & u_M^m \end{bmatrix}^T \tag{2.7}$$

Consider the system (2.1), satisfying Assumption 2.1. If the state feedback matrix F is chosen in such a way that the inequality [12]:

$$0 \leq h(t) \leq h^* \triangleq h^*(G) \triangleq \frac{\varphi}{\psi} \tag{2.8}$$

where

$$\varphi \triangleq \underline{\lambda}(G^T Q G) - 2(\frac{1-w}{2}\left\|G^T P B\right\| + \delta\left\|G^T P\right\|)\left\|FG\right\| - 2\left\|G^T P\right\|\left\|G\right\|(\beta_0 + \delta\beta_1) \tag{2.9}$$

$$\psi \triangleq 2\alpha(\psi_1 + \psi_2) \tag{2.10}$$

$$\psi_1 \triangleq \left\|G^T P A_1 (A_0 + \frac{1+w}{2} BF) G\right\| + \left\|G^T P A_1^2 G\right\| \tag{2.11}$$

$$\psi_2 \triangleq (\frac{1-w}{2}\left\|GPA_1 B\right\| + \delta\left\|GPA_1\right\|)\left\|FG\right\| + \left\|GPA_1\right\|\left\|G\right\|(\beta_0 + \beta_1) \tag{2.12}$$

is satisfied with:

$$A_F^T P + P A_F + Q = 0 \qquad A_F = A_0 + A_1 + \frac{1+w}{2} BF \tag{2.13}$$

where Q is a symmetric half positive matrix, $(\sqrt{Q}, A_0 + A_1)$ detectable pair, then the system (2.1)+(2.2) is robustly globally asymptotically stabilisable via memory less state feedback control (2.4) where G is any non singular matrix, and $\alpha = \sqrt{\dfrac{\overline{\lambda}(P)}{\underline{\lambda}(P)}}$.

3. The robust control

This paper propose a method to determine the matrices F and G, for robust control of the system (2.1)+(2.2) compatible with his uncertain. We establish $\varepsilon > 0$ for precision of calculus and $\rho > 0$ big enough for ensure the condition: sum of eigen value is positive in the case of 2×2 matrices, using the Viete relation. Let

$$n_n \triangleq n^2 \qquad n_s \triangleq \frac{n(n+1)}{2} \tag{3.1}$$

We establish a $n \times n$ matrices families:

$$N \triangleq \{N_i : i = \{1,...,n_n\}\} \qquad S \triangleq \{S_i = S_i^T : i = \{1,...,n_s\}\} \tag{3.2}$$

such that all the $n \times n$ matrices, M , can be represented:

$$M = \sum_{i=1}^{n_n} x_i N_i \tag{3.3}$$

respectively

$$M = \sum_{i=1}^{n_s} x_i S_i \tag{3.4}$$

in the symmetric case, where x_i are scalars. Let

$$M_1 \triangleq Q^{-1} \tag{3.5}$$

If we consider the inherent error of numerical representation, can be write

$$-\varepsilon I \le M_1 Q - I \le \varepsilon I \tag{3.6}$$

Let

$$M_1 = \sum_{i=1}^{n_s} x_i^{M_1} S_i \qquad Q = \sum_{i=1}^{n_s} x_i^{Q} S_i \tag{3.7}$$

where the variables $x_i^{M_1}$ and x_i^{Q} will be determine. So

$$M_1 Q = \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} x_i^{M_1} x_j^{Q} S_i S_j \tag{3.8}$$

Let the set of pairs of integer:

$$I^{M_1 Q} \triangleq \{(i,j) \in \{1,...,n_s\} \times \{1,...,n_s\} : S_i S_j \ne 0\} \tag{3.9}$$

For each $(i,j) \in I^{M_1 Q}$, will introduce a new variable, $t_{ij}^{M_1 Q}$, and define the block-diagonal matrix $A_{ij}^{M_1 Q}$, such that:

$$A_{ij}^1 = \begin{bmatrix} \rho & \rho x_i^{M_1} \\ x_j^Q & t_{ij}^{M_1 Q} + \varepsilon \end{bmatrix}, \; A_{ij}^2 = \begin{bmatrix} \rho & -\rho x_i^{M_1} \\ x_j^Q & -t_{ij}^{M_1 Q} + \varepsilon \end{bmatrix}, \; A_{ij}^{M_1 Q} = \begin{bmatrix} A_{ij}^1 & \\ & A_{ij}^2 \end{bmatrix} \tag{3.10}$$

Can be observed that $A_{ij}^{M_1Q} \geq 0$ is equivalent with $t_{ij}^{M_1Q} \cong x_i^{M_1} x_j^Q$. We can determine the coefficients $w_{ijk}^{M_1Q}$ such that:

$$M_1 Q = \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} \sum_{k=1}^{n_s} w_{ijk}^{M_1Q} t_{jk}^{M_1Q} S_i \tag{3.11}$$

Also, can be determined the symmetrical matrices, $S_{jk}^{M_1Q}$, such that:

$$M_1 Q = \sum_{j=1}^{n_s} \sum_{k=1}^{n_s} t_{jk}^{M_1Q} S_{jk}^{M_1Q} \tag{3.12}$$

Therefore

$$-\varepsilon I \leq \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} \sum_{k=1}^{n_s} w_{ijk}^{M_1Q} t_{jk}^{M_1Q} S_i - I \leq \varepsilon I \quad -\varepsilon I \leq \sum_{j=1}^{n_s} \sum_{k=1}^{n_s} t_{jk}^{M_1Q} S_{jk}^{M_1Q} - I \leq \varepsilon I \tag{3.13}$$

We define the matrix:

$$H^{(1)} \triangleq \begin{bmatrix} M_1 & G \\ G^T & t_1^{(2)} I \end{bmatrix} \tag{3.14}$$

Can be observed that $H^{(1)} \geq 0$ is equivalent with:

$$t_1^{(2)} I - G^T Q G \geq 0 \tag{3.15}$$

We define the matrices:

$$L_1^{(1)} \triangleq \begin{bmatrix} \rho & \rho t_1 \\ t_1 & t_1^{(2)} + \varepsilon \end{bmatrix} \qquad L_1^{(2)} \triangleq \begin{bmatrix} \rho & -\rho t_1 \\ t_1 & -t_1^{(2)} + \varepsilon \end{bmatrix} \qquad L_1 \triangleq \begin{bmatrix} L_1^{(1)} & \\ & L_1^{(2)} \end{bmatrix} \tag{3.16}$$

Can be observed that $L_1 \geq 0$ is equivalent with:

$$t_1^{(2)} - \varepsilon \leq t_1^2 \leq t_1^{(2)} + \varepsilon \text{ i.e. } t_1^2 \cong t_1^{(2)} \tag{3.17}$$

Let

$$G \triangleq \sum_{i=1}^{n_n} x_i^G N_i \tag{3.18}$$

where the variables x_i^G will be determined. So

$$G^T = \sum_{i=1}^{n_n} x_{\sigma(i)}^G N_i \tag{3.19}$$

where σ is a conveniently choice permutation of n_n order. We have the expression:

$$G^T Q = \sum_{i=1}^{n_n} \sum_{j=1}^{n_s} x_{\sigma(i)}^G x_j^Q N_i S_j \tag{3.20}$$

For each $(i,j) \in \{1,...,n_n\} \times \{1,...,n_s\}$ such that $N_i S_j \neq 0$ introduces the variable $t_{ij}^{G^TQ}$ and define the matrix $A_{ij}^{G^TQ}$ such that:

$$A_{ij}^{1G^TQ}=\begin{bmatrix}\rho & \rho x_{\sigma(i)}^{G}\\ x_j^Q & t_{ij}^{G^TQ}+\varepsilon\end{bmatrix}\quad A_{ij}^{2G^TQ}=\begin{bmatrix}\rho & -\rho x_{\sigma(i)}^{G}\\ x_j^Q & -t_{ij}^{G^TQ}+\varepsilon\end{bmatrix}\quad A_{ij}^{G^TQ}=\begin{bmatrix}A_{ij}^{G^TQ1} & \\ & A_{ij}^{G^TQ2}\end{bmatrix}\quad (3.21)$$

Can be observed that $A_{ij}^{G^TQ}\geq 0$ is equivalent with:

$$t_{ij}^{G^TQ}-\varepsilon\leq x_{\sigma(i)}^{G}x_j^Q\leq t_{ij}^{G^TQ}+\varepsilon \text{ i.e. } t_{ij}^{G^TQ}\cong x_{\sigma(i)}^{G}x_j^Q \quad (3.22)$$

Can be determined the coefficients $w_{ijk}^{G^TQ}$ such that

$$G^TQ=\sum_{i=1}^{n_n}\sum_{j=1}^{n_n}\sum_{k=1}^{n_s}w_{ijk}^{G^TQ}t_{jk}^{G^TQ}N_i \quad (3.23)$$

We observe that $w_{ijk}^{G^TQ}$ not depend on G or Q. We have:

$$G^TQG=\sum_{i=1}^{n_n}\sum_{j=1}^{n_n}\sum_{k=1}^{n_s}\sum_{l=1}^{n_n}w_{ijk}^{G^TQ}t_{jk}^{G^TQ}x_l^G N_i N_l \quad (3.24)$$

For each $(i,l)\in\{1,...,n_n\}\times\{1,...,n_n\}$ such that $N_iN_l\neq 0$ introduce the variable $t_{jkl}^{G^TQG}$ and define the matrices $A_{jkl}^{G^TQG}$ such that:

$$A_{jkl}^{1G^TQG}=\begin{bmatrix}\rho & \rho x_l^G\\ t_{jk}^{G^TQ} & t_{ij}^{G^TQG}+\varepsilon\end{bmatrix}\quad A_{jkl}^{2G^TQG}=\begin{bmatrix}\rho & -\rho x_l^G\\ t_{jk}^{G^TQ} & -t_{ij}^{G^TQG}+\varepsilon\end{bmatrix}$$

$$A_{jkl}^{G^TQG}=\begin{bmatrix}A_{jkl}^{G^TQG1} & \\ & A_{jkl}^{G^TQG2}\end{bmatrix} \quad (3.25)$$

Can be observed that the inequality $A_{jkl}^{G^TQG}\geq 0$ is equivalent with:

$$t_{jkl}^{G^TQG}-\varepsilon\leq t_{jk}^{G^TQ}x_l^G\leq t_{jkl}^{G^TQG}+\varepsilon \text{ i.e. } t_{jkl}^{G^TQG}\cong t_{jk}^{G^TQ}x_l^G \quad (3.26)$$

Can be determined the coefficients $w_{ijkl}^{G^TQG}$ such that:

$$G^TQG=\sum_{i=1}^{n_n}\sum_{j=1}^{n_n}\sum_{k=1}^{n_s}\sum_{l=1}^{n_n}w_{ijkl}^{G^TQG}t_{jkl}^{G^TQG}N_i \quad (3.27)$$

We observe that $w_{ijkl}^{G^TQG}$ not depend on G or Q. Also, can be determined the matrices $N_{jkl}^{G^TQG}$ such that:

$$G^TQG=\sum_{j=1}^{n_n}\sum_{j=1}^{n_s}\sum_{k=1}^{n_n}t_{jkl}^{G^TQG}N_{jkl}^{G^TQG} \quad (3.28)$$

Let

$$M_2\triangleq G^TQG \quad (3.29)$$

Define the matrix:

$$H^{(2)} \triangleq \begin{bmatrix} I & r_1 I \\ r_1 I & M_2 \end{bmatrix} \tag{3.30}$$

Can be observed that $H^{(2)} \geq 0$ is equivalent with:

$$G^T Q G - r_1^2 I \geq 0 \tag{3.31}$$

Let

$$M_3 = \sum_{i=1}^{n_s} x_i^{M_3} S_i \tag{3.32}$$

where the variables $x_i^{M_3}$ will be determined. We wish:

$$M_2 = M_3^T M_3 \text{ i.e. } -\varepsilon I \leq M_2 - M_3^T M_3 \leq \varepsilon I \tag{3.33}$$

We have:

$$M_3^T M_3 = \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} x_{\sigma(i)}^{M_3} x_j^{M_3} S_i S_j \tag{3.34}$$

Introduce the set:

$$I^{M_3^T M_3} \triangleq \{(i,j) \in \{1,...,n_s\} \times \{1,...,n_s\} : S_i S_j \neq 0\} \tag{3.35}$$

Can be observed that $I^{M_1 Q} = I^{M_3^T M_3}$. For each $(i,j) \in I^{M_3^T M_3}$ define the variable $t_{ij}^{M_3^T M_3}$ and build the matrix thus:

$$A_{ij}^{1M_3^T M_3} = \begin{bmatrix} \rho & \rho x_{\sigma(i)}^{M_3} \\ x_j^{M_3} & t_{ij}^{M_3^T M_3} + \varepsilon \end{bmatrix}$$

$$A_{ij}^{2M_3^T M_3} = \begin{bmatrix} \rho & -\rho x_{\sigma(i)}^{M_3} \\ x_j^{M_3} & -t_{ij}^{M_3^T M_3} + \varepsilon \end{bmatrix} \quad A_{ij}^{M_3^T M_3} = \begin{bmatrix} A_{ij}^{1M_3^T M_3} & \\ & A_{ij}^{2M_3^T M_3} \end{bmatrix} \tag{3.36}$$

Can be observed that $A_{ij}^{M_3^T M_3} \geq 0$ is equivalent with:

$$t_{ij}^{M_3^T M_3} - \varepsilon \leq x_{\sigma(i)}^{M_3} x_j^{M_3} \leq t_{ij}^{M_3^T M_3} + \varepsilon \text{ i.e. } t_{ij}^{M_3^T M_3} \cong x_{\sigma(i)}^{M_3} x_j^{M_3} \tag{3.37}$$

Observe that in this case can be considered σ the identical permutation, hence M_3 is symmetric. Can be determined the coefficients $w_{ijk}^{M_3^T M_3}$ such that:

$$M_3^T M_3 = \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} \sum_{k=1}^{n_s} w_{ijk}^{M_3^T M_3} t_{jk}^{M_3^T M_3} S_i \tag{3.38}$$

Mentioned that $w_{ijk}^{M_3^T M_3}$ not depend on M_3. Also, can be determined the symmetric matrices $S_{jk}^{M_3^T M_3}$ such that:

$$M_3^T M_3 = \sum_{j=1}^{n_s} \sum_{k=1}^{n_s} t_{jk}^{M_3^T M_3} S_{jk}^{M_3^T M_3} \tag{3.39}$$

From definition of M_2 we have:

$$-\varepsilon I \le \sum_{i=1}^{n_n}\sum_{j=1}^{n_n}\sum_{k=1}^{n_s}\sum_{l=1}^{n_n} w_{ijkl}^{G^TQG} t_{jkl}^{G^TQG} N_i - \sum_{i=1}^{n_s}\sum_{j=1}^{n_s}\sum_{k=1}^{n_s} w_{ijk}^{M_3^TM_3} t_{jk}^{M_3^TM_3} S_i \le \varepsilon I \qquad (3.40)$$

$$-\varepsilon I \le \sum_{j=1}^{n_n}\sum_{j=1}^{n_s}\sum_{k=1}^{n_n} t_{jkl}^{G^TQG} N_{jkl}^{G^TQG} - \sum_{j=1}^{n_s}\sum_{k=1}^{n_s} t_{jk}^{M_3^TM_3} S_{jk}^{M_3^TM_3} \le \varepsilon I \qquad (3.41)$$

Let:

$$H^{(3)} \triangleq \begin{bmatrix} s_{t_1} I & t_1 I - M_3 \\ t_1 I - M_3^T & s_{t_1} I \end{bmatrix} \qquad (3.42)$$

Observe that $H^{(3)} \ge 0$ is equivalent with:

$$s_{t_1}^2 I - (t_1 I - M_3)^T (t_1 I - M_3) \ge 0 \qquad (3.43)$$

If we continue in this manner obtain that the robust control problem of system (2.1)+(2.2) is equivalent with a feasibility problem $F(x) \ge 0$ where $F(x) = F_0 + \sum_{i=1}^{p} x_i F_i$ with $F_0, ..., F_p$ symmetric matrices. The solution of this problem represents the solution of robust control problem.

4. Computer simulation

Consider the uncertain time delay system (2.1)+(2.2) with:

$$A_0 = \begin{bmatrix} 1 & 0 \\ 2.85 & -2 \end{bmatrix} \qquad A_1 = \begin{bmatrix} 1 & 1.9 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \Delta B = .01 \begin{bmatrix} \chi \\ 0 \end{bmatrix} \qquad -1 \le \chi \le 0$$

$$h(t) = .2 + .1\sin t \qquad -15 \le u \le 20$$

$$f_0(x(t), t) = \begin{bmatrix} .01\sin(x_1(t)) \\ .01\sin(x_2(t)) \end{bmatrix} \qquad f_1(x(t-h), t) = \begin{bmatrix} .01\sin(x_1(t-h(t))) \\ .01\sin(x_2(t-h(t))) \end{bmatrix}$$

Let $w = .9$, according to procedure proposed find the matrices $F = [-3 \quad -2]$ and $G = \begin{bmatrix} 1 & \\ & .7 \end{bmatrix}$ such that $A_0 + A_1 + \frac{1+w}{2} BF = \begin{bmatrix} -.85 & \\ & -.15 \end{bmatrix}$. The memory less state feedback control law, $u(t) = sat(Fx(t))$, is a robust control law for uncertain time delay concerned system. For computer simulation we analyze inclusive the case of additional uncertain, i.e. $f_0^{disturb} = \mu f_0$ and $f_1^{disturb} = \mu f_1$, where μ is the measure of the relative additional uncertain. Considering the output of the system $y = x_1 + x_2$ can use the control structure from Fig. 1.

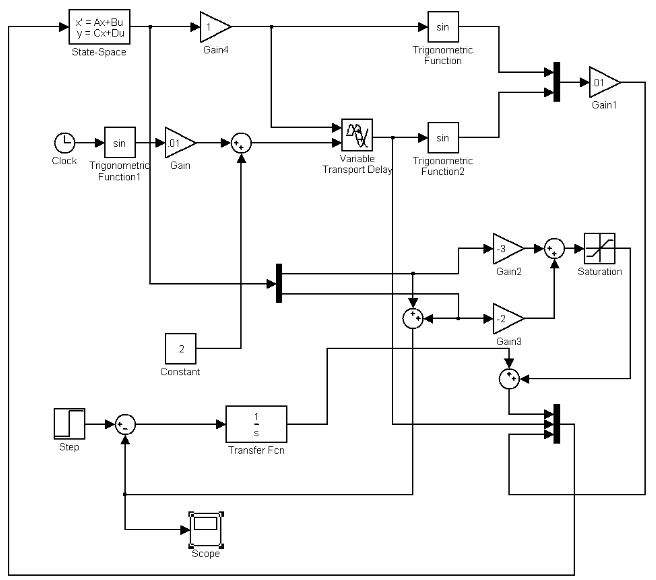

Fig. 1. SIMULINK scheme of control structured proposed

Also, consider some significant value for the parameter μ, thus: μ=0 for the case of the free uncertain system, μ=1 for the case of the system affected by the expected uncertain, and, μ=1.5 for the case of the system corrupted by the large uncertain. The corresponding results are represented in Fig. 2, Fig. 3 and Fig. 4.

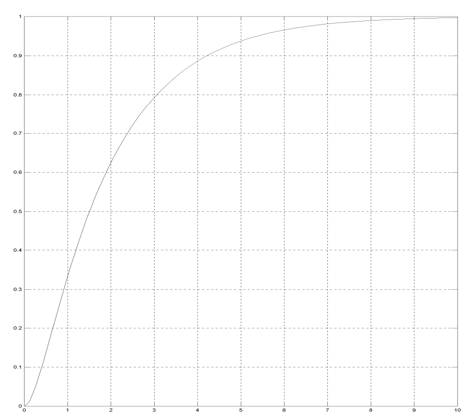

Fig. 2. Response of the free uncertain system ($\mu = 0$)

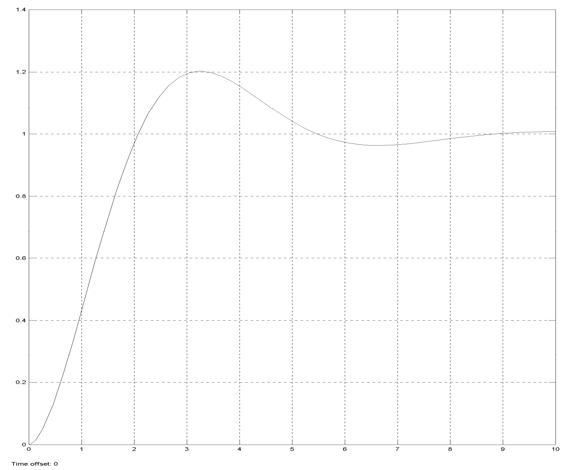

Fig. 3. Response of the system for uncertain expected ($\mu = 1$)

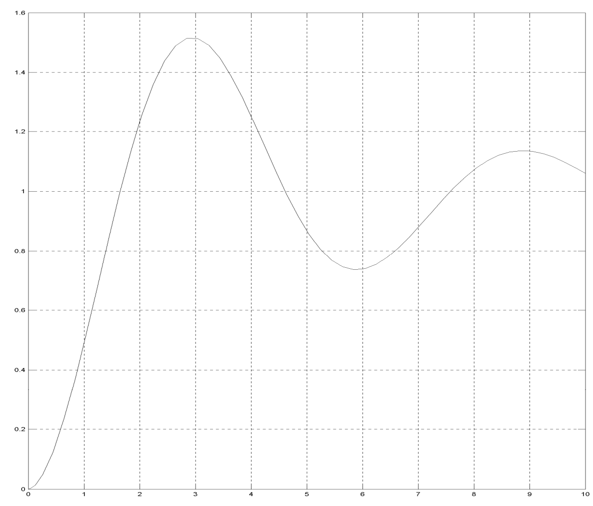

Fig. 4. Response of the system for more uncertain ($\mu = 1.5$)

The MATLAB language is used and the function of the CONTROL SYSTEM TOOLBOX package which allow the simple and efficient simulation for behavior of the systems concerned.

The results presented allow that the method proposed in this paper to be recommended for use to solve the concerned problem.

5. Conclusion

The problem of robust control for uncertain time delay systems, containing a saturating actuator has been addressed.

The uncertain time delay systems are of interest to control theorists and practitioners for various reasons. Control problems have been formulated and solved for such systems since the classical period and the mostly known result is that based on Smith predictor. The systems thus designed could be either non-robust or unstable. For this reason the more recent techniques based on state space, ensuring feedback stabilization and optimality of some quadratic criterion were applied

The saturating actuator and sensors often bring, in the system, a nonlinear input and raise obvious intricacy in synthesize of the control law. However, in practice, owing to physical limitation, there usually exist nonlinearities in the control input. Some common examples are mechanical connections, hydraulic servo-valves and electric servomotors, magnetic suspensions, and bearings, and some biomedical systems.

These effects of input nonlinearities usually result in the control performance degeneration or unstable for the control system. This paper solve the problem mentioned by to propose a procedure for determine a square criteria compatible with the uncertainness and a procedure for determine synthesize of robust control law for the concerned system.

For a proof the results proposed, in this paper, is used the Razumikhin's approach to the stability of uncertain systems with delay dependence, the Lyapunov theory to the stability of functional differential equations and the optimization technical via matrices linear inequalities.

Was considered an uncertain time delay system with nonlinear input, was used a parameter, μ, for characterize the level of uncertain, also was performed simulations for the checking of the performance and the applicability of the proposed procedure. The theoretical developments are illustrated by solve a robustness problem with respect to parametric uncertainty for a time delay system.

In conclusion, the results obtained allow that the method proposed in this paper is recommended for use in robust control of delay time plants.

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