U.P.B. Sci. Bull., Series C, Vol. 87, Iss. 4, 2025 ISSN 2286 – 3540

COMPARATIVE ANALYSIS OF MACROSCOPIC TRAFFIC FLOW MODELS ON HIGHWAYS

Alin Alexandru ȘERBAN [1*], Mădălin Corneliu FRUNZETE [1], Corneliu BURILEANU[1]

The problem of traffic in big cities represents one of the greatest challenges of this century because cities are becoming more and more populated with both people and vehicles. Therefore, solutions are needed to control traffic, algorithms or models that can model the main traffic parameters. This paper is based on speed-density models, and within this paper, the most important speed-density models were analyzed for the purpose of estimating travel speed based on a database containing data from highways. Seven different traffic flow models were analyzed in order to find the most suitable one for the highway conditions.

Keywords: traffic flow models, macroscopic models, mathematical model

1. Introduction and Related Work

As urbanization accelerates and the population continues to grow, traffic in major cities has become one of the most critical issues of the modern era. In many cases, the existing road infrastructure cannot cope with the increasing demands for transport, leading to severe congestion of the traffic. These bottlenecks not only affect the quality of life but also the environment and economic performance. Traffic congestion manifests itself through a significant slowdown in travel speeds, the formation of endless queues, and an exponential increase in travel times. Its causes are multiple and interconnected, including insufficient road capacity, high transport demand during peak hours, unforeseen road events (accidents, roadworks), poorly synchronized traffic lights, and the lack of efficient public transport alternatives [1]. Traffic congestion can be interpreted and compared to a Lorenz chaotic system, as traffic can sometimes exhibit chaotic behavior depending on the participants and external conditions that might occur [2]. Therefore, identifying and implementing effective traffic management solutions becomes an absolute priority.

Macroscopic traffic flow models are fundamental tools for analyzing and managing road traffic dynamics. They provide an overview of the collective behavior of vehicles on a road network, using parameters such as density, speed, and flow, without detailing the individual trajectories of vehicles.

---

* Corresponding author: alin.serban0507@upb.ro

[1] Faculty of Electronics and Telecommunications and Information Technology, National University of Science and Technology POLITEHNICA Bucharest, Romania, e-mails: alin.serban0507@upb.ro, madalin.frunzete@upb.ro, corneliu.burileanu@upb.ro

Macroscopic traffic flow models are based on three fundamental traffic parameters: traffic flow (the number of vehicles passing a specific point per unit of time), traffic density (the number of vehicles per unit length of the road) and the traffic speed (the average speed of vehicles in the traffic stream) [3].

The study in [4] proposes a new model to characterize the physiological and psychological response of drivers to changes in traffic flow. The authors introduce a regulation parameter that describes drivers' reactions to the conditions ahead, enabling a more realistic representation of traffic compared to existing models. The study shows that, for a slow response, traffic tends to form congestion, while a rapid response leads to a smoother flow. Additionally, it highlights that speed variations affect pollutant emissions and fuel consumption, suggesting that adjusting the regulation parameter for autonomous vehicles can reduce these negative effects. The proposed model provides a foundation for analyzing the behavior of autonomous vehicles, contributing to strategies for reducing fuel consumption and pollution. In the study from [5] the author proposes a new macroscopic traffic flow model that considers driver reactions and traffic stimuli. Unlike the Payne–Whitham (PW) model, which uses a constant velocity and can lead to unrealistic behavior in density and speed, the proposed model characterizes traffic behavior based on the distance between vehicles. The performance of both models was evaluated on a circular 300-meter route without active bottlenecks, with results indicating that the proposed model provides a more realistic representation of traffic behavior. Additionally, as highlighted in the study referenced in [6], traffic can be examined through the lens of channel information theory.

In this paper, we evaluate several macroscopic traffic flow models, selected after a comparative analysis of multiple approaches, and implement the most suitable ones under highway conditions. The focus is on identifying and validating the models that best reflect highway traffic dynamics through both theoretical analysis and practical implementation. The paper is organized as follows. Section 2 shows the used traffic flow models; section 3 shows the data description and model calibration and section 4 conclusions.

2. Traffic flow models

In this section the traffic flow models used and the boundary conditions for each one will be explained. The traffic models described in the following sections stand out due to their distinct characteristics, ranging from simple linear relations between speed and density to more sophisticated forms, including exponential, logarithmic, or combined approaches. These models provide a deeper understanding of traffic flow mechanisms, highlighting their importance in addressing congestion, improving efficiency, and designing effective transportation systems. The first one that will be described is the Greenshields model:

2.1 The Greenshields model

This model has been created and proposed by Greenshields in 1935, and it is a linear approach designed to examine the relation between speed, flow, and density [7]. While the model is straightforward and adheres to all boundary conditions ($\boldsymbol{u = 0}$ when $\boldsymbol{k = k_j}$ and $\boldsymbol{u = u_f}$ when $\boldsymbol{k = 0}$), its overall fit tends to be low, particularly when applied to freeway data. The formula for the Greenshields model is the following [8]:

$$u = uf\left(1 - \frac{k}{k_j}\right), \qquad (1)$$

where there are the following meanings: u is the speed, uf is the free flow speed, k represents the density and the k_j represents the jam density.

2.2 The Underwood model

Introduced in 1961, the Underwood model uses an exponential approach to describe the interaction between speed, flow, and density. It assumes that speed decreases exponentially as density rises, making it well-suited for traffic conditions with low to moderate densities [9]. However, its performance is less reliable in heavily congested situations. The Underwood model is as follows [8]:

$$u = u_f * e^{-\frac{k}{k_c}}, \qquad (2)$$

where $\boldsymbol{u}$ and $\boldsymbol{u_f}$ represents the speed and the free flow speed, $\boldsymbol{k}$ is the density and $\boldsymbol{k_c}$ is the critical density, the density at which the maximum flow occurs.

2.3 The Greenberg model

The Greenberg model, developed in 1959, uses a logarithmic equation to represent the interaction between speed, flow, and density [9]. It assumes that speed decreases logarithmically as density increases, making it particularly useful for describing traffic behavior in high-density conditions. The model adheres to the boundary condition $u = 0$ when $k = k_j$ (jam density). However, it struggles to represent free-flow conditions accurately, as it predicts infinitely high speeds at very low densities, limiting its effectiveness in such scenarios. The formula of the model is the following [8]:

$$u = u_c * \ln\frac{k_j}{k}, \qquad (3)$$

where u_c is the speed at capacity, which is the speed corresponding to the maximum flow of traffic and the other terms have the same meanings.

2.4 The Drake model

The Drake model builds upon traffic flow theory by utilizing a combined exponential-logarithmic relation to describe the interaction between speed, flow, and density [10]. It captures the behavior of traffic by assuming a logarithmic decrease in speed at high densities and an exponential decrease at low densities. The model adheres to the boundary conditions $u = 0$ at $k = k_j$ (jam density) and $u = u_f$ at $k = 0$ (free-flow density), ensuring it is applicable across a wide range of conditions. This adaptability makes the Drake model effective for analyzing both free-flow traffic at low densities and congested traffic at higher densities. The equation of the Drake model is the following [11]:

$$u = u_f * exp\left(-\frac{1}{2\left(\frac{k}{k_j}\right)^2}\right), \tag{4}$$

where all the terms have the same meaning as for the previous models.

2.5 The Robertson model

The Robertson traffic flow model aims to describe the relation between speed, flow, and density through an empirical approach designed for urban environments and traffic signal analysis [12]. It is particularly effective for capturing traffic behavior at intersections or within road networks where signal timing plays a critical role. The model highlights stop-and-go traffic dynamics, accounting for vehicle platooning and delays caused by traffic signals.

The boundary conditions of the model are as follows: $u = 0$ at $k = k_j$ (jam density), representing a complete halt in traffic, and $u = u_f$ at $k = 0$ (free-flow density), reflecting ideal conditions with no obstructions to traffic movement.

This model is most suitable for urban traffic systems where signal timings, congestion, and vehicle interactions significantly influence traffic patterns. Its ability to represent non-uniform traffic flow makes it highly effective in such contexts. However, for freeway or high-speed traffic, the model may require additional calibration to ensure accuracy. The equation associated with this model is as follows:

$$u = u_f * \left(1 - \left(\frac{k}{k_j}\right)^n\right), \tag{5}$$

where n is empirical exponent, typically ranging from 2 to 4, defining the shape of the speed-flow curve (its value determines the rate at which speed decreases with increasing flow).

2.6 The Van Aerde model

The Van Aerde traffic flow model offers a comprehensive mathematical framework to describe the relation between speed, flow, and density. Developed as a unified model, it is capable of capturing traffic behavior across the entire spectrum of traffic conditions, from free-flow to congested states [7]. The model combines both exponential and hyperbolic components to represent speed changes as density varies, ensuring accuracy under diverse traffic scenarios. This model is particularly well-suited for analyzing both freeway and urban traffic flows. Its versatility makes it a valuable tool for evaluating uninterrupted traffic, as well as congested scenarios where interactions between vehicles are significant. This model is defined by the following equation [14]:

$$\frac{1}{u} = \frac{1}{u_f} + \left(\frac{k}{k_j}\right) * \left(\frac{u_f - u_0}{u_f * u_0}\right), \tag{6}$$

where u_0 represents the minimum speed, the speed of vehicles at very high density, just before reaching complete congestion and the other terms have the same meaning as in the other models.

2.7 The Northwestern model

The Northwestern traffic flow model aims to describe the interplay between speed, density, and flow, focusing on the gradual reduction in speed as density rises [13]. It is especially effective for examining traffic scenarios involving a critical density (k_0)where maximum flow is achieved, while also accounting for stop-and-go traffic patterns in light to moderate congestion. In terms of boundary conditions, $u = u_f$ if the $k = 0$ and that means at zero density, traffic operates at free-flow speed, as there are no interactions between vehicles. Also, $u = 0$ as $k \to \infty$ and that means at very high densities approaching jam density (k_j), traffic speed gradually drops to zero due to congestion.

This model employs a Gaussian-inspired exponential decay function to represent speed, making it particularly useful for analyzing traffic behavior on freeways and highways. The Northwestern traffic model demonstrates strong efficiency in scenarios requiring a deep understanding of vehicle flow patterns and driver responses across different density conditions. This model is expressed as:

$$u = u_f * e^{-\frac{1}{2} * \left(\frac{k}{k_0}\right)^2}, \tag{7}$$

where k_0 is the critical density, the density at which traffic achieves its maximum flow. It represents the optimal balance between speed and density.

3. Data description and model calibration

The data used for this study comes from the National Highways of England website and the data related to A2 was used from the year 2014. This database contains logs about the date and time, the value of traffic flow (in vehicles per hour) and the speed value (in miles per hour, mph) [16]. The present data is for each day of the year and from 15 to 15 minutes. Because the data contains values about traffic flow and in the above models, we are using the density and the speed values, in the preparation part of the data, the values of the density were obtained based on the fundamental equation in traffic [17], [18]:

$$q = u * k, \quad (8)$$

where $\boldsymbol{q}$ represents the traffic flow, $\boldsymbol{u}$ is the speed and $\boldsymbol{k}$ is the density of the traffic. The results after running the traffic flow models can be seen in Table 1:

Table 1

Comparison of traffic flow models parameters

Model	u_f Km/h	k_j Veh/km	u_c Km/h	k_c Veh/km	R^2
Greenshields	119.2	150.4	59.6	75.2	0.76
Underwood	122.4	-	61.2	73.4	0.80
Greenberg	-	180.4	28.7	55.11	0.61
Drake	123.4	-	73.9	106.23	0.91
Robertson	120.5	140.3	64.2	84.5	0.74
Van Aerde	121.8	138.2	62.3	83.2	0.70
Northwestern	122.8	-	71.4	85.6	0.88

In the following figures, it can be seen model fit, showcasing how well each model aligns with the study data, through an analysis of how speed and density interact within each model.

In Fig. 1 the relation between speed and traffic density according to the Greenshields model can be seen. The coefficient of determination obtained is $\boldsymbol{R^2 = 0.76}$, indicating a strong correlation between the observed data and the proposed model, with a particularly good fit for medium and high densities. However, slight discrepancies from the regression line become noticeable at very low densities.

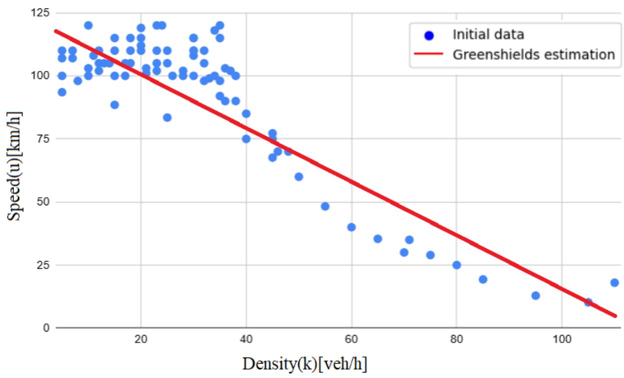

Fig. 1. Model fit for Greenshields model

For the Underwood model, in Fig. 2, the interaction between speed and density can be noticed. It is observed that at low densities, the speed remains almost constant; however, as the density increases, the speed decreases exponentially, reflecting the transition to congested traffic conditions.

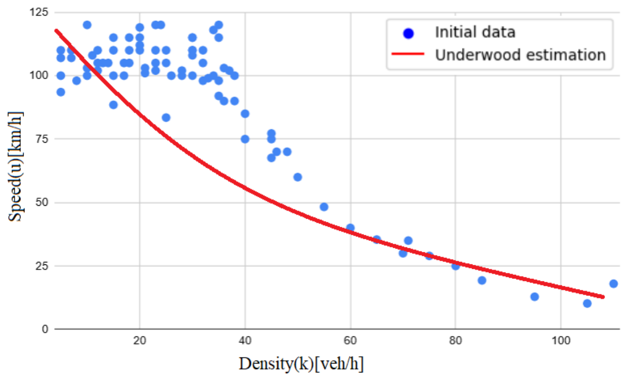

Fig. 2. Model fit for Underwood model

In the Fig. 3 it can be seen that the model Greenberg, with a low value for the coefficient of determination, $\boldsymbol{R^2} = \mathbf{0.61}$, indicating a weak correlation between the observed data and the proposed model. The Greenberg model shows a tendency to overestimate speeds at higher densities, likely due to its core assumption that speed decreases following a logarithmic trend as density increases.

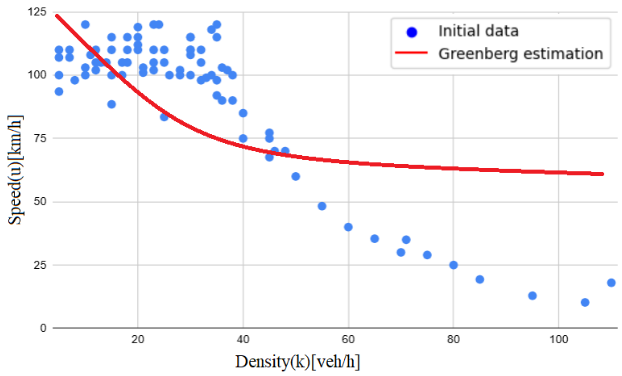

Fig. 3. Model fit for Greenberg model

Fig. 4 illustrates the dependency between speed and traffic density using the traffic model proposed by Drake, where an exponential relation is between the speed and density. The coefficient of determination, $\boldsymbol{R^2} = \mathbf{0.91}$, indicates a good correlation between the model and the observed data, making it one of the best-fitting models in this context. Compared with the other traffic flow models, the

Drake model shows the transition from free-flow conditions to the extreme conditions [19].

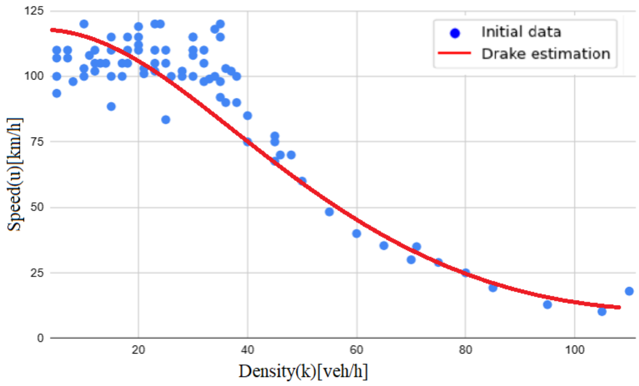

Fig. 4. Model fit for Drake model

In the case of the Robertson model, the graph in Fig. 5 illustrates the correlation between speed and density. Based on the coefficient of determination, $\boldsymbol{R^2 = 0.74}$, the model does not adequately fit the analyzed situation.

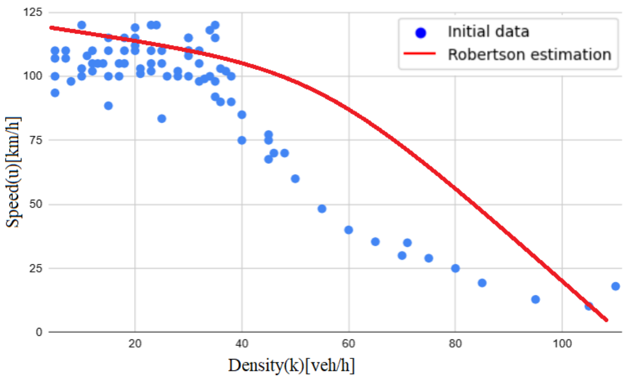

Fig. 5. Model fit for Robertson model

Similar to the Robertson model, Fig. 6 shows that the model proposed by Van Aerde exhibits a weak fit for the analyzed situation. This conclusion is supported by the coefficient of determination $\boldsymbol{R^2 = 0.70}$.

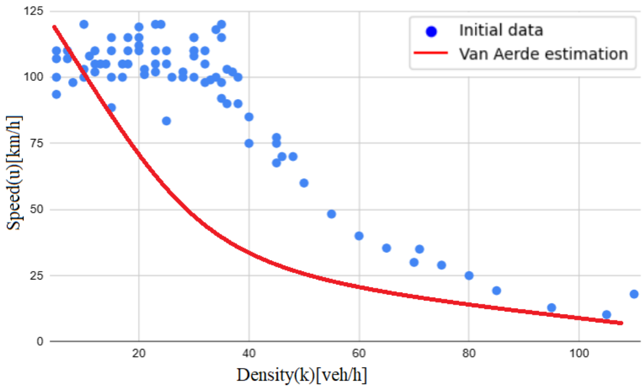

Fig. 6. Model fit for Van Aerde model

In the case of the last model that was analyzed, Northwestern model, in Fig. 7, a strong correlation can be seen between the data analyzed and the proposed model, this fact being confirmed by the coefficient of determination, $\boldsymbol{R^2 = 0.88}$.

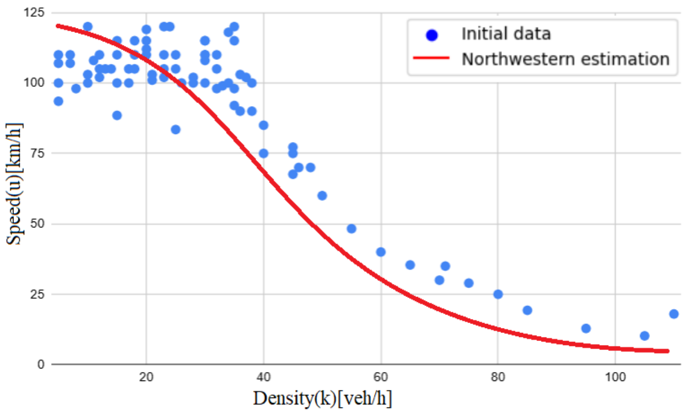

Fig. 7. Model fit for Northwestern model

The obtained results from all the traffic flow models were compared(and the results can be seen in the Table 1) in terms of values for the free flow speed($\boldsymbol{u_f}$), speed at density($\boldsymbol{u_c}$), density at capacity($\boldsymbol{k_c}$), jam density($\boldsymbol{k_j}$) and the value of the $\boldsymbol{R^2}$ parameter(this is a statistical measure used to assess how well a regression model explains the variation in the data and it can have values between 0 and 1, where 0 means no correlation between model and data and 1 means a perfect match for the variation of the data) [15]. Based on the values for $\boldsymbol{R^2}$ parameter, it can be seen that the Drake model obtained the best result, with $\boldsymbol{R^2 = 0.91}$. Based on these results, the Drake model can be recommended as the best macroscopic model that can be used for the freeway situation. The Drake model consistently achieves the highest observed R^2 value among the traffic models for the study while also providing realistic estimates for free-flow speed, as well as speed and density at capacity [20].

4. Conclusions

In the modern era, managing and monitoring traffic has become a crucial issue, as congestion impacts everyone by wasting valuable time. The quality of road infrastructure is a key factor in addressing this problem, as it can help streamline traffic flow. However, equally important are intelligent and adaptive traffic control systems that can analyze real-time traffic conditions and adjust traffic signals accordingly to enhance efficiency.

Based on the study and analysis conducted, it can be observed that the presented traffic flow models can be used to characterize a road, allowing for the determination of speed or density. Among all the analyzed traffic flow models (Greenshields, Underwood, Greenberg, Drake, Robertson, Van Aerde, Northwestern), the best fit for the analysis performed on data from the National Highways of England was achieved using the Drake model because it is applicable to scenarios involving both jam density and free-flow density conditions. It is important to consider that, as demonstrated, each traffic model has distinct characteristics and is best suited for specific road conditions. Speed and density dynamics in a highly congested metropolitan area differ significantly from those in a smaller city or highways [21]. This type of comparison highlights that macroscopic traffic flow models involve lower infrastructure and computational costs, as they require fewer sensors, less granular data, and simpler algorithms compared to video-based approaches. Furthermore, their reliance on aggregated flow trends increases robustness to data loss and sensor failures.

The future development of this project will focus on exploring advanced traffic flow models to identify the most suitable model for the specific scenarios from which the analyzed data originates. Machine learning algorithms are widely applied in domains such as healthcare, finance, and cybersecurity, and have shown significant impact in the traffic sector for tasks like traffic flow prediction, congestion detection, and anomaly identification [23]. Various machine learning algorithms can be applied to the analyzed scenarios, enabling both the prediction and optimization of critical traffic parameters, such as average speed and congestion levels for specific areas or road segments [22]. By leveraging historical data and real-time inputs, these algorithms provide accurate forecasts, offering valuable insights for optimizing traffic flow and improving urban mobility planning.

REFERENCES

[1] W. Yue, C. Li, G. Mao, N. Cheng and D. Zhou, "Evolution of road traffic congestion control: A survey from perspective of sensing, communication, and computation," in China Communications, vol. 18, no. 12, pp. 151-177, Dec. 2021, doi: 10.23919/JCC.2021.12.010.

[2] Dinu, A.; Frunzete, M. "The Lorenz chaotic system, statistical independence and sampling frequency", 2021 International Symposium on Signals, Circuits and Systems (ISSCS), 2021, pp. 1-4, doi: 10.1109/ISSCS52333.2021.9497431.

[3] J. Wang, L. Xu, Z. He and L. Wang, "Macroscopic Traffic Flow Modeling and Empirical Analysis of Multiple Modes of Travel in Urban Road Network," 2022 4th International Academic Exchange Conference on Science and Technology Innovation (IAECST), Guangzhou, China, 2022, pp. 513-517, doi: 10.1109/IAECST57965.2022.10061880.

[4] Khan, Z. H., & Gulliver, T. A. (2018). A macroscopic traffic model for traffic flow hrmonization. European Transport Research Review, 10, 1-12.

[5] H. Khan, Z., Imran, W., Azeem, S., S. Khattak, K., Gulliver, T. A., & Aslam, M. S. (2019). A macroscopic traffic model based on driver reaction and traffic stimuli. Applied Sciences, 9(14), 2848.

[6] Frunzete, M., & Popescu, A. A. (2016, June). Information channel theory for traffic control. In 2016 International Conference on Communications (COMM) (pp. 477-480). IEEE.

[7] Rakha, Hesham, and Brent Crowther. "Comparison of Greenshields, Pipes, and Van Aerde car-following and traffic stream models." Transportation Research Record 1802.1 (2002): 248-262.

[8] A. A. ŞERBAN and M. Frunzete, "Traffic flow models and statistical analysis using compressed data from acquisition module," 2024 Signal Processing: Algorithms, Architectures, Arrangements, and Applications (SPA), Poznan, Poland, 2024, pp. 183-187, doi: 10.23919/SPA61993.2024.10715618.

[9] Maelissa, Nelda, Antonetha Maitimu, and Sjafrudin Latar. "Analysis of Volume Relationship, Traffic Speed and Density in the Tulukabessy Street with the Greenberg and Underwood Methods." International Journal of Advanced Engineering Research and Science 5.12 (2018): 268269.

[10] Jabeena, M. "Comparative study of traffic flow models and data retrieval methods from video graphs." International Journal of Engineering Research and Applications 3.6 (2013): 1087-1093.

[11] G. E. Cantarella, S. de Luca, M. Di Gangi, R. Di Pace and S. Memoli, "Macroscopic vs. mesoscopic traffic flow models in signal setting design," 17th International IEEE Conference on Intelligent Transportation Systems (ITSC), Qingdao, China, 2014, pp. 2221-2226, doi: 10.1109/ITSC.2014.6958032.

[12] L. Shoufeng, L. Ximin and D. Shiqiang, "Revised MAXBAND Model for Bandwidth Optimization of Traffic Flow Dispersion," 2008 ISECS International Colloquium on Computing, Communication, Control, and Management, Guangzhou, China, 2008, pp. 85-89, doi: 10.1109/CCCM.2008.251.

[13] Romanowska, Aleksandra, and Kazimierz Jamroz. "Comparison of traffic flow models with real traffic data based on a quantitative assessment." Applied Sciences 11.21 (2021): 9914.

[14] D. C. Festa, G. Longo, G. Mazzulla and G. Musolino, "Experimental analysis of different simulation models for motorway traffic flow," ITSC 2001. 2001 IEEE Intelligent Transportation Systems. Proceedings (Cat. No.01TH8585), Oakland, CA, 2001, pp. 675-680, doi: 10.1109/ITSC.2001.948741.

[15] Williams, J. C., Mahmassani, H. S., & Herman, R. (1987). Urban Traffic Network Flow Models. Transportation Research Record, (1142), 78-88

[16] National Highways of England, Highways England Network Journey Time and Traffic Flow Data. Available at: https://www.data.gov.uk/dataset/9562c512-4a0b-45ee-b6ad-afc0f99b841f Accessed 5 January 2025.
[17] Fosu, G. O., Akweittey, E., Opong, J. M., & Otoo, M. E. (2020). Vehicular traffic models for speed-density-flow relationship. Journal of Mathematical Modeling, 8(3), 241-255.
[18] Y. Li, H. Lu, C. Bian and Y. G. Sui, "Traffic Speed-Flow Model for the Mix Traffic Flow on Beijing Urban Expressway," 2009 International Conference on Measuring Technology and Mechatronics Automation, Zhangjiajie, China, 2009, pp. 641-644, doi: 10.1109/ICMTMA.2009.312.
[19] Jabeena, M. (2013). Comparative study of traffic flow models and data retrieval methods from video graphs. International Journal of Engineering Research and Applications, 3(6), 1087-1093.
[20] Xie, S., & Wong, S. C. (2015). A Bayesian inference approach to the development of a multidirectional pedestrian stream model. Transportmetrica A: transport science, 11(1), 61-73.
[21] Kononov, J., Durso, C., Reeves, D., & Allery, B. K. (2012). Relationship between traffic density, speed, and safety and its implications for setting variable speed limits on freeways. Transportation research record, 2280(1), 1-9.
[22] Vişan, C., Boldeanu, M., Nicolae, G., Cucu, H., Burileanu, C., & Buzo, A. (2025). Evolutionary Bayesian Optimization for automated circuit sizing. Knowledge-Based Systems, 318, 113483.
[23] Meena, G., Sharma, D., & Mahrishi, M. (2020, February). Traffic prediction for intelligent transportation system using machine learning. In 2020 3rd International Conference on Emerging Technologies in Computer Engineering: Machine Learning and Internet of Things (ICETCE) (pp. 145-148). IEEE.