U.P.B. Sci. Bull., Series D, Vol. 83, Iss. 4, 2021 ISSN 1454-2358

THE MINIMAL ROTARY INERTIA MODIFIED MODEL WITH ROTATION TIME CONSTRAINT FOR THE PITCH CURVE WITH CONVEX CUSPS DEFECT FOR MULTI-LOBE NON-CIRCULAR GEAR

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Because the pitch curve with cuspidal points increase the manufacturing complexity of multi-lobe non-circular gear, so in order to reduce the manufacturing complexity of non-circular gear, a novel modified model of pitch curve with convex cusps is established. The model is driven by rotation time constraint and considered for minimal rotary inertia. Numerical example is implemented in MATLAB, and simulation results demonstrate the validity of the proposed model for the pitch curve design defect with convex cusps. Additionally, according to the analysis of modified result, the modified accuracy is higher with the decrease of the given rotation time constraint. Namely, the arbitrary modified accuracy requirement can be guaranteed by choosing appropriate rotation time constraint.

Keywords: Non-circular gear, pitch curve, convex cusps, modified mathematical model.

1. Introduction

Non-circular gears are used to transfer variable motion between two axes, compared with cam and link mechanisms. They possess large advantages, such as stable transmission, compact structure, high precision, excellent transmission power and efficiency. A variety of relevant mechanisms with non-uniform transmission characteristic have been developed in the last ten years. For example, non-circular planetary gear mechanism for gear pumps and index device for automation industry have been designed by Zheng, Hua and et al [1-2]. A pair of gears comprised of the non-circular gear and its conjugated face gear with orthogonal axes has been developed by Lin, Gong and Nie [3]. A new differential composed of non-circular bevel gears for off-road vehicle has been investigated by Jia, Gao and Suo [4]. In order to prevent the resonance oscillations, a new design method of non-circular gear mechanisms has been presented by Karpov, Nosko and et al [5]. A variable gear pump used the non-circular bevel gear, which

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possesses higher delivery and wider range of variation, has been developed by Lin, Hou and Gong [6].

With the extensive application of non-circular gears in mechanical products, non-circular gears with design defects may severely affect reliability and safety of transmission system and even have potential dangerous problems, for example, causing huge property losses and heavy casualties. Thus, it is imperative to study modified theories and methods for design defects in non-circular gears, which will be of great theoretical significance and practical value for non-circular gears.

The shape of pitch curve determines the transmission performance of non-circular gear. A constructing method of N-lode non-circular gear pitch curve by using Bézier and B-spline nonparametric curve has been proposed by Quintero Riaza, Foix and Nebot [7]. Identical multi-lobe non-circular gear pitch curves have been obtained by Tong and Yang [8]. In order to extend the applications of non-circular gears, a design method of pitch curve with epicycle constraint constructed by plane regular multi-curve polygon for multi-lobe non-circular gear has been proposed by Yao [9]. Recently, two novel design methods of the non-circular gear pitch curve with minimal rotary inertia and steepest rotation characteristics have been proposed by Zhang and Fan [10, 11].

Researchers at home and abroad have made some research in the modified method of pitch curve design defect. For example, based on the design idea of Ref. [11], the steepest rotation modified method of the pitch curve design defect with discontinuity points can be proposed by Zhang and Fan [12]. Based on the manufacturing principle of gear shaper and multi-lobe non-circular bevel gear, a section pitch surface of the cutter can be used to replace the concave or convex pitch surface of multi-lobe non-circular bevel gears [13].

We know that the transmission ratio function curve possesses cusps at junctions between lobes of pitch curves with cusp points for multi-lobe non-circular gears. Namely, there is a sudden change in the output angular acceleration during the non-circular gears transmission [9, 14-15]. Therefore, in order to improve the transmission stability of multi-lobe non-circular gears with convex cusps pitch curves, the calculus of variations is used to establish a minimal rotary inertia modified model of the pitch curve design defect with convex cusps for multi-lobe non-circular gear with rotation time constraint in this paper. Numerical example is shown to validate the feasibility and reasonability of the proposed modified model.

2. Minimal rotary inertia modified model with rotation time constraint

As shown in Fig. 1, the fixed coordinate system Γ(o-xy) rigidly connected with the rotation center o of the pitch curve $r(\theta)$ with convex cusps v_1, v_2, ..., v_N for multi-lobe non-circular gear, and the polar angle θ is measured counterclockwise from the positive direction x-axis, then the polar equation for the pitch curve $r(\theta)$ with convex cusps and its constraint conditions can be represented as [12]

$$r(\theta)=\begin{cases} r_1(\theta), \theta\in[0,2\pi/N] \\ \vdots \\ r_k(\theta)=r_1(\theta-2(k-1)\pi/N), \theta\in[2(k-1)\pi/N, 2k\pi/N] \end{cases} \tag{1}$$

$$\begin{cases} r_k(2k\pi/N)=r_{(k+1)}(2k\pi/N) \\ r_k{}'(2k\pi/N)\neq r_{(k+1)}{}'(2k\pi/N),\ r_k{}'(2k\pi/N)>0, r_{(k+1)}{}'(2k\pi/N)<0 \end{cases} \tag{2}$$

where N is the number of lobes of pitch curve $r(\theta)$ for multi-lobe non-circular gear, k=1,2,...,N and the following discussion relating to subscript k, if k=N, then (k+1)=1.

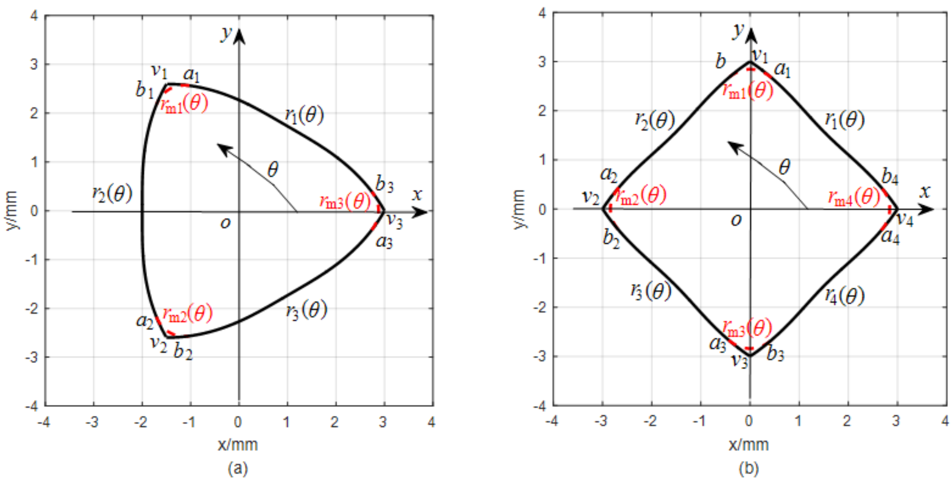

Fig. 1 Pitch curves with convex cusps and its modified model for multi-lobe non-circular gear

Referring to Fig. 1, we know that these convex cusps v_1, v_2, ..., v_N have certain period $2\pi/N$. In order to simplify the calculation, we just need to study the modified model of convex cusp v_1. Assuming the pitch curve $r_{m1}(\theta)$ can be used to replace two tiny section pitch curves $\overset{\frown}{a_1v_1}$ and $\overset{\frown}{v_1b_1}$, a_1 and b_1 are tangency points that the modified pitch curve $r_{m1}(\theta)$ makes with pitch curves $r_1(\theta)$ and $r_2(\theta)$, their corresponding polar angles are θ_{a1} and θ_{b1}, respectively.

As shown in Fig. 2, assuming infinitesimal $dT(\theta)$ is the rotation time of non-circular gear through the arc length $dS(\theta)$ on the modified pitch curve $r_{m1}(\theta)$

under the condition of input angular velocity ω, then infinitesimal $dT(\theta)$ and rotation time constraint T can be, respectively, represented as

$$dT(\theta)=\frac{dS(\theta)}{\omega r_{\mathrm{m1}}(\theta)}=\frac{\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}}{\omega r_{\mathrm{m1}}(\theta)}d\theta,\ s.t.\ dS(\theta)=\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}d\theta \tag{3}$$

$$T=\int_{\theta_{\mathrm{a1}}}^{\theta_{\mathrm{b1}}}dT(\theta)=\int_{\theta_{\mathrm{a1}}}^{\theta_{\mathrm{b1}}}\frac{\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}}{\omega r_{\mathrm{m1}}(\theta)}d\theta \tag{4}$$

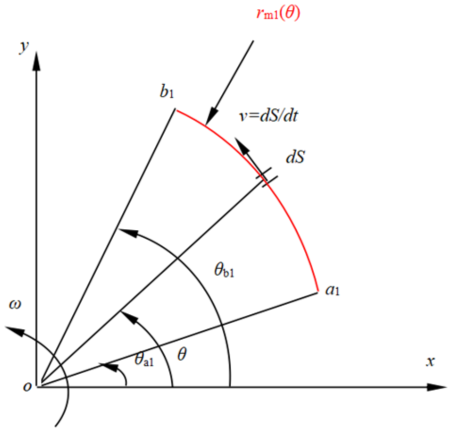

Fig. 2 Solution of the minimum mean kinetic energy modified model with rotation time constraint

According to the kinematics principle, assuming the mass of the modified pitch curve $r_{m1}(\theta)$ is Q, then the mass of an infinitesimal arc length dS on the modified pitch curve $r_{m1}(\theta)$ is dQ ($dQ=LdS$), where L ($L=Q/S$) is linear density, then the rotary inertia dJ corresponding to the infinitesimal arc length dS can be expressed as [10, 14]

$$dJ=r_{\mathrm{m1}}^{2}(\theta)dQ=r_{\mathrm{m1}}^{2}(\theta)LdS=\frac{Qr_{\mathrm{m1}}^{2}(\theta)\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}}{\int_{\theta_{\mathrm{a1}}}^{\theta_{\mathrm{b1}}}\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}d\theta}d\theta \tag{5}$$

The integral to both sides of Eq. (5) at the same time, one obtains

$$J=\int_{\theta_{\mathrm{a1}}}^{\theta_{\mathrm{b1}}}\frac{Qr_{\mathrm{m1}}^{2}(\theta)\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}}{\int_{\theta_{\mathrm{a1}}}^{\theta_{\mathrm{b1}}}\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}d\theta}d\theta \tag{6}$$

From the Eq. (6), we know that the rotary inertia J under the constraint of rotation time T is different for different modified pitch curve $r_{m1}(\theta)$, the minimum $J_{\min}$ of rotary inertia J can be established according to the variational method with integral constraint [16]

$$J_{\min}=\min\int_{\theta_{\mathrm{a1}}}^{\theta_{\mathrm{b1}}}\left(\frac{Qr_{\mathrm{m1}}^{2}(\theta)\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}}{\int_{\theta_{\mathrm{a1}}}^{\theta_{\mathrm{b1}}}\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}d\theta}+\mu\frac{\sqrt{r_{\mathrm{m1}}^{2}(\theta)+r_{\mathrm{m1}}'^{2}(\theta)}}{\omega r_{\mathrm{m1}}(\theta)}\right)d\theta \tag{7}$$

where $J_{\min}$ is the minimal rotary inertia of non-circular gear through the modified pitch curve $r_{m1}(\theta)$ under the rotation time constraint T, μ is an undetermined Lagrange multiplier.

Meanwhile, the following equation can also be represented as

$$\frac{Q{r_{m1}}^{4}(\theta)}{\sqrt{{r_{m1}}^{2}(\theta)+{r'_{m1}}^{2}(\theta)}\int_{\theta_{a1}}^{\theta_{b1}}\sqrt{{r_{m1}}^{2}(\theta)+{r'_{m1}}^{2}(\theta)}d\theta}+\mu\frac{r_{m1}(\theta)}{\omega\sqrt{{r_{m1}}^{2}(\theta)+{r'_{m1}}^{2}(\theta)}}=f \tag{8}$$

where f is an undetermined integral constant.

Assuming $r'_{m1}(\theta)=r_{m1}(\theta)\tan\alpha$, $\alpha\in(-\pi/2, \pi/2)$, the following equations can be represented as

$$ {r_{m1}}^{3}(\theta)=\frac{\omega f_1\sec\alpha-f_2}{\omega},\alpha\in(-\frac{\pi}{2},\frac{\pi}{2}) \tag{9}$$

$$ dr_{m1}(\theta)=\frac{f_1}{3r_{m1}^{3}(\theta)}\sec\alpha\tan\alpha d\alpha \tag{10}$$

$$ dr_{m1}(\theta)=\frac{f_1}{3r_{m1}^{3}(\theta)}\sec\alpha\tan\alpha d\alpha \tag{11}$$

where f_1 and f_2 are undetermined parameters and they can be represented as

$$ f_1=\frac{f}{Q}\cdot\int_{\theta_{a1}}^{\theta_{b1}}\sqrt{{r_{m1}}^{2}(\theta)+{r'_{m1}}^{2}(\theta)}d\theta,\ f_2=\frac{\mu}{Q}\cdot\int_{\theta_{a1}}^{\theta_{b1}}\sqrt{{r_{m1}}^{2}(\theta)+{r'_{m1}}^{2}(\theta)}d\theta \tag{12}$$

According to Eq. (9)-Eq. (11), the second-derivative $r''_{m1}(\theta)$ and curvature radius ρ_J of minimum rotary inertia modified pitch curve $r_{m1}(\theta)$ with rotation time constraint T can be represented as

$$ r''_{m1}(\theta)=r_{m1}(\theta)(4\sec^2\alpha-1-\frac{3f_2\sec\alpha}{\omega f_1}),\alpha\in(-\frac{\pi}{2},\frac{\pi}{2}) \tag{13}$$

$$ \rho_J=\frac{r_{m1}(\theta)\sec^2\alpha}{\frac{3f_2}{\omega f_1}-2\sec\alpha},\alpha\in(-\frac{\pi}{2},\frac{\pi}{2}) \tag{14}$$

Integrating both sides of Eq. (11), we can obtain

$$\theta(\alpha)=\begin{cases}\frac{2\omega f_1}{3(\omega f_1-f_2)}\sqrt{\frac{\omega f_1-f_2}{\omega f_1+f_2}}\arctan(\sqrt{\frac{\omega f_1+f_2}{\omega f_1-f_2}}\tan(\alpha/2))+f_3,\frac{(\omega f_1)^2}{(f_2)^2}>1\\ \frac{\omega f_1}{3(\omega f_1-f_2)}\sqrt{\frac{f_2-\omega f_1}{f_2+\omega f_1}}\ln\left|\frac{\tan(\alpha/2)+\sqrt{\frac{f_2-\omega f_1}{f_2+\omega f_1}}}{\tan(\alpha/2)-\sqrt{\frac{f_2-\omega f_1}{f_2+\omega f_1}}}\right|+f_3,\frac{(\omega f_1)^2}{(f_2)^2}<1\end{cases} \tag{15}$$

where f_3 is an undetermined integral constant.

Assuming

$$\theta(\alpha_{a1})=\theta_{a1},\ \theta(\alpha_{b1})=\theta_{b1} \tag{16}$$

where α_{a1} and α_{b1} are undetermined values of parameter α corresponding to polar angles θ_{a1} and θ_{b1}, respectively.

Along with Eq. (4), Eq. (9) and Eq. (15), the minimal rotary inertia modified model with rotation time constraint T for the pitch curve with convex cusps and its constraint conditions can be represented as

$$r_{\mathrm{m1}}(\theta)=\begin{cases} r_{\mathrm{m1}}(\alpha)=\sqrt[3]{\dfrac{\omega f_1\sec\alpha-f_2}{\omega}} \\ \theta(\alpha)=\begin{cases} \dfrac{2\omega f_1}{3(\omega f_1-f_2)}\sqrt{\dfrac{\omega f_1-f_2}{\omega f_1+f_2}}\arctan(\sqrt{\dfrac{\omega f_1+f_2}{\omega f_1-f_2}}\tan(\alpha/2))+f_3, (\dfrac{\omega f_1}{f_2})^2>1 \\ \dfrac{\omega f_1}{3(\omega f_1-f_2)}\sqrt{\dfrac{f_2-\omega f_1}{f_2+\omega f_1}}\ln\left|\dfrac{\tan(\alpha/2)+\sqrt{\dfrac{f_2-\omega f_1}{f_2+\omega f_1}}}{\tan(\alpha/2)-\sqrt{\dfrac{f_2-\omega f_1}{f_2+\omega f_1}}}\right|+f_3, (\dfrac{\omega f_1}{f_2})^2<1 \end{cases} \end{cases} \tag{17}$$

and

$$\begin{cases} \theta\in[\theta_{\mathrm{a1}},\theta_{\mathrm{b1}}],\ \alpha\in[\alpha_{\mathrm{a1}},\alpha_{\mathrm{b1}}],\ \theta(\alpha_{\mathrm{a1}})=\theta_{\mathrm{a1}},\ \theta(\alpha_{\mathrm{b1}})=\theta_{\mathrm{b1}} \\ r_{\mathrm{m1}}(\alpha_{\mathrm{a1}})=r_1(\theta_{\mathrm{a1}}),\ r_{\mathrm{m1}}'(\alpha_{\mathrm{a1}})=r_1'(\theta_{\mathrm{a1}}) \\ r_{\mathrm{m1}}(\alpha_{\mathrm{b1}})=r_2(\theta_{\mathrm{b1}}),\ r_{\mathrm{m1}}'(\alpha_{\mathrm{b1}})=r_2'(\theta_{\mathrm{b1}}) \\ T=\dfrac{2f_2}{3\omega^2 f_1}(\theta_{\mathrm{b1}}-\theta_{\mathrm{a1}})+\dfrac{1}{3\omega}\ln\left|\dfrac{\sec\alpha_{\mathrm{b1}}+\tan\alpha_{\mathrm{b1}}}{\sec\alpha_{\mathrm{a1}}+\tan\alpha_{\mathrm{a1}}}\right| \end{cases} \tag{18}$$

3. Parameter analysis of the minimal rotary inertia modified model with rotation time constraint

According to Eq. (2), the first order derivatives $r_1'(\theta_{\mathrm{a1}})$ and $r_2'(\theta_{\mathrm{b1}})$ around convex cusp v_1 should satisfy the following constraints:

$$r_1'(\theta_{\mathrm{a1}})>0,\ r_2'(\theta_{\mathrm{b1}})<0 \tag{19}$$

According to $r'_{\mathrm{m1}}(\theta)=r_{\mathrm{m1}}(\theta)\tan\alpha$, $\alpha\in(-\pi/2, \pi/2)$ and Eq. (18), the ranges of undetermined parameters α_{a1} and α_{b1} can be represented as

$$\alpha_{\mathrm{a1}}\in(0,\pi/2),\ \alpha_{\mathrm{b1}}\in(-\pi/2,0) \tag{20}$$

According to the meshing principle of non-circular gear, we know that the value of modified pitch curve $r_{\mathrm{m1}}(\theta)$ should be greater than zero, and the parameter α is monotone decreasing at the range of $[\alpha_{\mathrm{a1}}, \alpha_{\mathrm{b1}}]$, therefore, Eq. (17) should satisfy the following inequations

$$\left(\omega f_1\sec\alpha-f_2\right)_{\min}>0 \text{ and } \left[\frac{\omega f_1}{3(\omega f_1-f_2\cos\alpha)}\right]_{\max}<0 \tag{21}$$

Then, the ranges of undetermined parameters f_1 and f_2 can be solved

$$\begin{cases} f_1 f_2>0 \\ \dfrac{\omega f_1}{\omega f_1-f_2\cos\alpha_{\mathrm{a1}}}<0,\ \dfrac{\omega f_1}{\omega f_1-f_2\cos\alpha_{\mathrm{b1}}}<0 \end{cases} \text{ or } \begin{cases} f_1 f_2<0 \\ \dfrac{\omega f_1}{\omega f_1-f_2}<0 \end{cases} \tag{22}$$

Referring to Fig. 1, it is only when the modified pitch curve $r_{\mathrm{m1}}(\theta)$ is outer convex, the convex point v_1 can be modified within a small margin of error, Eq. (14) should satisfy the following inequation

$$\left(\frac{3f_2}{\omega f_1}-2\sec\alpha\right)>0,\ \forall\alpha\in[\alpha_{\mathrm{a1}},\ \alpha_{\mathrm{b1}}] \tag{23}$$

Then, the undetermined parameters f_1 and f_2 should also satisfy the following constraint

$$\frac{f_2}{\omega f_1} > \frac{2}{3}\sec\alpha_{a1},\ \frac{f_2}{\omega f_1} > \frac{2}{3}\sec\alpha_{b1} \tag{24}$$

Along with Eq. (22), therefore, the ranges of undetermined parameters α_{a1}, α_{b1}, f_1 and f_2 can be represented as

$$\alpha_{a1} \in (0,\frac{\pi}{2}),\ \alpha_{b1} \in (-\frac{\pi}{2},0),\ f_1 < 0,\ f_2 < 0,\ \frac{f_2}{\omega f_1} > \sec\alpha_{a1},\ \frac{f_2}{\omega f_1} > \sec\alpha_{b1} \tag{25}$$

Along with Eq. (17), the minimal rotary inertia modified model with rotation time constraint for the pitch curve with convex cusps of non-circular gear can be represented as

$$r_{m1}(\theta)=\begin{cases} r_{m1}(\alpha)=\sqrt[3]{\dfrac{\omega f_1\sec\alpha - f_2}{\omega}} \\ \theta(\alpha)=\dfrac{\omega f_1}{3(\omega f_1 - f_2)}\sqrt{\dfrac{f_2-\omega f_1}{f_2+\omega f_1}}\ln\left|\dfrac{\tan(\alpha/2)+\sqrt{\dfrac{f_2-\omega f_1}{f_2+\omega f_1}}}{\tan(\alpha/2)-\sqrt{\dfrac{f_2-\omega f_1}{f_2+\omega f_1}}}\right|+f_3 \end{cases},\ \theta\in[\theta_{a1},\theta_{b1}] \tag{26}$$

According to Eq. (1), Eq. (18) and Eq. (26), the modified multi-lobe non-circular pitch curve $R(\theta)$ can be represented as

$$R(\theta)=\begin{cases} \begin{cases} r_1(\theta),\theta\in[\theta_{b1}-\dfrac{2\pi}{N},\theta_{a1}] \\ r_{m1}(\theta),\theta\in[\theta_{a1},\theta_{b1}] \end{cases} \\ \quad\vdots \\ \begin{cases} r_k(\theta)=r_1(\theta-\dfrac{2(k-1)\pi}{N}),\theta\in[\theta_{b1}+\dfrac{2(k-2)\pi}{N},\theta_{a1}+\dfrac{2(k-1)\pi}{N}] \\ r_{mk}(\theta)=r_{m1}(\theta-\dfrac{2(k-1)\pi}{N}),\theta\in[\theta_{a1}+\dfrac{2(k-1)\pi}{N},\theta_{b1}+\dfrac{2(k-1)\pi}{N}] \end{cases} \end{cases} \tag{27}$$

4. Modified example of the minimal rotary inertia modified model with rotation time constraint

Assuming the 3-lobe non-circular gear pitch curve $r_v(\theta)$ with convex cusps satisfied the given transmission ratio $i_v(\theta)$ of Eq. (29) can be expressed as

$$r(\theta)=\begin{cases} r_1(\theta)=4-\sqrt{3}\sin\theta-\cos\theta,\ \theta\in[0,\ 2\pi/3] \\ r_2(\theta)=r_1(\theta-2\pi/3),\ \theta\in[2\pi/3,\ 4\pi/3] \\ r_3(\theta)=r_1(\theta-4\pi/3),\ \theta\in[4\pi/3,\ 2\pi] \end{cases} \tag{28}$$

$$i_v(\theta)=\begin{cases}\dfrac{r_1(\theta)}{4.7741-r_1(\theta)}, \theta\in[0,2\pi/3]\\ \dfrac{r_2(\theta)}{4.7741-r_2(\theta)}, \theta\in[2\pi/3,4\pi/3]\\ \dfrac{r_3(\theta)}{4.7741-r_3(\theta)}, \theta\in[4\pi/3,2\pi]\end{cases} \tag{29}$$

Fig. 3 and Fig. 4 are transmission ratio $i_V(\theta)$ of Eq. (29) and 3-lobe non-circular gear pitch curve $r_V(\theta)$ with convex cusps and its conjugated external meshing non-circular gear pitch curve $r_e(\theta_e)$, respectively. The transmission ratio curve of multi-lobe non-circular gears with convex cusps pitch curves is not smooth, so there is a sudden change in the output angular velocity during the non-circular gears transmission, as shown in Fig. 3. It is necessary to modify the pitch curve with convex cusps for multi-lobe non-circular gear in order to improve transmission stability.

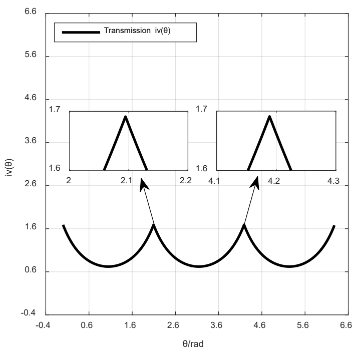

Fig. 3 Given transmission ratio $i_V(\theta)$ of Eq. (29)

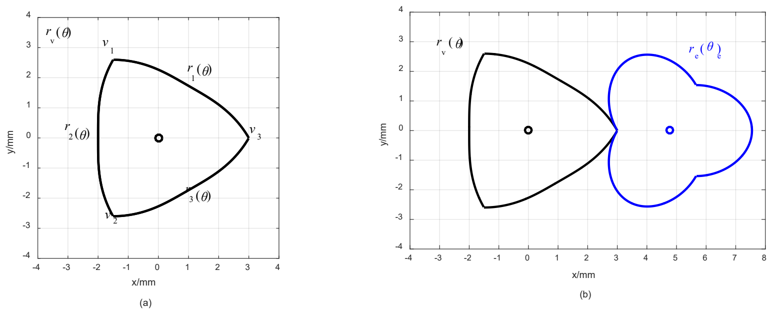

Fig. 4 Non-circular gear pitch curve $r_V(\theta)$ with convex cusps and its conjugated external meshing non-circular gear pitch curve $r_e(\theta_e)$

According to the description of section 2 and section 3, the minimal rotary inertia modified parameters with rotation time constraint for the pitch curve $r_V(\theta)$ with convex cusps are listed in Table 1, the center distance a_E and the modified transmission ratio function $i_{EV}(\theta)$ of external meshing non-circular gears can be solved and they are listed in Table 2. For convenience, the units of parameters listed in Table 1 and Table 2 are adopted by standard international unit (SIU).

Table 1

Minimal rotary inertia modified parameters with rotation time constraint

Constraint T and angular velocity ω	Undetermined parameters f_1, f_2, f_3, θ_{a1}, θ_{b1}, α_{a1} and α_{b1}	Minimal rotary inertia modified model for the pitch curve with convex cusps
$\begin{cases} T=0.08 \\ \omega=1 \end{cases}$	$\begin{cases} f_1=-18.1267, f_2=-41.9562, f_3=2\pi/3 \\ \theta_{a1}=2\pi/3-0.1430, \theta_{b1}=2\pi/3+0.1430 \\ \alpha_{a1}=0.5172, \alpha_{b1}=-0.5172 \end{cases}$	$r_{m1}(\theta)=\begin{cases} r_{m1}(\alpha)=\sqrt[3]{-18.1267\sec\alpha+41.9562} \\ \theta(\alpha)=-0.1597\ln\left\lvert\frac{\tan(\alpha/2)+0.6298}{\tan(\alpha/2)-0.6298}\right\rvert+\frac{2\pi}{3} \end{cases}$
$\begin{cases} T=0.09 \\ \omega=1 \end{cases}$	$\begin{cases} f_1=-18.8926, f_2=-42.5779, f_3=2\pi/3 \\ \theta_{a1}=2\pi/3-0.1500, \theta_{b1}=2\pi/3+0.1500 \\ \alpha_{a1}=0.5165, \alpha_{b1}=-0.5165 \end{cases}$	$r_{m1}(\theta)=\begin{cases} r_{m1}(\alpha)=\sqrt[3]{-18.8926\sec\alpha+42.5779} \\ \theta(\alpha)=-0.1650\ln\left\lvert\frac{\tan(\alpha/2)+0.6207}{\tan(\alpha/2)-0.6207}\right\rvert+\frac{2\pi}{3} \end{cases}$
$\begin{cases} T=0.1 \\ \omega=1 \end{cases}$	$\begin{cases} f_1=-19.6490, f_2=-43.1910, f_3=2\pi/3 \\ \theta_{a1}=2\pi/3-0.1570, \theta_{b1}=2\pi/3+0.1570 \\ \alpha_{a1}=0.5158, \alpha_{b1}=-0.5158 \end{cases}$	$r_{m1}(\theta)=\begin{cases} r_{m1}(\alpha)=\sqrt[3]{-19.6490\sec\alpha+43.1910} \\ \theta(\alpha)=-0.1703\ln\left\lvert\frac{\tan(\alpha/2)+0.6121}{\tan(\alpha/2)-0.6121}\right\rvert+\frac{2\pi}{3} \end{cases}$
$\begin{cases} T=0.11 \\ \omega=1 \end{cases}$	$\begin{cases} f_1=-20.3962, f_2=-43.7960, f_3=2\pi/3 \\ \theta_{a1}=2\pi/3-0.1640, \theta_{b1}=2\pi/3+0.1640 \\ \alpha_{a1}=0.5150, \alpha_{b1}=-0.5150 \end{cases}$	$r_{m1}(\theta)=\begin{cases} r_{m1}(\alpha)=\sqrt[3]{-20.3962\sec\alpha+43.7960} \\ \theta(\alpha)=-0.1754\ln\left\lvert\frac{\tan(\alpha/2)+0.6038}{\tan(\alpha/2)-0.6038}\right\rvert+\frac{2\pi}{3} \end{cases}$
$\begin{cases} T=0.12 \\ \omega=1 \end{cases}$	$\begin{cases} f_1=-21.1346, f_2=-44.3930, f_3=2\pi/3 \\ \theta_{a1}=2\pi/3-0.1710, \theta_{b1}=2\pi/3+0.1710 \\ \alpha_{a1}=0.5143, \alpha_{b1}=-0.5143 \end{cases}$	$r_{m1}(\theta)=\begin{cases} r_{m1}(\alpha)=\sqrt[3]{-21.1346\sec\alpha+44.3930} \\ \theta(\alpha)=-0.1805\ln\left\lvert\frac{\tan(\alpha/2)+0.5958}{\tan(\alpha/2)-0.5958}\right\rvert+\frac{2\pi}{3} \end{cases}$

Table 2

Solved center distance a_E

Constraint T and angular velocity ω	$\begin{cases} T=0.08 \\ \omega=1 \end{cases}$	$\begin{cases} T=0.09 \\ \omega=1 \end{cases}$	$\begin{cases} T=0.1 \\ \omega=1 \end{cases}$	$\begin{cases} T=0.11 \\ \omega=1 \end{cases}$	$\begin{cases} T=0.12 \\ \omega=1 \end{cases}$
External meshing maximum polar angle θ_{Ee} of one lobe non-circular gear pitch curve	$\begin{cases} \theta_{Ee}=\frac{2\pi}{3}=\int_0^{\frac{2\pi}{3}}\frac{R_1(\theta)}{a_E-R_1(\theta)}d\theta \\ R_1(\theta)=\begin{cases} r_1(\theta)=4-\sqrt{3}\sin\theta-\cos\theta, \theta\in[\theta_{b1}-\frac{2\pi}{N}, \theta_{a1}] \\ r_{m1}(\theta), \theta\in[\theta_{a1}, \theta_{b1}] \end{cases} \end{cases}$				
Solved center distance a_E	4.7566	4.7550	4.7534	4.7516	4.7499

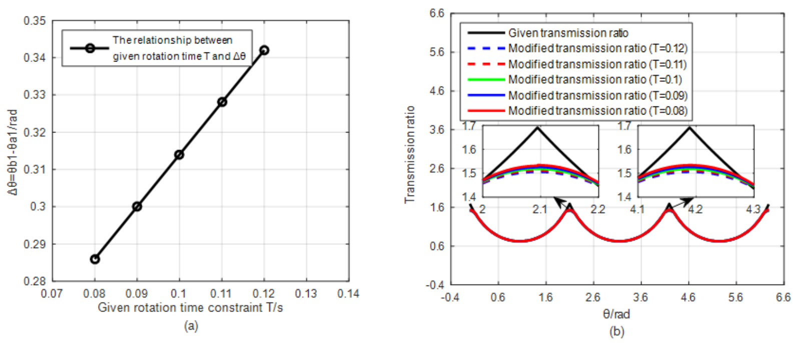

Fig. 5 Changes of $\Delta\theta$ and modified transmission ratio functions for the minimal rotary inertia modified with different rotation time constraints

Fig. 5(a)~(b) are the changes of $\Delta\theta$ and modified transmission ratio functions for the minimal rotary inertia modified with different rotation time constraints of the pitch curve with convex cusps, respectively. Referring to Fig. 5, we known that the $\Delta\theta$ is decreasing with gradual decrease of the rotation time constraint T and the modified transmission ration function $i_{Ev}(\theta)$ is closer to the original transmission ration function $i_v(\theta)$ of Eq. (29). It means that the modified accuracy is higher with the decrease of the given rotation time constraint T. Therefore, the modified accuracy can be guaranteed by choosing appropriate rotation time constraint T in practical engineering application.

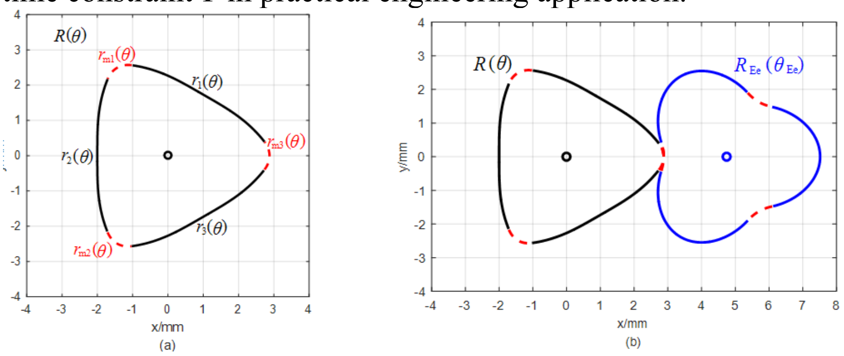

Fig. 6 Modified pitch curve $R(\theta)$ and its conjugate external meshing non-circular gear pitch curve $R_{Ee}(\theta_{Ee})$ of the minimal rotary inertia modified

In this paper, the rotation time constraint (T=0.08) is chosen as the minimal rotary inertia modified example to illustrates the feasibility of the minimal rotary inertia modified method. Fig. 6(a)~(b) are, respectively, the modified pitch curve $R(\theta)$ and its conjugate external meshing pitch curve $R_{Ee}(\theta_{Ee})$, there corresponding modified parameters are listed in Table 1 and Table 2. And the pitch curve equations $R(\theta)$ and $R_{Ee}(\theta_{Ee})$ are shown in Eq. (30) and Eq. (31), respectively.

$$R(\theta)=\begin{cases}\begin{cases}r_1(\theta)=4-\sqrt{3}\sin\theta-\cos\theta,\theta\in[0.1430,\dfrac{2\pi}{3}-0.1430]\\ r_{m1}(\theta)=\begin{cases}r_{m1}(\alpha)=\sqrt[3]{-18.1267\sec\alpha+41.9562}\\ \theta(\alpha)=-0.1597\ln\left|\dfrac{\tan(\alpha/2)+0.6298}{\tan(\alpha/2)-0.6298}\right|+2\pi/3\end{cases},\theta\in[\dfrac{2\pi}{3}-0.1430,\dfrac{2\pi}{3}+0.1430]\end{cases}\\ \begin{cases}r_2(\theta)=r_1(\theta-\dfrac{2\pi}{N}),\theta\in[\dfrac{2\pi}{3}+0.1430,\dfrac{4\pi}{3}-0.1430]\\ r_{m2}(\theta)=r_{m1}(\theta-\dfrac{2\pi}{N}),\theta\in[\dfrac{4\pi}{3}-0.1430,\dfrac{4\pi}{3}+0.1430]\end{cases}\\ \begin{cases}r_3(\theta)=r_1(\theta-\dfrac{4\pi}{3}),\theta\in[\dfrac{4\pi}{3}+0.1430,2\pi-0.0.1430]\\ r_{m3}(\theta)=r_{m1}(\theta-\dfrac{4\pi}{3}),\theta\in[2\pi-0.0.1430,2\pi+0.0.1430]\end{cases}\end{cases} \tag{30}$$

$$\begin{cases} R_{\mathrm{Ee}}(\theta_{\mathrm{Ee}}) = 4.7566 - R(\theta) \\ \theta_{\mathrm{Ee}} = \int_0^{\theta} \frac{R(\theta)}{4.7566 - R(\theta)} d\theta \end{cases} \tag{31}$$

5. Conclusions

The minimal rotary inertia modified model with rotation time constraint of pitch curve design defect with convex cusps for multi-lobe non-circular gear is proposed. A modified example is implemented in MATLAB, and its results demonstrate the validity of the proposed modified method. Due to the modified accuracy is higher with the decrease of the given rotation time constraint, so the modified model driven by rotation time constraint is obviously superior to the unconstrained modified model. Moreover, the modified accuracy can be guaranteed by choosing appropriate rotation time constraint in practical engineering application.

The appropriate rotation time constraint has two engineering applications:

1) Rotation time constraint can be used as an evaluation index of modified accuracy of the minimal rotary inertia modified model for the pitch curve design defect with convex cusps.

2) According to the different engineering applications of non-circular gears transmission, the rotation time constraint can be arbitrarily selected in a certain range under the condition of satisfying the modified accuracy.

Previous research on pitch curve design defect of non-circular gear focus on design defects with discontinuity points [12] and concave cusps [14-15], and these modified models are unconstrained driven modified model. However, the pitch curve design defect with convex cusps can be effectively modified by the proposed modified model with rotation time constraint in this paper, and the modified accuracy can be guaranteed according to the rotation time constraint. The modified models and theories of the design defects for the pitch curve with non-differentiable points, including discontinuous points, concave cusps, and convex cusps, are further improved.

Acknowledgment

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the Basic Research Program of Natural Science in Shaanxi Province, China (grant number: 2021JQ-809), and the 2021 Service technology innovation project of Wenzhou Association for Science and Technology (grant number: kjfw40).

REFERENCES

[1]. *F.Y. Zheng, L. Hua and X. H. Han,* "The mathematical model and mechanical properties of variable center distance gears based on screw theory", Mechanism and Machine Theory, **vol. 101**, 2016, pp. 116-139.

[2] *F.Y. Zheng, L. Hua and et al,* "Synthesis of indexing mechanisms with non-circular gears", Mechanism and Machine Theory, **vol. 105**, no. 4, 2016, pp. 108-128.

[3] *Lin C., Gong H. and Nie N.,* "Geometry design, three-dimensional modeling and kinematic analysis of orthogonal fluctuating gear ratio face gear drive", Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, **vol. 232**, no. 4, 2013, pp. 779-793.

[4] *J. Jia, B. Gao and W. Suo,* "Study on a new type of differential with variable ratio for off-road vehicle", China Mechanical Engineering, **vol. 23**, no. 23, 2012, pp. 2844-2847.

[5] *O. Karpov, P. Nosko and et al,* "Prevention of resonance oscillations in gear mechanisms using non-circular gears",Mechanism and Machine Theory, **vol. 114**, 2017, pp. 1-10.

[6] *C. Lin, Y. Hou and H. Gong,* "Flow characteristics of high-order ellipse bevel gear pump", Journal of Drainage and Irrigation Machinery Engineering, **vol. 29**, no. 5, 2011, pp. 379-385.

[7] *H. F. Quintero Riaza, S. C. Foix and L. J. Nebot,* "The synthesis of an *N*-lobe noncircular gear using Bézier and B-spline nonparametric curve on the design of its displacement law", Journal of Mechanical Design, Transactions of the ASME, **vol. 129**, no. 9, 2006, pp. 981-985.

[8] *S. H. Tong and D. C. H. Yang,* "Generation of identical noncircular pitch curves", Journal of Mechanical Design, Transactions of the ASME, **vol. 120**, no. 2, 1998, pp. 337-341.

[9]. *W. X. Yao*, (Noncircular gears design), China Machine Press, Beijing, China, 2013.

[10]. *X. Zhang, S. W. Fan,* "Study on the pitch curve with minimal rotary inertia for the noncircular gears", Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, **vol. 232**, no. 15, 2018, pp. 2666-2673.

[11]. *X. Zhang and S. W. Fan*, "Synthesis of the Steepest Rotation Pitch Curve Design for Noncircular Gear", Mechanism and Machine Theory, **vol. 102**, 2016, pp. 16-35.

[12]. *X. Zhang, S. W. Fan,* "Classification identification and modification of the pitch curve with nondifferentiable points for *N*-lobed noncircular gear", Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, **vol. 230**, no. 19, 2016, pp. 3556-3567.

[13]. *G. Lv, S. W. Fan and X. Zhang*, "Study on the design and the shaping of the pitch surface of a multi-lobed non-circular bevel gear", Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering, **vol. 230**, no. 4, 2016, pp. 542-553.

[14]. *X. Zhang*, *S. Fan,* "Modification design with minimal rotary inertia for the pitch curve with concave cusps of *N*-LNG", Academic Journal of Manufacturing Engineering, **vol. 17**, no. 3, 2019, pp. 113-121.

[15]. *X. Zhang*, *S. Fan,* "Minimal mean kinetic energy modification for the design defect of pitch curve with concave cusps of *N*-lobe noncircular gear", UPB Scientific Bulletin, Series D: Mechanical Engineering, **vol. 82**, no. 4, 2020, pp. 191-198.

[16]. *D. Z. Lao*, (Fundamentals of the calculus of variations), National Defense Industry Press, Beijing, China, 2015.