DISCRETE-TIME CONTROL STATEGIES FOR HORIZONTAL AXIS WIND TURBINES

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This paper introduces two discrete-time controllers for horizontal axis wind turbines (HAWTs): a Linear Quadratic Gaussian (LQG) one and a model predictive (MPC) one. The LQG controller aims to strongly attenuate the disturbances influence on the system output. The control objective consists of keeping the output power constant, despite the wind variation, and thus reducing the fatigue that involves damaging the turbine components. The MPC provides an integrated solution for controlling systems with interacting variables, complex dynamics and constraints. Both controllers have the advantage that can easily be implemented and in case of HAWTs.

Keywords: wind turbine, linear model, discret-time controller, LQG, MPC

1. Introduction

In 2010, Romania has increased the energy production using wind turbines with 448MW, ensuring a total of 462MW. Although this only represents 1.6% of the totally produced energy in Romania, many private companies are developing projects to build more wind farms. The research in wind turbine control has established as an objective the maximization of the power produced when the wind speed is in the range between the cut in and the cut out wind speed. This goal is usually achieved by controlling the electromagnetic torque of the generator, in order to obtain the optimal rotor speed for optimum power coefficient [1]. The problem that arises in this framework is the turbine grid integration. Thus, in many cases, it is important to obtain an optimal value, rather than the maximum available amount [2]. In order to reach for this goal, in the past years, different control strategies have been analysed, from classical control ones (using PI) to RST [3], optimal [4] and predictive [5] control ones, for different types of wind turbines.

The turbine considered within this article is an onshore horizontal axis wind turbine (HAWT) with variable rotor speed. This paper introduces two

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discrete controllers: a discrete Linear Quadric Gaussian (LQG) one and a Model Predictive (MPC) one. The objective for these controllers is to keep the output power at its nominal value, in presence of disturbances (wind speed variations). The LQG controller modifies the value of the pitch angle, in order to maintain the turbine output power nearby the constant nominal value (referred to as *rated power*) and rejects the disturbances influence on the system output. The MPC predicts the future state of the plant and computes a new control signal in order to obtain the desired closed-loop performances.

2. General model of wind power

As the wind is the power source for wind turbines, it is important to know the amount of energy available on site. The electrical energy that can be obtained from the wind (without considering the limitations of the physical system) is:

$$E(t) = \frac{1}{2}\rho_{air} A v_w^3 t \quad , \forall t \in \mathbb{R}_+ , \tag{1}$$

where A is the area swept by the wind, ρ_{air} is the air density and v_w is the wind speed. However, the wind turbine is a complex system, consisting of mechanical and electrical components that introduce losses in the energy conversion process. Thus, the power extraction efficiency of a wind turbine is defined in terms of a factor refered to as *power coefficient*, C_P. According to A. Betz [6], the theoretical upper bound of C_P is 0.593. The power produced by a wind turbine is defined as:

$$P_{wt} = \frac{1}{2}\rho_{air} A v_w^3 C_P\left(\lambda, \beta\right) , \tag{2}$$

where λ is the relative speed (the ratio between peripheral speed of blades and the wind speed), while β is the pitch angle of blades.

As the output power is proportional to the cube of the wind speed, it is necessary to determine a wind speed model, in order to integrate it into the process model. The wind speed v_w is generally considered as a non-stationary random process expressed by:

$$v_w = v_s + v_t , \tag{3}$$

where v_s is a low frequency component and v_t is a high frequency turbulent component. The first one refers to the long term variations of wind currents, whereas the second one corresponds to the fast, unexpected changes of wind direction and/or speed. In this paper v_s will be considered as a constant and v_t as a zero-mean Gaussian white noise process.

The wind speed can produce several damages on the turbine, depending on its value and blades orientation against the air flow. Thus, it is important to consider aspects like structural dynamic loads and reliability of the wind speed components such as drive trains, blades and tower, in the controller design process. It is suitable to divide the turbine operation into three different operating regions depending on the value of wind speed [10, 11]. Within this article, the third region is under concern. Here, the wind speed is above the rated value. Therefore, the blades pitch should be controlled such that the optimally captured wind power involves minimal fatigue of the physical components.

3. Wind Turbine Mathematical Model

The plant model used in this paper is one of a variable speed HAWT with two blades. The mathematical model will be built considering the main components with as mnay degrees of freedom as possible, in order to obtain an accurate representation of this system. To avoid the implementation constraints of high order model control algorithms using physical controllers, the following elements will be considered: the first mode of the drive train, the first mode of tower bending dynamics, the first mode of the blades flapping, the two blades as a whole facing same forces acting on them [7].

It is important to define first the two important factors of a wind turbine: the power coefficient $C_P\left(\lambda,\beta\right)$ (see eq. (2)) and the thrust coefficient $C_a(\lambda,\beta)$, both depending on two specific variables of the wind turbine: λ and β. The thrust coefficient $C_a(\lambda,\beta)$, which depends on the thrust force exerted by the wind on the turbine rotor is determined empirically. The definition used in this paper can be found in Reference [10].

The next step consists of modelling the HAWT mechanical equations of its components. The wind turbine is an assembly of interconnected subsystems: aerodynamic, mechanical and electrical [8]. The Lagrange equations are suitable to express the mathematical model:

$$\frac{d}{dt}\left(\frac{\partial E_C}{\partial \dot{\mathbf{q}}_i}\right)-\frac{\partial E_C}{\partial \mathbf{q}_i}+\frac{\partial E_D}{\partial \dot{\mathbf{q}}_i}+\frac{\partial E_P}{\partial \mathbf{q}_i}=\mathbf{Q}, \tag{4}$$

where E_C is the kinetic energy, E_P is the potential energy, E_D is the dissipated energy of the system, $\mathbf{Q}$ is the vector of the generalized forces acting on the system and $\mathbf{q}$ is the vector of generalized coordinates.

The three energies can be expressed as sums of energies specific to the wind turbine components considered for the model,

$$
\begin{aligned}
E_C &= \frac{J_T}{2}\omega_T^{\,2} + \frac{J_G}{2}\omega_G^{\,2} + \frac{M_T}{2}\dot{y}_T^{\,2} + M_P\left(\dot{y}_T + r_P\cdot\dot{\zeta}\right)^2, \\
E_P &= \frac{k_A}{2}\left(\theta_T - \theta_G\right)^2 + k_P\left(r_P\zeta\right)^2 + \frac{k_T}{2}y_T^{\,2}, \\
E_D &= \frac{d_A}{2}\left(\omega_T - \omega_G\right)^2 + d_P\left(r_P\dot{\zeta}\right)^2 + \frac{d_T}{2}\dot{y}_T^{\,2},
\end{aligned} \tag{5}
$$

where J_T, J_G, M_T, M_P, k_A, k_P, k_T, d_A, d_P and d_T are coefficients specific to the wind turbine components as defined in Appendix, ω_G is the generator angular speed and r_P is the distance from the rotor hub to the point on the blade where the generalized thrust force is applied. The values of all parameters can be found in the Appendix as well.

For the wind turbine model, the generalized coordinates vector is:

$$
\mathbf{q} = \left(\theta_T \;\; \theta_G \;\; \zeta_1 \;\; \zeta_2 \;\; y_T\right), \tag{6}
$$

where θ_T is the angular position of the rotor, θ_G represents the angular position of the generator, ζ_1 and ζ_2 are the blades bending angles, while y_T represents the horizontal movement of the tower. The vector Q representing the generalized forces acting on the system is:

$$
\mathbf{Q} = (C_{aero} \;\; -C_{em} \;\; F_{aero,1} \;\; F_{aero,2} \;\; 2F_{aero}), \tag{7}
$$

where C_{aero} is the aerodynamic torque, C_{em} represents is the electromagnetic torque and $F_{aero,1}$, $F_{aero,2}$ stand for the thrust forces acting on the blades.

As previously stated, some simplifying hypotheses were made, in order to decrease the model order. The thrust forces acting on the blades can be considered equal. Consequently, as the blades are similar, it can be assumed that the blades equaly bent under the action of the same thrust forces. The mathematical expressions for the aerodynamic torque and the thrust force can be written as follows:

$$
\begin{aligned}
C_{aero} &= \frac{1}{2}\rho\pi R^2 \frac{v^3}{\omega_T} C_P(\lambda,\beta), \\
F_{aero} &= \frac{1}{2}\rho\pi R^2 v^2 C_a(\lambda,\beta).
\end{aligned} \tag{8}
$$

Another important component of the mathematical model is the wind speed profile. According to [5]:

$$
\dot{v}_w(t) = -\frac{1}{T_v}v_w(t) + v_t(t), \quad \forall t \in \mathbb{R}_+, \tag{9}
$$

where T_v is the time constant, calculated in [8], and v_t is the turbulent component of the wind speed.

The last expression of the mathematical model corresponds to the output power of the wind turbine:

$$P_{el} = \omega_G C_{em} \,. \tag{10}$$

Based on the mathematical models expressed so far, the state space representation of the wind turbine can be obtained. The command vector is represented by $\mathbf{u} = (\beta \ C_{em})$, the state vector is represented by $\mathbf{x}_m{}^T = \left(\theta_T - \theta_G \ \zeta \ y_T \ \omega_T \ \omega_G \ \dot{\zeta} \ \dot{y}_T \ \beta \ v_w\right)^T$ while the system output is $y = P_{el}$. Since the mathematical expressions of C_{aero} and F_{aero} introduce a certain level of nonlinearity to the system, it is necessary to perform a linearization on the model, around some operating point $P_{op}\left(\omega_{T,op}, \beta_{op}, v_{med}\right)$.

The linear state space continuous-time model is then:

$$\begin{aligned} \dot{\mathbf{x}}_m(t) &= \mathbf{F}\mathbf{x}(t) + \mathbf{G_2}\mathbf{u}(t) + \mathbf{G}_1 w_x(t) \\ y(t) &= \mathbf{H}\mathbf{x}_m(t) + \mathbf{M}\mathbf{u}(t) + w_y\left(t\right) \end{aligned} \quad , \forall t \in \mathbb{R}_+ , \tag{11}$$

where v_x and v_y represent the disturbances of the system: v_x is the wind speed variatio. The matrices $\mathbf{F}$, $\mathbf{G}_1$, $\mathbf{G}_2$, $\mathbf{H}$ and $\mathbf{M}$ corresponding to this model were computed with the numerical values from the Appendix.

4. Discrete-time LQG controller design

In this section, the discrete-time LQG controller with integral action is designed for the considered HAWT, in order to minimize the effect of wind variations on the produced electrical power. The Fig. 1 illustrates the structure of such cotroller.

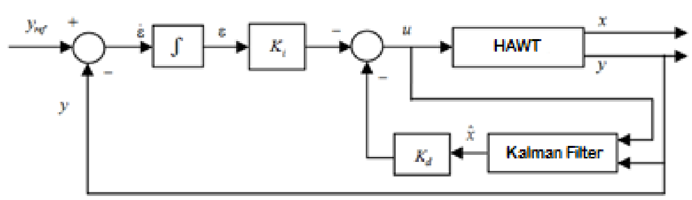

Fig.1. LQG controller with integral action.

The integrator is included in the control loop in order to cancel the tracking error $y_{ref} - y$. The design of LQG controllers with integral action is reduced to the standard LQG design introducing the augmented state vector

$\mathbf{z} = \begin{bmatrix} \mathbf{x}^T & \varepsilon \end{bmatrix}^T$. The standard LQG control problem for (11) consists in finding a control law $\mathbf{u}^*$ that minimizes the quadratic cost function $J(\mathbf{u})$.

A discrete-time turbine model for sample time $T_d = 0.01$ s is first obtained from (11):

$$\begin{cases} \mathbf{x}[n+1] = \bar{\mathbf{F}}\mathbf{x}[n] + \bar{\mathbf{G}}_2\mathbf{u}[n] + \bar{\mathbf{G}}_1 w_x[n] \\ y[n] = \mathbf{H}\mathbf{x}[n] + \mathbf{M}\mathbf{u}[n] + w_y[n] \end{cases}, \forall n \in \mathbb{N}. \quad (12)$$

For the LQG controller design, the augmented model is:

$$\begin{cases} \mathbf{z}[n+1] = \mathbf{A}\mathbf{z}[n] + \mathbf{B}\mathbf{u}[n] + \mathbf{E}v_t[n] + \mathbf{E}v_s[n] \\ y[n] = \mathbf{C}\mathbf{z}[n] + \mathbf{D}\mathbf{u}[n] + w[n] \end{cases}, \forall n \in \mathbb{N}. \quad (13)$$

and the corresponding quadratic cost function can be expressed like below:

$$J(\mathbf{u}[n]) = \lim_{N\to\infty} E\left\{ \frac{1}{N} \sum_0^N (y^T[n]\mathbf{Q}_1[n]y[n] + \mathbf{u}^T[n]\mathbf{R}_1[n]\mathbf{u}[n]) \right\} =$$

$$= \lim_{N\to\infty} E\left\{ \frac{1}{N} \sum_0^N (\mathbf{z}^T[n]\mathbf{Q}[n]\mathbf{z}[n] + \mathbf{u}^T[n]\mathbf{R}[n]\mathbf{u}[n] + 2\mathbf{z}^T[n]\mathbf{S}[n]\mathbf{u}[n]) \right\}, \quad (14)$$

where $\mathbf{z}[n] = \begin{bmatrix} \mathbf{x}^T[n] & \varepsilon[n] \end{bmatrix}^T$, $\varepsilon[n] = y_{ref} - y[n]$, $\mathbf{Q} = \mathbf{C}^T\mathbf{Q}_1\mathbf{C}$, $\mathbf{R} = \mathbf{R}_1 + \mathbf{D}^T\mathbf{Q}_1\mathbf{D}$ and $\mathbf{S} = \mathbf{C}^T\mathbf{Q}_1\mathbf{D}$, while $\mathbf{Q}_1$ and $\mathbf{R}_1$ are positive definite weighting matrices selected by the user.

The discrete-time LQG control law is:

$$\mathbf{u}^*[n] = \mathbf{K}\hat{\mathbf{z}}[n], \text{ with } \hat{\mathbf{z}}[n] = \begin{bmatrix} \hat{\mathbf{x}}[n] & \varepsilon[n] \end{bmatrix}^T, \quad (15)$$

where $\hat{\mathbf{x}}[n]$ is the optimal estimate of $\mathbf{x}[n]$, as given by the Kalman filter:

$$\begin{cases} \hat{\mathbf{x}}[n+1] = \mathbf{A}\hat{\mathbf{x}}[n] + \mathbf{B}\mathbf{u}[n] + \mathbf{K}_f\left(y[n] - \hat{y}[n]\right) \\ \hat{y}[n] = \mathbf{C}\hat{\mathbf{x}}[n] + \mathbf{D}\mathbf{u}[n] \end{cases}, \forall n \in \mathbb{N}. \quad (16)$$

The gain matrix $\mathbf{K} = \begin{bmatrix} -\mathbf{K_d} & \mathbf{0} \\ 0 & K_i \end{bmatrix}$ in (15) is computed as:

$$\mathbf{K} = \left(\mathbf{R} + \mathbf{B}^T\mathbf{P}\mathbf{B}\right)^{-1}\left(\mathbf{B}^T\mathbf{P}\mathbf{A} + \mathbf{S}^T\right), \quad (17)$$

where $\mathbf{P}$ is the non negative definite solution of a discrete matrix Riccati equation. The gain matrix $\mathbf{K}_f$ of the Kalman filter (16) is determined as:

$$\mathbf{K}_f = \mathbf{A}\mathbf{P}_f\mathbf{C}^T\left(\mathbf{W} + \mathbf{C}\mathbf{P}_f\mathbf{C}^T\right)^{-1}, \quad (18)$$

where $\mathbf{P}_f$ is the non negative definite solution of another Riccati equation.

5. Discrete MPC design

The general design objective of model predictive control is to compute a trajectory of a future manipulated variable $\mathbf{u}$ to optimize the future behavior of the plant output y. The optimization is performed within a limited time window by giving plant information at the begining of that window [9].

The model described in (11) must be transformed from continuous-time into a discrete-time one. The sample time chosen for this controller is $T_d = 0.02$ s. Using the discrete model, the matrices corresponding to the augmented model are computed:

$$\begin{aligned} \mathbf{x}[n+1] &= \mathbf{A}_e\mathbf{x}[n] + \mathbf{B}_e\Delta\mathbf{u}[n] + \mathbf{B}_\varepsilon\varepsilon[n] \\ y[n] &= \mathbf{C}_e\mathbf{x}[n] \end{aligned}, \forall n \in \mathbb{N}, \quad (19)$$

where $\mathbf{x}[n] = \left[\Delta\mathbf{x}_m[n] \quad y[n]\right]^T$, $\Delta\mathbf{u}[n] = \mathbf{u}[n] - \mathbf{u}[n-1]$ and ε is the input disturbance corresponding to the wind speed variation, assumed to be a sequence of integrated white noise. This means that the input disturbance w is related to a zero mean, white noise sequence ε by the difference equation $w[n] - w[n-1] = \varepsilon[n]$, $\forall n \in \mathbb{N}$.

The system signals will be referred as m - number of inputs ($m = 2$), n_1 - the number of states ($n_1 = 9$) and q - the number of outputs ($q = 1$). Then the dimension of the augmented state-space equation is $n = n_1 + q\left(=10\right)$.

The strategy of MPC design implies that the plant output has to be predicted with the future control signals as the adjustable variables, within an optimization window length N_P. In order to proceed, the future control trajectory is denoted by: $\Delta\mathbf{u}[n], \Delta\mathbf{u}[n+1], ..., \Delta\mathbf{u}[n+N_C-1]$, where N_C is the control horizon length. This parameter sets the number of samples used to capture the future control trajectory. Considering that the state vector $\mathbf{x}[n]$ is known, the future state variables - $\mathbf{x}[n+1|n], \mathbf{x}[n+2|n],..., \mathbf{x}[n+N_P|n]$ - are predicted for N_P number of samples, where N_P referred to as prediction horizon length. In the future state variables representation, the term $\mathbf{x}[n+i|n]$ stands for the predicted state variable at $n+i$, with given current plant information $\mathbf{x}[n]$. The control horizon length N_C is chosen to be less than (or equal to) the prediction horizon length N_P. The following vectors can then naturally be defined as:

$$\begin{aligned} \Delta\mathbf{U} &= \left[\Delta\mathbf{u}[n]^T \; \Delta\mathbf{u}[n+1]^T \; ... \; \Delta\mathbf{u}[n+N_C-1]^T\right]^T \\ \mathbf{Y} &= \left[y[n+1|n] \; \mathrm{y}[\mathrm{n}+2|n] \; ... \; \mathrm{y}[n+N_P|n]\right]^T \end{aligned}, \forall n \in \mathbb{N}. \quad (20)$$

Based on the state-space model $\left(\mathbf{A}_e,\mathbf{B}_e,\mathbf{C}_e,\mathbf{B}_\varepsilon\right)$, the future state variables are recursively computed using the set of future control parameters:

$$\begin{aligned}\mathbf{x}[n+N_P\,|\,n] = {} & \mathbf{A}^{N_P}\mathbf{x}[n] + \mathbf{A}^{N_P-1}\mathbf{B}\Delta\mathbf{u}[n] + \mathbf{A}^{N_P-2}\mathbf{B}\Delta\mathbf{u}[n+1] \\ & + \mathbf{A}^{N_P-N_C}\mathbf{B}\Delta\mathbf{u}[n+N_C-1] + \mathbf{A}^{N_P-1}\mathbf{B}_\varepsilon\varepsilon[n] \\ & + \mathbf{A}^{N_P-2}\mathbf{B}_\varepsilon\varepsilon[n+1\,|\,n] + \ldots + \mathbf{B}_\varepsilon\varepsilon[n+N_P-1\,|\,n]. \end{aligned} \quad (21)$$

With the assumption that ε is a zero-mean white noise sequence, the predicted value of $\varepsilon(n+i\,|\,n)$ at future sample i is assumed to be null. The predicted state and output variables are calculated as this mean values. Hence, the noise effect to the predicted values is minimal.

Effectively, one obtaines:

$$\mathbf{Y} = \mathbf{F}\mathbf{x} + \mathbf{L}\Delta\mathbf{U}, \quad (22)$$

where:

$$\mathbf{F} = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \mathbf{CA}^3 \\ \vdots \\ \mathbf{CA}^{N_P} \end{bmatrix}; \mathbf{L} = \begin{bmatrix} \mathbf{CB} & 0 & \cdots & 0 \\ \mathbf{CAB} & \mathbf{CB} & \cdots & 0 \\ \mathbf{CA}^2\mathbf{B} & \mathbf{CAB} & \cdots & 0 \\ \vdots & & & \\ \mathbf{CA}^{N_P-1}\mathbf{B} & \mathbf{CA}^{N_P-2}\mathbf{B} & \cdots & \mathbf{CA}^{N_P-N_C}\mathbf{B} \end{bmatrix}. \quad (23)$$

Let $\mathbf{r}$ be a given set-point signal within a prediction horizon. Then, the objective of the predictive control system is to bring the predicted output as close as possible to the set-point signal (which is kept constant in the optimization window). This objective involves solving the problem to find the optimal control parameter vector $\Delta\mathbf{U}$, such that an error function between the set-point and the predicted output is minimized.

Assuming that the data vector that contains the set-point information is: $\mathbf{R}_S^T[n] = \overbrace{\begin{bmatrix}1 & 1 & \ldots & 1\end{bmatrix}}^{N_P}\mathbf{r}[n]$, for each sample time n, the cost function J that reflects the control objective is:

$$J = \left(\mathbf{R}_s - \mathbf{Y}\right)^T\left(\mathbf{R}_s - \mathbf{Y}\right) + \Delta\mathbf{U}^T\bar{\mathbf{R}}\Delta\mathbf{U}. \quad (24)$$

The first term in (24) is linked to the objective of minimizing the errors between the predicted output and the set-point signal, while the second term purpose is to limit the variation of $\Delta\mathbf{U}$, when minimizing J, in order to avoid operation shocks applied to the plant. Also, $\bar{\mathbf{R}}$ is a diagonal matrix such as $\bar{\mathbf{R}} = r_\omega\mathbf{I}_{N_C\times N_C}$ $\left(r_\omega \geq 0\right)$, where r_ω is used as a tuning parameter for the desired closed-loop performance.

The incremental optimal control within the optimization window is linked to the current set-point signal $\mathbf{r}$ and state variable $\mathbf{x}$, via the following equation:

$$\Delta\mathbf{U} = \left(\mathbf{L}^{\mathrm{T}}\mathbf{L} + \bar{\mathbf{R}}\right)^{-1}\left(\mathbf{L}^{\mathrm{T}}\bar{\mathbf{R}}_s\mathbf{r} - \mathbf{L}^{\mathrm{T}}\mathbf{F}\mathbf{x}\right). \quad (25)$$

Applying the receding horizon control principle, the first m elements of $\Delta\mathbf{U}$ are considered to form the incremental optimal control:

$$\begin{aligned}\Delta\mathbf{u}[n] &= \overbrace{\left[\mathbf{I}_m \quad \ldots \quad \mathbf{0}_m\right]}^{N_C}\left(\mathbf{L}^{\mathrm{T}}[n]\mathbf{L}[n] + \bar{\mathbf{R}}\right)^{-1} \times \left(\mathbf{L}^{\mathrm{T}}[n]\bar{\mathbf{R}}_s\mathbf{r}[n] - \mathbf{L}^{\mathrm{T}}[n]\mathbf{F}\mathbf{x}[n]\right) \\ &= \mathbf{K}_y[n]\mathbf{r}[n] - \mathbf{K}_{mpc}[n]\mathbf{x}[n]\end{aligned}. \quad (26)$$

The mesurable state of the process, will be estimated using the following observer:

$$\hat{\mathbf{x}}_m[n+1] = \overbrace{\mathbf{A}_m\hat{\mathbf{x}}_m[n] + \mathbf{B}_m\mathbf{u}[n]}^{\text{model}} + \overbrace{\mathbf{K}_{ob}\left(y[n] - \mathbf{C}_m\hat{\mathbf{x}}_m[n]\right)}^{\text{correction term}}, \quad (27)$$

where $\mathbf{K}_{ob}$ is the observer gain matrix, while $\mathbf{A}_m$ and $\mathbf{B}_m$ correspond to the plant model. Note that $\mathbf{K}_{ob}$ was computed using the Matlab `place` function.

6. Simulation results

The simulation environment used for performance analysis of the designed controllers is MATLAB/ SIMULINK.

The turbine model (11) was implemented for the operating point $P_{op} = (8 \text{ rad/s}, 1°, 17 \text{ m/s})$. The presented simulation results were obtained for the wind speed profile given in Fig. 2.

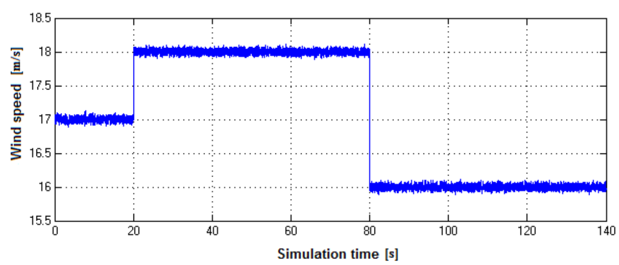

Fig. 2. Wind speed profile.

As one can observe, the wind speed has a deviation from the initial value, 17 m/s, at instant $t_1 = 20 \text{ s}$, when it becomes 18 m/s and even a bigger one at instant $t_2 = 80 \text{ s}$, when it becomes 16 m/s.

6.1 Simulation results for the wind turbine controlled by the discrete-time LQG controller with integral action

The simulation results of the controller described on Chapter 4 are described in this paragraf.

The two control signals: pitch angle β and generator electomagnetic torque C_{em} provided by the LQG controller are shown in Fig. 3.

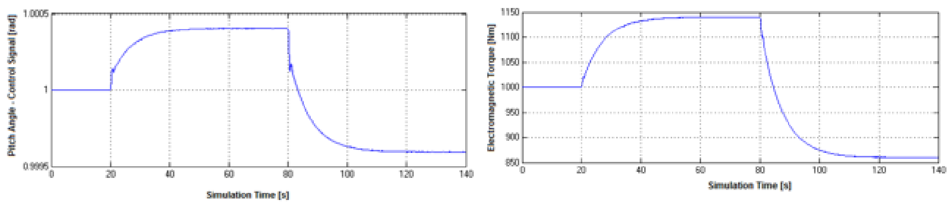

Fig. 3. Control signals: pitch angle and generator electromagnetic torque.

At instants $t_1 = 20$ s and $t_2 = 80$ s, small deviation from the initial values can be observed in both command signals, corresponding to wind speed variation. The average stabilization time is 30 s, although this is not a fast reaction, the deviation is very small and it does not affect the HAWT components. In turn, Fig. 4 illustrates the output power of the wind turbine, obtained when using the designed discrete-time LQG controller.

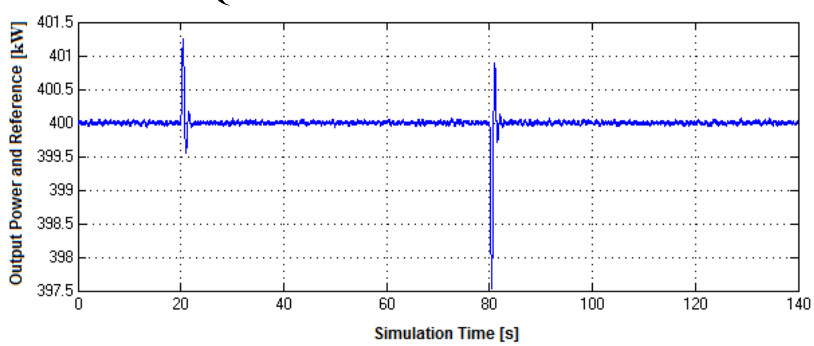

Fig. 4. Electrical power produced by the wind turbine

The nominal power of the considered HAWT is 400 kW. One can observe that, at instants t_1 and t_2, the output power has two spikes produced by the variation of wind speed. At t_1, the deviation is of 1.25 kW and, at t_2 the deviation reises to 2.4 kW. The controller manages to reject the disturbances in 2 s at t_1 and in 3 s at t_2.

6.2 Simulation results for the wind turbine controlled by the Discrete-time MPC controller

The the control signals provided by the MPC strategy described in Chapter 5 are illustrated in Fig. 5. At instant $t = 4\ s$, a small variation from the initial values can be observed in both command signals, corresponding to wind speed variation. The average stabilization time is 0.5 s, wich is quite a fast reaction. Since the variation is small enough, it does not affect the HAWT components.

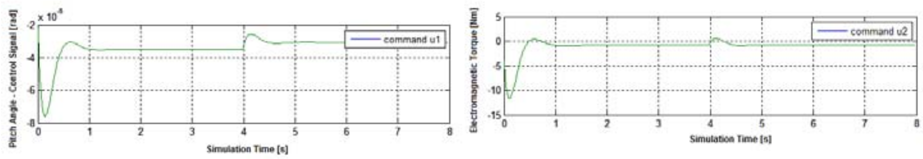

Fig. 5. Control signals: pitch angle variation (left) and generator electromagnetic torque variation (right).

In turn, Fig. 6 illustrates the output power of the wind turbine obtained using the designed discrete-time MPC controller.

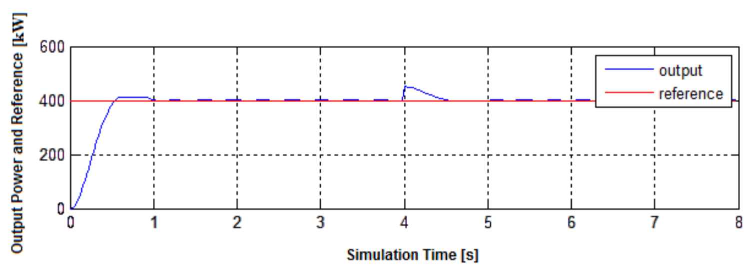

Fig. 6. Electrical power produced by the wind turbine with MPC control.

At instant $t_1 = 4$ s, while facing the input disturbance corresponding to the wind speed variation, the output has quite a small variation, but the predictive control is effective at instant $t_2 = 4.5$ s, as the output signal is stabilized at its nominal value.

7. Conclusions

For this study, the discrete-time approach has been chosen, in order to provide two control strategies that can easily be implemented in digital control systems especially in case of HAWTs. The LQG approach presented has showed very good results in disturbance (the wind speed variation) rejection. Regarding the MPC strategy, one can observe that, based on the current plant information represented by the state variable vector, the prediction of the future behavior of the plant output relies on the state-space model where the optimal control trajectory is captured by the set of parameters that define the incremental control movement. The MPC approach presented has showed sufficiently good results for the output power of the wind turbine. Nevertheless, improvements are posible, by cosidering more performant predictors (e.g. of ARMAX class) and a finer tunning for r_ω parameter in definition (24). Future developments can include the implementation of the two controllers on a digital processor or on a microcontroller. These control strategies can be easily implemented on a real wind turbine with the proper configuration of the parameters according to the physical characteristics of the turbine.

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APPENDIX

Table 1

Significance and numerical values of the wind turbine parameters

Symbol.	Physical meaning	Value	Symbol.	Physical meaning	Value
J_T	Turbine inertia	$214\,000\ \mathrm{kg}\times\mathrm{m}^2$	d_T	Tower Damping coefficient	$50\,000\ \mathrm{kg}\times\mathrm{m/s}$
J_G	Generator inertia	$41\ \mathrm{kg}\times\mathrm{m}^2$	d_A	Drive shaft damping coefficient	$60\,000\ \mathrm{kg}\times\mathrm{m}^2/s$
M_T	Tower and nacelle mass	35000 kg	r_P	Distance from the rotor hub	8 m
M_P	Blade mass	3000 kg	N	Number of blades	2
k_P	Blade Stiffness coefficient.	$1000\ \mathrm{kg}\times\mathrm{m}^2/\mathrm{s}^2$	$\omega_{T,nom}$	Nominal rotor speed	$4\ \mathrm{rad/s}$
k_T	Tower Stiffness coefficient.	$8500\ \mathrm{kg}\times\mathrm{m/s}^2$	$\omega_{T,op}$	Rotor speed – operational point	$8\ \mathrm{rad/s}$
k_A	Drive Shaft Stiffness coefficient.	$11000\ \mathrm{kg}\times\mathrm{m}^2/\mathrm{s}^2$	β_{op}	Pitch angle - operational point	9 rad
d_P	Blade Damping coefficient.	$10000\ \mathrm{kg}\times\mathrm{m}^2/\mathrm{s}$	λ_{op}	Speed trip ratio – operational point	8