PERFORMANCE ANALYSIS OF LARGE SCALED BEAMFORMING MU-MIMO SYSTEM

Ammar Ali SAHRAB[1], Ion MARGHESCU[2]

Large scaled Multi-User Multi-Input-Multi-Output (MU-MIMO) wireless communication systems referring to a Base Station (BS) equipped with very large number of antennas (users) gain very high performance consideration due to high data rates, spectral efficiency and reliability that they can achieve. In multi-user wireless communication system with independent and identically distributed (i.i.d.) Complex Gaussian (Rayleigh Fading) Channels, multiple antennas allow independent users equipment (UE) to transmit their own data stream in the uplink at the same time or the BS to transmit the multiple user data streams to be decoded by each UE in the downlink. In this paper Bit-Error-Rate (BER) performance for large scaled beamforming MU-MIMO system with different types of precoding and modulation schemes is evaluated as a function of Signal-to-Noise Ratio (SNR). Two methods for designing beamforming and channel precoding selection to get a better SNR performance are considered.

Keywords: Large Scaled MU-MIMO; Multiuser; MIMO; Beamforming

1. Introduction

Multiple Input Multiple Output (MIMO) technology in wireless systems has been widely studied during the last years and applied to many wireless standards due to the fact that it improves channel capacity and reliability [6]. In the single-user MIMO (SU-MIMO) system, a point-to-point link high data rate transmission can be supported by spatial multiplexing while providing spatial diversity gain. Also, in SU-MIMO systems the multiplexing gain disappear when the signal power is low, relative to interference and noise, or in propagation environments with dominating line-of-sight or low number of scatters. Practical size limitations on terminals also limit the number of antennas that could be used and thereby the achievable multiplexing gain [13]. However, most communication systems deal with multiple users who are sharing the same radio resources in which multiple mobile stations are served by a single base station. The use of large-scale MIMO technology is being considered as a potential technology for the fifth generation (5G) wireless systems due to very high spectral

[1] Assist. Lect., Dept. of Electrical Engineering, University of Mustansiriyah, Baghdad, Iraq; PhD Student, e-mail: sohrab722007@gmail.com

[2] Prof., Dept. of Electronic Telecommunication and Information Technology, University POLITEHNICA of Bucharest, Romania, e-mail: ion.marghescu@gmail.com

efficiency/sum rates, increased reliability, and power efficiency [8]. MU-MIMO wireless system has gained considerable amount of interest since it can significantly increase data throughput and achieve higher diversity gain. Considering a multiple-stream-per-user MU-MIMO system, a Base Station (BS) communicates with multiple users in the same frequency and time slots, and each user receives multi-stream [4]. Fig. 1 illustrates a typical multi-user MIMO (MU-MIMO) communication environment in which multiple Mobile Stations (MS) or users each with multiple antennas are served by a single BS with multiple antennas in the communication system. Due to multi-user diversity, the performance of MU-MIMO systems is generally less sensitive to the propagation environment than in the point-to-point MIMO case [6]. To achieve high system capacity, energy and spectral efficiency, User Equipment (UE) is communicating over orthogonal channels. The interference cancellation can be achieved by using strong downlink precoding techniques. Thus, the suppression of interference is crucial in designing the transmit beamforming. The uplink benefits could be inspected through UEs transmitter detection algorithms like Zero Forcing (ZF) and Minimum Mean Square Error (MMSE). The downlink benefits (signal transmission in massive MIMO) is analyzed and the results are compared for Dirty Paper Coding (DPC), Block Diagonalization (BD) and Tomlinson-Harashima Precoding (THP) [7].

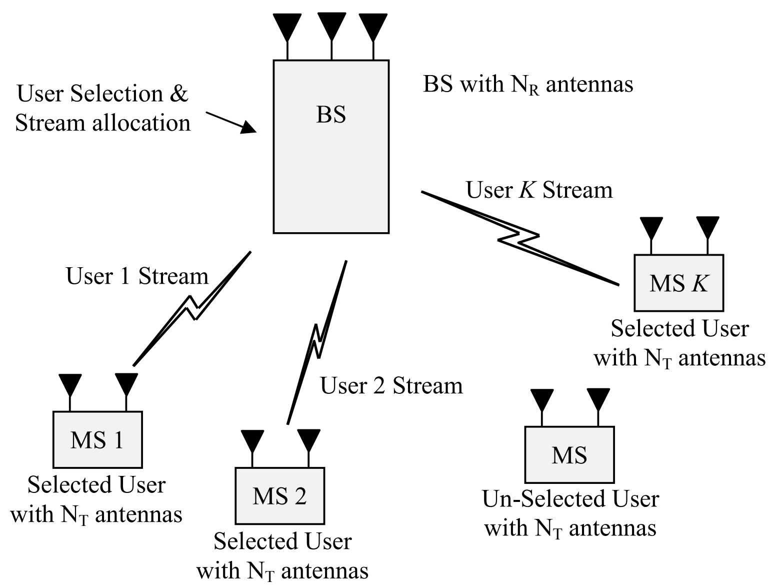

Fig. 1. Multi-User MIMO communication systems: K=4.

In Fig. 1, K-1 out of K users are selected and allocated communication resource such as time, frequency, and spatial stream. Suppose that the BS and each MS are equipped with N_R and N_T antennas, respectively. As K independent users form a virtual set of $K \cdot N_T$ antennas which communicate with a single BS with N_R antennas, the end-to-end configuration can be considered as a $(K \cdot N_T) \times N_R$ MIMO system for downlink, or $N_R \times (K \cdot N_T)$ MIMO system for uplink. In this multi-user communication system, multiple antennas allow the independent users to transmit their own data stream in the uplink (many-to-one) at the same time using ZF and MMSE algorithms or the BS to transmit the multiple user data streams to be decoded by each user in the downlink (one-to-many) using DPC, BD and THP. This is attributed to the increase in degrees of freedom with multiple antennas compared with the single-user MIMO system. In the multi-user MIMO system, downlink and uplink channels are referred to as Broadcast Channel (BC) and Multiple Access Channel (MAC), respectively.

In [6], massive MIMO systems from several different perspectives were described. By equipping a BS with a large number of antennas, spectral and energy efficiency can be improved. To make the benefits of massive MIMO a reality, significant additional research is needed on a number of issues, including channel correlation, hardware implementations and impairments, interference management, and modulation.

In [7], large scaled antenna systems have capabilities to fulfill the future needs of the wireless communication systems and also promise to give throughput even more than that of 4G systems. The author gave an idea to use massive MIMO systems for higher throughput in the communication systems. Massive MIMO systems with multiple antennas of hundred positioned at the BS improves the spectral efficiency largely. An efficient deployment scheme for massive MIMO is the concentrated antenna design. Moreover, the DPC technique showed better performance when compared to BD and THP.

In [8], low complexity detection algorithms were proposed for large-scale Spatial Modulation MIMO (SM-MIMO) systems. An interesting observation from the simulation results is that SM-MIMO outperforms massive MIMO by several dBs for the same spectral efficiency. Similar performance gains were shown in favor of SM-MIMO on frequency selective fading. The SNR advantage of SM-MIMO over massive MIMO is attributed to the following reasons: (i) because of the spatial index bits, SM-MIMO can use a lower-order Quadrature Amplitude Modulation (QAM) alphabet compared to that in massive MIMO to achieve the same spectral efficiency, and (ii) for the same spectral efficiency and QAM size, massive MIMO will need more spatial streams per user which leads to increased spatial interference.

In [9], The pilot contamination problem in multicell massive MIMO systems under correlated channels was studied, and it is simulated by the

MATLAB software. Simulation results show that the performance of the massive MIMO systems becomes worse when the channels are correlated. When the channel correlation coefficient is large enough, the performance of the massive MIMO systems is too bad to work. And when the channel correlation coefficient is smaller than a certain value, the performance of the massive MIMO systems will be enhanced effectively with the increase of the number of BS antennas; while the channel correlation coefficient is larger than a certain value, the performance of the massive MIMO systems is enhanced slowly by increasing the number of BS antennas.

In [13], linear pre-coding performance for very-large MIMO downlink channels is studied. It is found that the user channels, in the studied residential-area propagation environment, can be decorrelated by using reasonably large antenna arrays at the base station. With linear pre-coding, sum rates as high as 98% of DPC capacity for two single-antenna users and 20 antennas BS were achieved. This have shown that even in realistic propagation environments and with a relatively limited number of antennas, benefits of an large number of antennas used at BS could be noticed.

In [14], It is seen that for a limited feedback MU-MIMO system, residual multi-user interference is created even in ZF beamforming (ZFBF) transmission thereby causing performance loss. When each user is unaware of other users channels, optimizing performance by accurate Channel Quality Index (CQI) estimation, receive beamformer and channel codeword selection is quite challenging. Simulated results have shown that the performance improvement can be obtained by jointly designing the receive beamformer and channel codeword to optimize the SINR/CQI. In addition, the BS can further improve performance by adjusting each user's CQI based on the feedback channel of the actual co-scheduled users. The system level results have shown that, even with limited feedback in practical MU-MIMO systems, multi-user MIMO gains can be improved.

In [15], having in mind that standard THP algorithm for downlink MU-MIMO does not provide the flexibility for user power allocation, a modified THP algorithm which accepts arbitrary effective linear precoder was proposed. The authors studied the special case of zero-forcing THP (ZF-THP) with arbitrary user power allocation and derived optimal user power allocation to maximize the weighted sum-rate of users.

In our paper, linear precoding techniques performance considering two modulation schemes (QPSK and 16 QAM) for large-scaled beamforming MU-MIMO systems are studied by simulation. We consider a single-cell environment in which a BS with a large-scaled antenna array serves a number of multi-antenna users simultaneously.

2. System Model

A. Single User MIMO (SU-MIMO)

Consider a SU-MIMO (point-to-point) transmission model equipped with N_T transmit antennas and N_R receive antennas. The system model for communication from one device to another is expressed as follows [1] [2]:

$$y = \sqrt{\rho}\mathbf{H}x + n \tag{1}$$

where $Y_k \in C^{N_R \times 1}$ is the received signal vector, $X_k \in C^{N_T \times 1}$ is the transmit signal vector, $\mathbf{H}_k \in C^{N_R \times N_T}$ is constant channel matrix, $n_k \in C^{N_R \times 1}$ represent independent identical distribution (i.i.d.) zero mean cyclic symmetric complex Gaussian random noise and interference vector with covariance matrix $\sigma^2 I$, and the scalar ρ is the transmit power.

The instantaneous achievable rate (capacity) can be expressed as:

$$C = \log_2 \det\left(I + \frac{\rho}{N_T}\mathbf{H}^H\mathbf{H} \right) \tag{2}$$

The actual achievable rate depends on the distribution of the singular values of $\mathbf{H}^H\mathbf{H}$.

B. Multi User MIMO (MU-MIMO)

Consider a MU-MIMO transmission model with N_T transmit antennas at each UE, N_R receive antennas at BS and N_{UE} number of user equipment as shown in Fig. 1.

B1. Uplink: The system model for transmission from one BS to N_{UE} user is expressed as follows [6]:

$$y_u = \sqrt{\rho_u}\mathbf{H}_u x_u + n_u \tag{3}$$

where $y_u \in C^{N_R \times 1}$ is the received signal vector, **u**=1,2,..., *K*, $x_u \in C^{N_T \times 1}$ is the transmit signal vector at the **u**-th user, $\mathbf{H}_u \in C^{N_R \times N_T}$ is the MIMO uplink channel matrix gain between the **u**-th user MS and BS, $n_u \in C^{N_R \times 1}$ is independent identical distribution (i.i.d.) zero mean cyclic symmetric complex Gaussian random noise vector with covariance matrix $\sigma^2 I$. the scalar ρ_u is the uplink transmit power.

The symbol transmit signal energy from the k-th user is set to be $\mathbf{E}\{|x_k|^2\}=1$ for $\mathbf{x}_u=[x_1,...,x_k]^T$. Without loss of generality, it is assumed that x_k, coefficients in $\mathbf{n}_u$ and in $\mathbf{H}_u$ are mutually independent and zero-mean. Additionally, coefficients in $\mathbf{H}_u$ represent independent and identically Rayleigh fading channels with normalized channel energy.

The overall achievable rate (capacity) of all users can be expressed as:

$$C = \log_2 \det\left(I + \rho_u \mathbf{H}^H \mathbf{H}\right) \tag{4}$$

B2. Downlink: Downlink channel, known as a Broadcast Channel (BC), where the system model for transmission from N_{UE} user to one BS is expressed as follows:

$$y_d = \sqrt{\rho_d}\,\mathbf{H}_d^T x_d + n_d \tag{5}$$

where $y_d \in C^{N_T \times 1}$ is the received signal vector at all users, $x_d \in C^{N_R \times 1}$ is the transmit signal vector by the BS; the scalar $\rho_\mathbf{d}$ is the downlink transmit power; $\mathbf{H}_d = \mathbf{H}_u$ and $n_d \in C^{N_T \times 1}$ is additive noise defined before. To normalize transmit signal energy one assumes $\mathbf{E}\{|x_\mathbf{d}|^2\}=1$.

The BS performs power allocation to all users to maximize the sum transmission rate, and the system capacity can be expressed as:

$$C = \max_p \log_2 \det\left(I + \rho_d \mathbf{H}^H \mathbf{H}\right) \tag{6}$$

where p is a positive diagonal matrix with the power allocations $(p_1,...,p_{N_T})$ as its diagonal elements and $\sum_{k=1}^{N_T} p_\mathbf{k}=1$.

3. Linear Precoding

A. ZF receiver

ZF receiver in MIMO system is important from a practical viewpoint because of its ability to significantly reduce complexity, with tolerable performance degradation. ZF precoding method eliminates the interference by transmitting the signals towards the proposed user with nulls in the direction of other users. The ZF precoder is given as [7]:

$$W_{ZF} = \mathbf{H}^H \left(\mathbf{H}^H \mathbf{H} \right)^{-1} \tag{7}$$

where: $\mathbf{H}^H \left(\mathbf{H}^H \mathbf{H} \right)^{-1}$ is the pseudo inverse of the channel matrix.

B. MMSE Receiver

The receiver based on MMSE criterion is well-known for single-user MIMO with spatial multiplexing transmission scheme. The MMSE receive matrix is the one that minimizes the MSE (mean square error) between the recovered signal and the desired signal. MMSE precoding can suppress mutual interference against signal power efficiency. The mathematical expression MMSE precoder is given by [7]:

$$W_{MMSE} = \mathbf{H}^H \left(\alpha I + \mathbf{H}^H \mathbf{H} \right)^{-1} \tag{8}$$

where the new parameter ‘α’ can be given as $\alpha = N_T/\rho$. If we keep small value of α, then MMSE precoder will reach the ZF precoder.

4. Non-Linear Precoding

A. Block Diagonalization (BD)

In the block diagonalization (BD) method, the interference from other user signals is canceled in the process of precoding. Then, the inter-antenna interference for each user can be canceled by various signal detection methods.

For the *u*-th user signal the received signal is given as [2][5]:

$$y_u = \mathbf{H}_u^{DL} W_u \tilde{x}_u + n_u \tag{9}$$

where $\mathbf{H}_u^{DL}$ is the uplink channel matrix defined before, $W_u \in C^{N_T \times N_R}$ is the precoding matrix for the *u*-th user, and $\tilde{x}_u \in C^{N_R \times 1}$ is the precoded transmitted signal, and $\{ \mathbf{H}_u^{DL} W_k \}$ form an effective channel matrix for the *u*-th-user receiver and the *k*th-user transmit signal. In other words, the interference-free transmission will be warranted as long as the effective channel matrix can be block-diagonalized, that is,

$$\mathbf{H}_u^{DL} W_k = 0_{N_T \times N_R}, \forall u \neq k \tag{10}$$

where $0_{N_{M,u} \times N_{M,u}}$ is a zero matrix. Then the received signal of the *u*-th user in term of transmitted signal is given as:

$$y_u = \mathbf{H}_u^{DL} x_u + n_u \tag{11}$$

B. Dirty Paper Coding (DPC)

The dirty paper coding (DPC) is based on the idea that an interference-free transmission can be realized by subtracting the potential interferences before transmission. In theory, DPC would be implemented when channel gains are completely known on the transmitter side. DPC is a method of precoding the data such that the effect of the interference can be canceled subject to some interference that is known to the transmitter. More specifically, the interferences due to the first up to (k–1)th user signals are canceled in the course of precoding the *k*-th user signal [2][5].

C. Tomlinson-Harashima Precoding (THP)

DPC on the transmitter side is very similar to Decision Feedback Equalization (DFE) on the receiver side. In fact, combination of DPC with symmetric modulo operation turns out to be equivalent to Tomlinson-Harashima Precoding (THP). THP was originally invented for reducing the peak or average power in the DFE, which suffers from error propagation. The original idea of THP in DFE is to cancel the post-cursor Inter-Symbol-Interference (ISI) in the transmitter, where the past transmit symbols are known without possibility of errors. In fact, it requires a complete knowledge of the channel impulse response, which is only available by a feedback from the receiver for time-invariant or slowly time-varying channel [2][5].

5. Spectral Efficiency

Spectral efficiency (SE) refers to the information rate that can be transmitted over a given bandwidth in a specific communication system. It is a measure of how efficiently a limited frequency spectrum is utilized by the physical (and sometimes MAC) layer protocol. The SE of a digital communication system is measured in bit/s/Hz. It is the net bit rate (useful information rate excluding error-correcting codes) or maximum throughput divided by the bandwidth in hertz of a communication channel or a data link. Alternatively, the SE may be measured in bit/symbol, which is equivalent to bits per channel use (bpcu), implying that the net bit rate is divided by the symbol rate (modulation rate) or line code pulse rate. SE is typically used to analyze the efficiency of a digital modulation method or line code, sometimes in combination with a forward error correction (FEC) code and other physical layer overhead.

The MIMO channel capacity can be given by its time average. In practice, we assume that the random channel is an ergodic process. Then, we should consider the following statistical notion of the MIMO channel capacity:

$$\overline{C} = E\{C(\mathbf{H})\}E\left\{\max_{Tr(\mathbf{R}_{xx})=N_T} \log_2 \det\left(I_{N_R} + \frac{E_x}{N_T N_0}\mathbf{H}R_{xx}\mathbf{H}^H\right)\right\} \quad (12)$$

which is frequently known as an ergodic channel capacity. A simple MATLAB program is used to simulate the ergodic channel capacity concerning of ergodic theory and the behavior of a dynamical system when it is allowed to run for a long time.

6. Simulation Results

We evaluated the performance of different types of receivers mentioned in the previous sections using a MATLAB program. The parameters of the program are shown in table 1.

Table 1

MATLAB Program Simulation Parameters

Parameters	Specification
Number of transmitted bits	80000
Frame length	200
Number of packets	1000
Modulation type	QPSK & QAM16
Noise	AWGN
Channel	Rayleigh and Rician Fading
Channel precoding	BD, DPC & THP
Detection techniques	ZF & MMSE

The results refer to the large scaled beamforming MU-MIMO system behavior in a flat-fading Rayleigh channel, under different precoding techniques: BD, DPC, THP and with ZF and MMSE detection techniques. The results can be observed in the graphs shown in Fig's. 2 and 3. Another results can that can be observed in the graphs given in Fig. 4 refer to the large scaled beamforming MU-

MIMO system channel capacity for different number of transmit and receive antennas. The channel undergoes independent fading between the multiple transmit-receive antenna pairs.

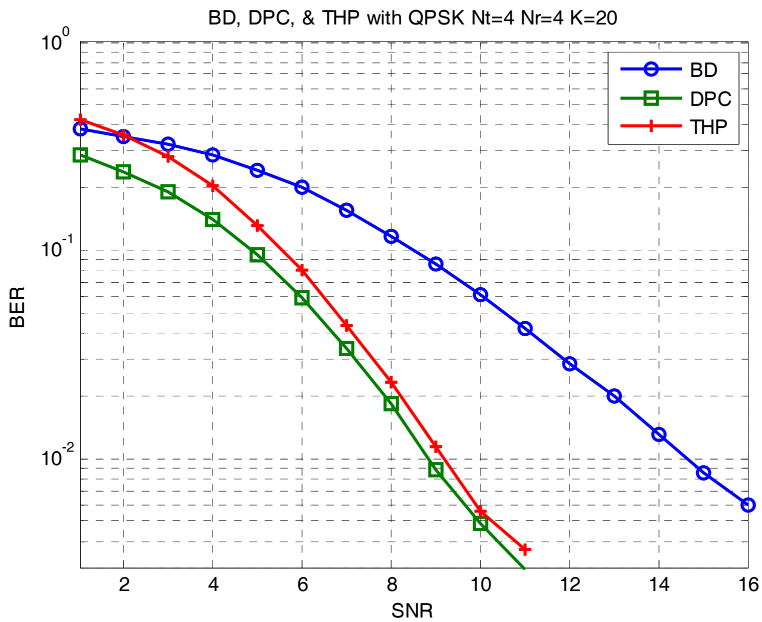

Fig. 2. Performance analysis of beamforming MU-MIMO QPSK combined with BD, DPC and THP precoders and MMSE detection.

In Fig. 2, one can see that using QPSK modulation the DPC system has a better performance than that of THP and BD systems. The simulation results obtained in the case of using $N_T = N_R = 4$, in which four users with the highest channel norm values are selected out of K = 20, give a comparison between BD, DPC and THP precoders. At 10^{-1} BER one can observe that DPC has a 0.5 dB advantage when compared to THP and 4 dB advantage when compared to BD. This is because DPC remove the potential interference before transmission and obtains high data rate with an interference-free reception, while BD and THP cancel the interferences at the receiver side.

In Fig. 3, one can see that using QAM16 modulation the DPC system has a better performance than that of THP and BD systems. The simulation results obtained in the case of using $N_T = N_R = 4$, in which four users with the highest channel norm values are selected out of K = 20, give a comparison between BD, DPC and THP precoders. At 10^{-1} BER one can observe that DPC has a 0.25 dB advantage when compared to THP and 4 dB advantage when compared to BD. This can be explained using the same arguments as in the previous case.

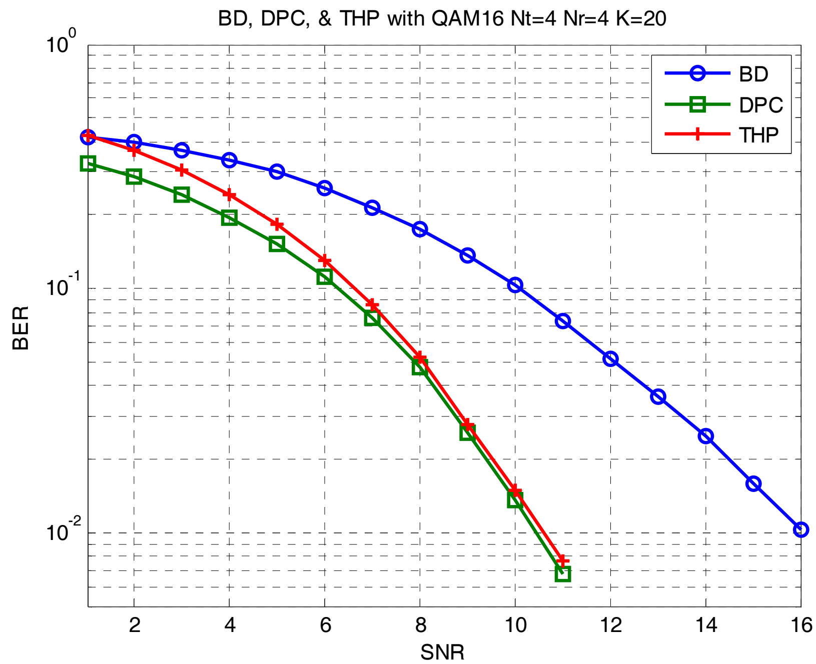

Fig. 3. Performance analysis of beamforming MU-MIMO QAM16 combined with BD, DPC and THP precoders and MMSE detection.

Making a comparison between Fig. 2 and Fig. 3 at 10^{-1} BER one can observe that, as an example, DPC has a 5 dB SNR for QPSK modulation and 6.3 dB SNR for QAM16 modulation. A similar analysis for DPC is presented in [7] considering ZF receiver, QPSK modulation, no path loss in the channel, N_T=300 and N_R=20. For BER=10^{-1}, SNR=3dB was obtained. The advantage of QPSK modulation is due to the fact that QAM16 modulation scheme is considerably less resilient to noise and interference and, as it is well known, one can achieve high reliability with QPSK modulation and high data rate with QAM16 modulation.

Next we evaluated the ergodic capacity of the multi-antenna, and the results are shown in Fig. 4. It can be remarked that using MU-MIMO systems the SE increases by increasing the number of antennas so that for large number of antennas very high values for SE can be obtained. It can be seen that for 10 dB SNR, a gain of 50 bps/Hz in SE is realized when comparing the performance of large number of antennas systems with the performance of small number of antennas ones. We should mention that for SNR=10dB, with N_T=8 and N_R=8, in [13] a SE of 18 bps/Hz (in our case cca 21 bps/Hz) was obtained while in [16] with N_T=4 and N_R=4 and flat Rayleigh fading, the obtained value is about 10bps/s/Hz (in our case cca 11 bps/Hz). The difference can be explained by the fact that, in our program, we considered correlated channel and uncorrelated antennas. The obtained results can be explained by the fact that larger number of antennas systems experiences a narrower beamwidth compared to a single antenna system and the resource allocation and transmit power control action could be more effective.

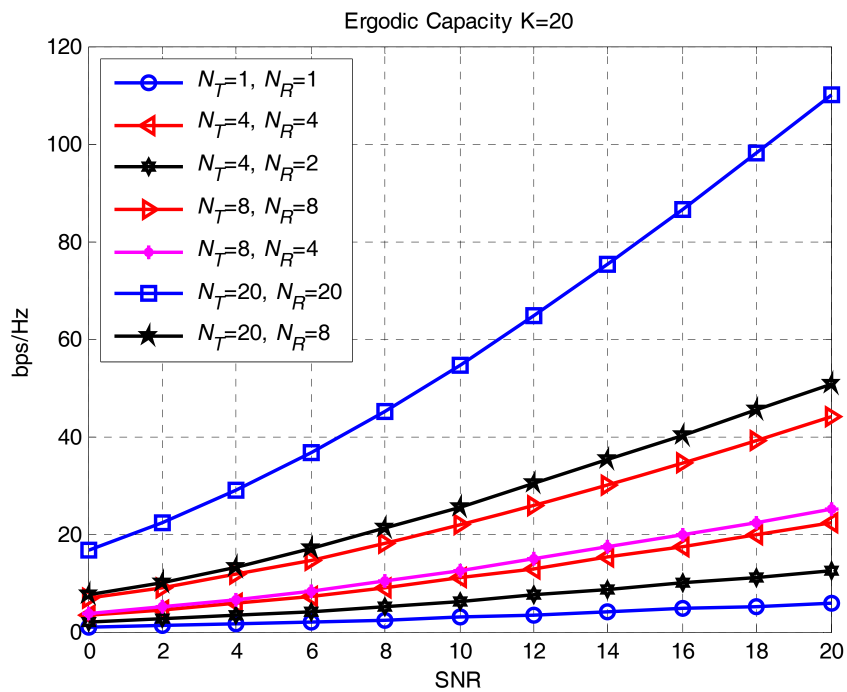

Fig. 4. Performance analysis of spectral efficiency improvement for large scale MU-MIMO system

7. Conclusions

The performance of the large scaled beamforming MU-MIMO communication systems is evaluated on the basis of BER. We used BD, DPC and THP precoders at the transmitter, and ZF and MMSE detections schemes at the receivers, in order to eliminate ISI and improve overall performance. This way we have extended the analysis given in [6][8][9][14] where no precoding with either ZF or MMSE detections schemes are used and in [13][15] where DPC or THP precoding method is used. The performance of the system under different modulation schemes and considering AWGN and Rayleigh Fading channels were studied using MATLAB programs extending the analysis given in [13][15] where Gaussian channel is used, in [7][8] where no path loss in the channel is assumed. It was shown that further performance enhancement can be achieved through the use of BD, DPC and THP with preference for DPC compared to the others. We observe that using detection algorithms like ZF and MMSE enhance the system performance with preference for ZF compared to MMSE. Comparing the results given in Fig's 2 and 3 it can observed that the modulation type (QPSK and QAM16) are clearly affecting the performance of the system compared to [7][9][13][15] where nothing is mentioned about modulation. At the same time, comparing the graphs, it can be seen that the performance of the QPSK modulation technique is better than the one of QAM16 especialy in the range of low SNR. The simulation results show that using multiple transmit antennas and multiple receive antenna with multiple users provides a good BER performance with respect to SNR especially for large number of antennas. Finnaly it was checked that the spectral efficiency of a MIMO system increases if the number of antennas increases, in our study the largest values were obtained for N_T=20 and N_R=20. The result of spectral efficiency can be explained by the fact that systems with a large number of antennas have a narrower beamwidth compared to the beamwidth of a single antenna system and the resource allocation and transmit power control action could be more effective. Finnaly let's mention that the values obtained for SE represents only limit ones that can be achieved in case of perfect knowledge of the Channel State Information and if both the scattering objects and the antennas can be considered uncorrelated.

R E F E R E N C E S

[1]. *George Tsoulos*, "MIMO System Technology for Wireless Communications", Taylor & Francis Group, United States of America, 2006.

[2]. *Ezio Biglieri, Robert Calderbank, Anthony Constantinides, Andrea Goldsmith, Arogyaswami Paulraj and H. Vincent Poor*, "MIMO Wireless Communications", Cambridge University Press, USA New York, 2007.

[3]. *Jeffrey G. Andrews, Arunabha Ghosh and Rias Muhamed*, "Fundamentals of WiMAX Understanding Broadband Wireless Networking", Pearson Education, Inc., United States, 2007.

[4]. *Erik Dahlman, Stefan Parkvall & Johan Sköld*, 4G LTE/LTE-Advanced for Mobile Broadband, Elsevier Ltd., First Published, 2011.

[5]. *H. Bolcskei, D. Gesbert, C. B. Papadias, and A.-J. Van Der Veen*, "Space-Time Wireless Systems From Array Processing to MIMO Communications", Cambridge University Press, UK, 2006.

[6]. *Lu Lu, Geoffrey Ye Li, A. Lee Swindlehurst, Alexei Ashikhmin and Rui Zhang*, "An Overview of Massive MIMO: Benefits and Challenges", IEEE Journal of Selected Topics in Signal Processing, **vol. 8**, No. 5, pp. 742-758, October 2014.

[7]. *Y. Mehmood, W. Afzal, F. Ahmad, U. Younas, I. Rashid and I.* Mehmood, "Large Scaled Multi-User MIMO System so called Massive MIMO Systems for Future Wireless

Communication Networks", Proceedings of the 19th International Conference on Automation & Computing, Brunel University, London, UK, 13-14 September 2013.

[8]. *T. Lakshmi Narasimhan, P. Raviteja, and A. Chockalingam*, "Large-Scale Multiuser SM-MIMO Versus Massive MIMO", Available Online at http://ita.ucsd.edu/workshop/14/files/paper/paper_2751.pdf. Accessed on 12.02.2015

[9]. *Xin Fang, Sheng Fang, Na Ying, Haiyan Cao and Chao Liu*, "The Performance of Massive MIMO Systems under Correlated Channel", IEEE ICON, 978-1-4799-2084-6/13/$31.00 ©2013 IEEE, 2013.

[10]. *Jakob Hoydis, Stephan ten Brink and M´erouane Debbah*, "Massive MIMO in the UL/DL of Cellular Networks: How Many Antennas Do We Need?", IEEE Journal on Selected Areas in Communications, **vol. 31**, No. 2, pp. 160-171, February 2013.

[11]. *Keke Zu*, "Novel Efficient Precoding Techniques for Multiuser MIMO Systems", Ph.D. thesis, Department of Electronics University of York, May 2013.

[12]. *Titus Ikechukwu Eneh*, "Adaptive MMSE Multiuser Receivers MIMO OFDM Wireless Communication Systems", Ph.D. thesis, Mobile and Wireless Communication Engineering, University of Greenwich, United Kingdom, June 2011.

[13]. *Xiang Gao, Ove Edfors, Fredrik Rusek, and Fredrik Tufvesson*, "Linear pre-coding performance in measured very-large MIMO channels", IEEE Vehicular Technology Conference (VTC Fall), ISBN: 978-1-4244-8328-0, pp. 1-5, 5-8 Sept. 2011.

[14]. *J. C. Mundarath and J. H. Kotecha*, "Multi-User Multi-Input Multi-Output (MU-MIMO) Downlink beamforming Systems with Limited Feedback", IEEE GLOBECOM Global Telecommunications Conference, ISBN: 978-1-4244-2324-8, pp. 1-6, Nov. 30-Dec. 4 2008.

[15]. *Hamid Farmanbar and Hadi Baligh*, "Modified Tomlinson-Harashima Precoding for Downlink MU-MIMO Channel with Arbitrary Precoder", IEEE Vehicular Technology Conference (VTC Fall), E-ISBN: 978-1-4673-1879-2, pp. 1-5, 3-6 Sept. 2012.

[16]. *Chengshan Xiao and Yahong R. Zheng, "*Ergodic Capacity, Capacity Distribution and Outage Capacity of MIMO Time-Varying and Frequency-Selective Rayleigh Fading Channels" web.mst.edu/~zhengyr/Publications/Allerton03.pdf.