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TOWARDS INTERACTIONS THROUGH DIFFERENTIABLE–NON–DIFFERENTIABLE SCALE TRANSITIONS IN SCALE RELATIVITY THEORY

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It is shown that, in the framework of Scale Relativity Theory, correlations of type informational entropy/cross entropy – probability density, in the description of the dynamics of any complex system, can be perceived as interactions. Explaining these interactions for a Gaussian – type probability density, implies both attractive forces (of Newtonian type) and repulsive forces (oscillatory harmonic type).

Keywords: informational entropy, cross entropy, multifractals, Scale Relativity Theory

1. Introduction

Usually, models used to describe complex system dynamics are based on a combination of basic theories derived especially from physics and computer simulations [1-4]. Whilst the description of the complex system dynamics implies computational simulations based on specific algorithms [4 – 6] or developments on the standard theory from various classes of models:

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i) based on the usual conservation laws, developed on spaces with integer dimensions, i.e. the ones from the differentiable class of models (differentiable models) [1 – 3];

ii) based on conservation laws, developed on spaces with non – integer dimensions and explicitly written through fractional derivatives, i.e. the ones from the non – differentiable class of models (Fractal or Multifractal models) [5, 6].

Recently, a new class of models has arisen, based on Scale Relativity Theory, either in the monofractal dynamics as in the case of Nottale [7], or in the multifractal dynamics as in the case of the Multifractal Theory of Motion [8].

Both in the context of Scale Relativity Theory [7], as well as in the one of Fractal Theory of Motion [8], supposing that any complex system dynamics are assimilated both structurally and functionally to a multifractal object, said dynamics can be described through motions of the complex system's structural units (dependent on the chosen scale resolution) on continuous and non – differentiable curves (multifractal curves). Since for a large temporal scale resolution with respect to the inverse of the highest Lyapunov exponent [9], the deterministic trajectories of any structural units belonging to the complex system, can be replaced by a collection of potential ("virtual") trajectories, the concept of definite trajectory can be substituted by the one of probability density.

Then, the multifractality expressed through stochasticity becomes operational and correlations of type informational entropy/cross entropy – probability density, in the description of the dynamics of any complex system, can be established. This means that, instead of "working" with a single variable described by a strict non – differentiable function, it is possible to "work" only with approximations of this mathematical function, obtained by averaging them on different scale resolutions. As a consequence, any variable purposed to describe the complex system dynamics will perform as the limit of a family of mathematical functions, this being non – differentiable for null scale resolutions and differentiable otherwise [7].

In the present paper, it is shown that, correlations of type informational entropy/cross entropy – probability density, in the description of the dynamics (by means of Scale Relativity Theory) of any complex system, can be perceived as interactions. Explaining these interactions, both attractive- and repulsive-type forces are found.

2. Conservation laws at various scale resolutions

Assuming that any complex system can be assimilated to a multifractal object, its dynamics in the multifractal Theory of Motion are described through continuous but non-differentiable curves (multifractal curves). According to this theory, the following covariant derivative [8]:

$$\frac{\hat{d}}{dt} = \partial_t + \hat{V}^l \partial_l + D^{lp} \partial_l \partial_p \tag{1}$$

where

$$\hat{V}^l = V_D^l - iV_F^l \tag{2a}$$

$$D^{lp} = \frac{1}{4}(dt)^{\frac{2}{f(\alpha)}-1}\left(d^{lp} + i\bar{d}^{lp}\right) \tag{2b}$$

$$d^{lp} = \lambda_+^l \lambda_+^p - \lambda_-^l \lambda_-^p \tag{2c}$$

$$\bar{d}^{lp} = \lambda_+^l \lambda_+^p - \lambda_-^l \lambda_-^p \tag{2d}$$

$$f(\alpha) = f[\alpha(D_F)] \tag{2e}$$

$$\partial_t = \frac{\partial}{\partial_t}, \partial_l = \frac{\partial}{\partial X^l}, \partial_l \partial_p = \frac{\partial}{\partial X^l}\frac{\partial}{\partial X^p}, i = \sqrt{-1}, i, l, p = 1,2,3 \tag{2f}$$

becomes operational in the writing of conservation laws.

In Eq. (1) and Eqs. $(2a)$, $(2b)$, $(2c)$, $(2d)$, $(2e)$, and $(2f)$, t is the non-multifractal time with the role of affine parameter of the motion curves, X^l are the multifractal spatial coordinates, dt is the scale resolution, $\hat{V}^l$ is the complex velocity field, V_D^l is the differentiable part of the velocity field independent of scale resolution, V_F^l is the non-differentiable part of the velocity field and dependent on the scale resolution; $\lambda_\pm^i$ are constant coefficients associated to differential-non-differential transition, $f(\alpha)$ is the singularity spectrum of order α, α is the singularity index and D_F is the fractal dimension of the "movement curves" [9,10].

There are many modes, and thus a varied selection of definitions of fractal dimensions: more precisely, the fractal dimension in the sense of Kolmogorov, the fractal dimension in the sense of Hausdorff-Besikovitch etc. [10]. In the case of many models, selecting one of these definitions and operating it in the context of any complex system dynamics, the value of the fractal dimension must be constant and arbitrary for the entirety of the dynamical analysis: for example, it is regularly found that $D_F < 2$ for correlative processes in the dynamics of complex systems, $D_F > 2$ for non – correlative processes etc. [10].

Accepting the scale covariant principle in the describing of any complex system dynamics, the conservation law of the specific momentum (i.e. geodesic equations on a multifractal manifold) takes the form:

$$\frac{\hat{d}\hat{V}^i}{dt} = \partial_t \hat{V}^i + \hat{V}^l \partial_i \hat{V}^i + \frac{1}{4}(dt)^{\left[2/f(\alpha)\right]-1} D^{lp} \partial_l \partial_p \hat{V}^i = 0 \tag{3}$$

The explicit form of D^{lp} depends on the type of multifractalization used. It can be admitted that the multifractalization process can take place through stochastic Markovian (thus, memoryless) processes; however, since natural processes exhibit memory-like qualities, it is then necessary to employ a stochastic non-Markovian process. In this case, wherein it is possible to generalize many of the previous results [9,11], the following constraints are admitted:

$$\begin{aligned}\frac{1}{4}(dt)^{\left[{}^{2}/_{f(\alpha)}\right]-1}d^{lp} &= \alpha\delta^{lp}\\ \frac{1}{4}(dt)^{\left[{}^{2}/_{f(\alpha)}\right]-1}\bar{d}^{lp} &= \beta\delta^{lp}\end{aligned} \tag{4}$$

where α and β are two constant coefficients associated to the differential-nondifferential transition, and δ^{lp} is Kronecker's pseudotensor. Thus, (3) with the restriction (4) yield:

$$\partial_t\hat{V}^i + \hat{V}^l\partial_i\hat{V}^i + (\alpha + i\beta)\partial_l\partial^l\hat{V}^i = 0 \tag{5}$$

After (5), the separation of complex system dynamics on various scale resolutions implies either a conservation law for the specific momentum at differentiable scale resolutions:

$$\left(\partial_t + V_D^l\partial_l + \alpha\partial_l\partial^l\right)V_D^i = \left[V_F^l\partial_l - \beta\partial_l\partial^l\right]V_F^i, \tag{6}$$

or a conservation law for the specific momentum at non-differentiable scale resolutions:

$$\left(\partial_t + V_D^l\partial_l + \alpha\partial_l\partial^l\right)V_F^i = \left[V_F^l\partial_l - \beta\partial_l\partial^l\right]V_D^i, \tag{7}$$

Thus, any geodetic motion on multifractal manifolds (i.e non-constrained free motions on multifractal manifolds – see (5)) is found as correlated with non-geodetic motions on Euclidian manifolds (i.e constrained motions on Euclidian manifolds – see (6) and (7)), induced either through a specific multifractal force at differentiable scale resolution:

$$f_D^i = \left(V_F^l\partial_l - \beta\partial_l\partial^l\right)V_F^i, \tag{8}$$

or, through a specific multifractal force at non-differentiable scale resolution:

$$f_F^i = -\left(V_F^l\partial_l - \beta\partial_l\partial^l\right)V_D^i, \tag{9}$$

In order to correlate the non-geodetic dynamics on Euclidian manifolds, constraints arising from the multifractal-non-multifractal transition must be exploited. In this case, the velocity field associated to the differentiable – non – differentiable scale transition (multifractal – non – multifractal scale transition):

$$\bar{V}^l = V_D^l - V_F^l \tag{10}$$

satisfies, by subtracting (6) and (7), the conservation law of the relative specific momentum:

$$\left[\partial_t + \hat{V}^l\partial_i + (\alpha + i\beta)\partial_l\partial^l\right]\hat{V}^i = 2\left(V_F^l\partial_l - \beta\partial_l\partial^l\right)V_F^i \tag{11}$$

Now, according with the self-similarity property of the movement curves (through which also dynamics on Euclidian manifolds should be geodetic or free), the supplementary constraint:

$$f^i = 2\left(V_F^l\partial_l - \beta\partial_l\partial^l\right)V_F^i \equiv 0 \tag{12}$$

correlated with the incompressibility of the multifractal fluid at non-differentiable scale resolution:

$$\partial_i V_F^i = 0 \tag{13}$$

will function as an intrinsic property of any complex system. The differential equations (12) and (13) are constituted as stationary Navier-Stokes type systems at non-differentiable scale resolution. This system of differential equations in dimensionless plane coordinates, with adequate initial and boundary conditions admits the following solutions [12]:

$$U = \frac{1.5}{(\nu\xi)^{\frac{1}{3}}}\operatorname{sech}^2\left[\frac{0.5\eta}{(\nu\xi)^{\frac{2}{3}}}\right] \tag{14a}$$

$$V = \frac{1.9}{(\nu\xi)^{\frac{1}{3}}}\left\{\frac{\eta}{(\nu\xi)^{\frac{2}{3}}}\operatorname{sech}^2\left[\frac{0.5\eta}{(\nu\xi)^{\frac{2}{3}}}\right] - \tanh\left[\frac{0.5\eta}{(\nu\xi)^{\frac{2}{3}}}\right]\right\} \tag{14b}$$

where ξ and η are nondimensional spatial coordinates, U and V are the nondimensional components of the velocity field along the $O\xi$ and $O\eta$ axes, and ν is the multifractality degree.

Therefore, the velocity field along the $O\xi$ axis is described by the multifractal soliton (14a), while the velocity field along the $O\eta$ axis is described by the multifractal soliton – kink (14b).

In such a context, when investigating the dynamic of a complex fluid expansion in a multifractal medium, there are two types of scales that need to be considered.

Firstly, there are the internal interaction scales, which is an amalgam of dynamics induced by the properties of the complex fluid and by its nature. For example, if the complex fluid is considered as a multi element transient plasma [13-18], this scale will be dominated by collision, chemical processes, molecular formation, ionization processes, excitations, etc. The external interaction scales contain the dynamic between the complex fluid and the multifractal medium in which the fluid is embedded. Keeping the same example as before for the plasma as a complex fluid, this scale can relate to the overall dynamics of the plasma, gas-plasma interactions or plasma confinement. These interactions can also be investigated on an interface separating the two fractal objects meaning one could potentially investigate just the double layer separating a flowing transient plasma and the background gas and explore all the phenomena mentioned before. In the following, let the influence of the fractalization degree on each of the two components (*U* and *V*) of the complex fluid for a 2D flow be explored. In Figure 1 in 3D and contour plot are represented the velocity component (*U*) on the $O\xi$ for three fractalization degrees (0.3, 1 and 3). For a low fractality degree it is noticed a very directional flow mainly across the $O\xi$ with little spatial expansion. The enhancement of the fractality in the system leads to a decrease of the velocity and a strong lateral expansion. It is important to note that the main expansion direction does not change, only the contributions on the $O\eta$ direction. The fractalization degree of the system on this velocity component acts as a fractal-like dispersion phenomenon. In Figure 2 in 3D and contour plot are represented the velocity component (*V*) on the $O\eta$ for three fractalization degrees (0.3, 1 and 3). Let it be noted that this component of the velocity is not influenced by the fractalization degree when investigating the absolute value of the velocity, thus remaining quasi constant. There is however a strong influence on the direction of the component.

For low fractality degree there is a small angle with respect to the $O\xi$ axis. Higher values of fractalization degree induce a change in the expansion angle transitioning towards higher angles. The fractalization degree of the system on this velocity component works towards the uniformization of the *V* component as the distribution tends to reach the maximum expansion velocity available for the system.

The same considerations as with plasma, may be applied to other complex fluids (i.e. blood, polymers, biocomposites etc.) [19-27], or at organic materials [28] and liquid crystals (like behaviour of some fatty acids mixtures) [29].

The algorithms employed for the theoretical fit of the empirical data, based on the multifractal analysis, were first used in the articles [30-35].

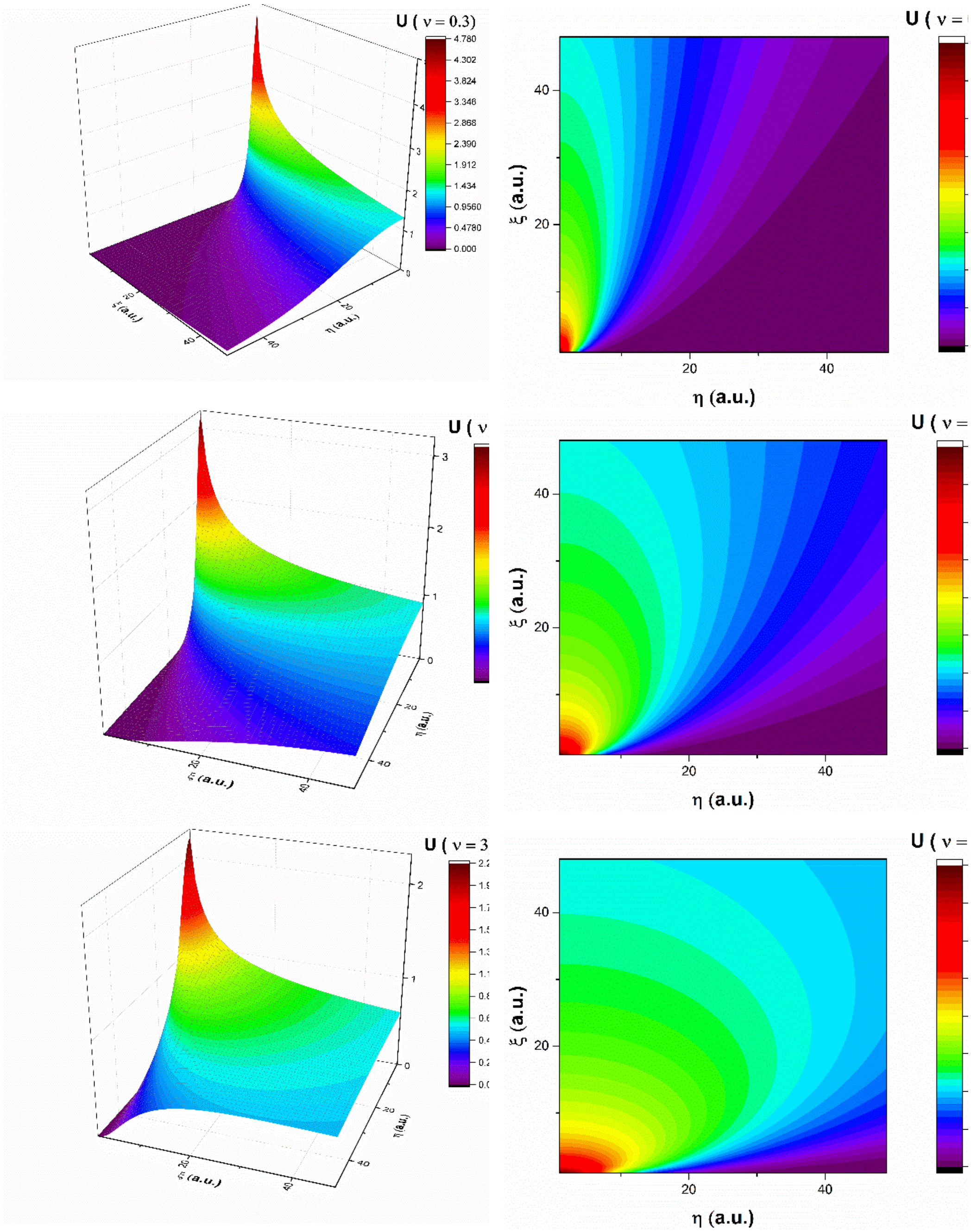

Fig. 1. 3D and contour plot representation of the velocity component on the Oξ for three fractalization degrees (0.3, 1 and 3)

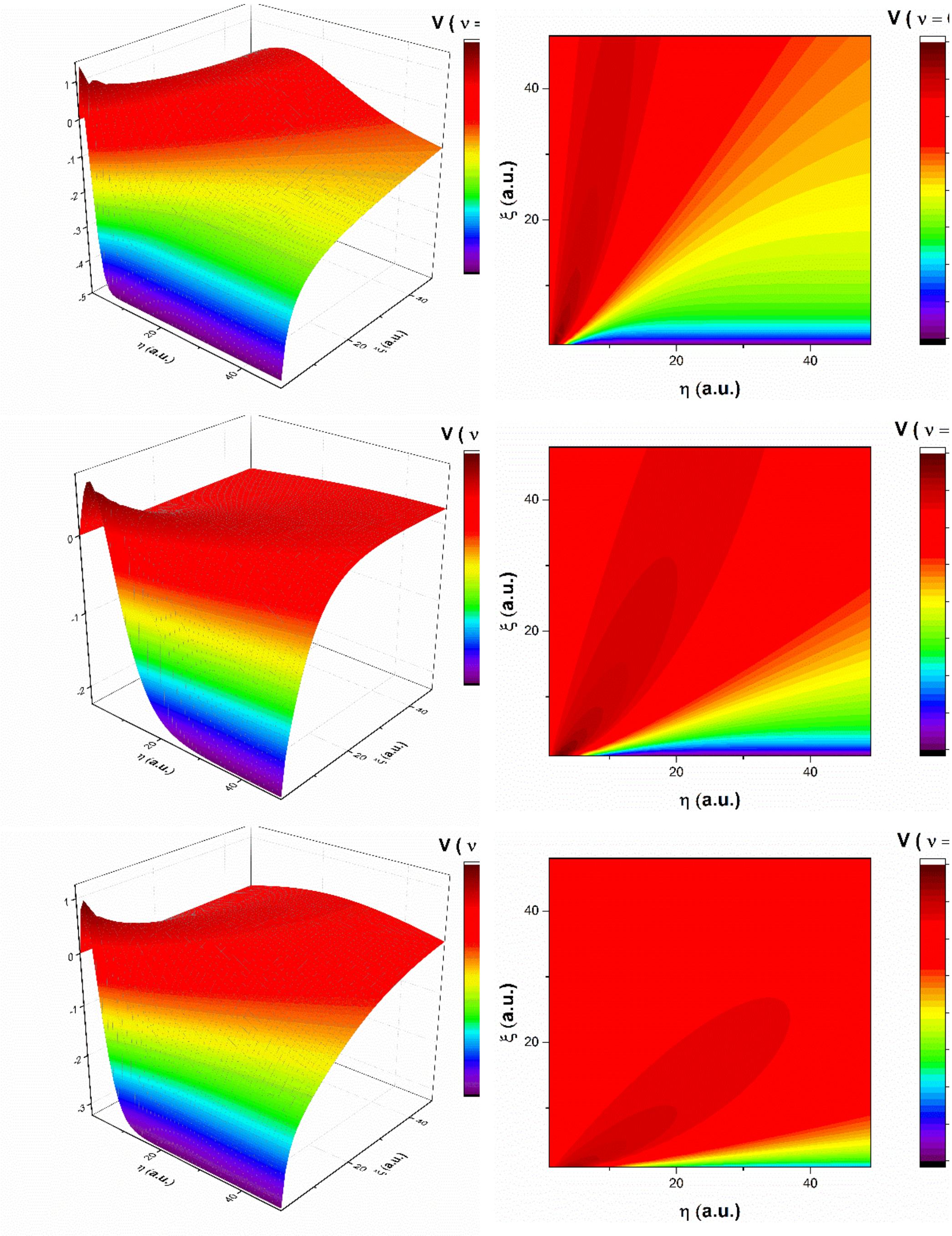

Fig. 2. 3D and contour plot representation of the velocity component on the Oη for three fractalization degrees (0.3, 1 and 3)

3. Correlation between informational entropy and cross entropy

The informational entropy of a repartition is defined by the relation [36]:

$$H = -\int \rho(x) \ln \rho(x) dx \tag{15}$$

where $\rho(x)$ is the probability density and x denotes (globally), the random variables of the problem, dx being the elementary measure of the domain.

In such a context, admitting that the maximum of the informational entropy in the inference upon the probabilities (when only a partial information is available), is equivalent to frankly admitting the fact that no more knowledge is possible. The obtained distributions must be as such, the ones which are the least deriving from the real ones, because no restrictive hypothesis is inferred upon the missing information.

The partial information which, in most cases, is available, is given in the form of the average of a function $f(x)$, or in the form of multiple functions:

$$\bar{f} = \int \rho(x) f(x) dx \tag{16}$$

Relation (16) together with the relation of the measurement of distribution of densities

$$\int \rho(x) dx = 1 \tag{17}$$

are now constraints to which the functional variation (15) must be subjected to, in order to offer the repartition density corresponding to the maximum of the informational entropy. In this case, Lagrange's method of undetermined multipliers leads directly to the exponential repartition

$$\rho(x) = exp[-a - bf(x)] \tag{18}$$

which can be multivariant as well, in the case in which multiple constraints of type (16) are dealt with. If, besides these types of constraints, the following variance is further specified:

$$(\Delta f)^2 = \int \rho(x) \left[f(x) - \bar{f}\right]^2 dx \tag{19}$$

then the nature of the repartition changes. It becomes the Gaussian:

$$\rho(x) = exp[a - bf(x) - cf^2(x)] \tag{20}$$

From the point of view of group theory, (15) is not invariant, as it can be observed. The informational entropy can be however rewritten in a manifest invariant form, through the introduction of a measure $m(x)$, to which (15) becomes:

$$H(\rho, m) = \int \rho(x) \ln \left[\frac{\rho(x)}{m(x)}\right] dx \tag{21}$$

This functional is usually determined through the “cross entropy”/variation of entropy term. Now, the minimization of the entropy variation leads to the same type of repartitions, which differ from one another through the change of the elementary measure of the random variables, as the entropy maximization does:

$$dx \to m(x)dx \tag{22}$$

As such, (18) and (20) become, for example:

$$\rho(x) = m(x)exp[-a - bf(x)] \tag{23}$$

and

$$\rho(x) = m(x)exp[-a - bf(x) - cf^2(x)] \tag{24}$$

Therefore, the principle of the minimal entropy variation generalizes the principle of maximal informational entropy, them being identical only in the case in which $m(x)$ is a constant, meaning the operation with uniform repartitions.

Because many times this is indeed the case, the discussion will revolve around one principle or the other, without making any difference between them. Regarding the same aspect, $m(x)$ is presented as a “candidate” for apriori probabilities, produced with the help of measurable continuous groups; as such, it can be taken as a invariant function on these groups. This is, for example, the case of the $SL(2R)$ group [37], which admits as an integral invariant function the unity.

From a stochastic point of view, it can be said that the variables pertaining to this group are distributed, so the discussion is linked to one of the previously – mentioned cases, in which the principle of the minimal variation of entropy is identified with the one of the maximal informational entropy.

4. The interactions as differentiable – non – differentiable scale transitions

According with the previous considerations - in the sense that the multifractality expressed through stochasticity becomes operational and correlations of type informational entropy/cross entropy – probability density, in the description of the dynamics of any complex system can be established, in what follows, it will be shown that the differentiable – non – differentiable scale transitions are responsible for the generation of interactions.

In such a context, let it be considered for irrotational motions of the complex system dynamics. Then, the complex velocity fields (2a) become:

$$\hat{V}^i = -2i\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial^i \ln \Psi \tag{25}$$

where Ψ is the function of states. If it is chosen Ψ of the form:

$$\Psi = \sqrt{\rho}e^{is}, \tag{26}$$

where $\sqrt{\rho}$ is the amplitude and s is the phase, the complex velocity fields (25) take the explicit form:

$$\hat{V}^i = 2\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial^i s - i\lambda(dt)^{\left[\frac{2}{f(\alpha)}\right]-1}\partial^i \ln \rho \tag{27}$$

such that, the specific multifractal potential Q and the specific multifractal force F^i, become:

$$Q = -2\lambda^2(dt)^{\left[\frac{4}{f(\alpha)}\right]-2}\frac{\partial^l\partial_l\sqrt{\rho}}{\sqrt{\rho}} \tag{28}$$

respectively

$$F^i = -\partial^i Q = -2\lambda^2(dt)^{\left[\frac{4}{f(\alpha)}\right]-2}\partial^i\frac{\partial^l\partial_l\sqrt{\rho}}{\sqrt{\rho}} \tag{29}$$

In such a framework, (26) for $f(x) \equiv r$ and a convenient choice of constants a, b and c, will take the form:

$$\rho(r) = \frac{1}{(2\pi)^{\frac{1}{2}}\sigma} exp\left[-\frac{(r-r_0)^2}{\sigma^2}\right] \tag{30}$$

with r_0 is the average and σ is the variance. From here, using (29), it results the multifractal specific force:

$$\boldsymbol{F}(r) = \frac{-\partial Q}{\partial \boldsymbol{r}} = -\frac{\mu^2 r_0}{\sigma^2}\frac{\boldsymbol{r}}{r^3} + \frac{\mu^2(r-r_0)}{2\sigma^4}\frac{\boldsymbol{r}}{r} \tag{31}$$

where

$$\mu^2 = 2\lambda^2(dt)^{\left[\frac{4}{f(\alpha)}\right]-2} \tag{32}$$

Thus, both the multifractal specific force of attractive type/of Newtonian type

$$\boldsymbol{F}_N(r) = -\frac{\mu^2 r_0}{\sigma^2 r^3}\boldsymbol{r} \tag{33}$$

and the multifractal force of repulsive type

$$\boldsymbol{F}(r) = \frac{\mu^2(r - r_0)}{2\sigma^4}\frac{\boldsymbol{r}}{r} \tag{34}$$

are natural consequences of the information variation.

5. Conclusions

In the framework of Scale Relativity Theory, dynamics of any complex system on a multifractal manifold are analyzed. Thus, the momentun Conservation Laws both at differentiable and fractal scale are obtained.

Furthermore, the same Conservation Law is obtained for the differentiable–non–differentiable scale transition. Explaining the correlations of type informational entropy/cross entropy–probability density induce various types of interactions. Moreover, explaining these interactions for a Gaussian–type probability density imply attractive-and repulsive-type forces.

The differentiable – non – differentiable scale transition is the one which can allow the explaining of interactions for various types of given probabilities. It can be concluded that the force is just a model which can be found as differentiable–non–differentiable scale transitions, based on correlations of type informational entropy/cross entropy–probability density.

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