COVARIANCE BASED MIMO RADAR BEAMFORMING FOR PATTERNS WITH DEEP NULLS

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Multiple Input-Multiple Output radar is an emerging technology and has exclusive advantages and flexibilities. In this paper, we propose a method for beam-pattern synthesis in order to construct a desired pattern which has specific nulls. It is shown that arbitrary cross correlation matrix (R) can be approximated to achieve a desired beam-pattern with a null in a specific direction. Besides constrained convex optimization problem is investigated and it is shown that reconstructed pattern by this matrix requires null at arbitrary angle. Penalty function and barrier methods are applied to solve this constrained convex optimization problem. Finally, power and advantages of our method for beam-pattern synthesizing has been depicted through simulation results.

Keywords: MIMO Radar Beamforming, Convex Optimization, Penalty Function Method, Barrier Method

1. Introduction

Multiple input-multiple output radars have attracted researcher's attention in recent years. These radars have been characterized with multi antennas for transmitting different waveforms and receiving reflected signals. MIMO radars like MIMO communications proposed new approaches in signal processing. Such structures have good potentials in fading mitigation, resolution improvement and jamming and deception suppression [1]. In this type of radars unlike phased array counterpart , signals can be chosen so that the power density near arbitrary target is maximized or reflected signal cross correlation is minimized[2, 3]. MIMO radars ability causes resolution improvement [3, 4], high sensitivity in low velocity target detection [5] and increasing parameter identifiability [6]. Generally MIMO radars are classified into two main categories of MIMO radars with widely separated antennas[3] and MIMO radars with collocated antennas[7]. In widely separated antenna MIMO radars, transmitters are so far such that each one shows different aspect of a target. This Specification of MIMO radars increases spatial diversity [8, 9]. In the case of collocated antenna MIMO radars, the transmitters are so close to

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each other that special aspect of a desired target can be shown. High ability in interference suppression [10, 11], improvement of parameter identifiability and flexibility increasing in waveform designation are some properties of this kind of radars [12]. Generally, MIMO radars waveform designing can be divided into three main classes. Covariance based methods [13, 14], ambiguity function based methods [15, 16] and extended targets based methods [17, 18] are these three classes. In covariance based approaches signal cross correlation matrix is considered instead of entire waveform. In [15, 16] signal cross correlation matrix is designed such that transmitted power is distributed in desired range of angles. In addition in [19] cross correlation matrix is designed to control spatial power. In contrast with the covariance based approaches, in the second method, the entire signal is optimized. These methods try to find a set of signals which construct a desired ambiguity function. They are more complicated than covariance based methods. Angular resolution, Doppler and range of radar are described with radar ambiguity function [20]. Extended target based methods like latter methods use ambiguity function unless they estimate and detect extended targets. Another way for transmit beampattern designation for MIMO radar is minimizing the radiation powers of antenna in the selected directions with optimization variables constrained to the waveform phases [21]. Also, the eigen decomposition method is applied to calculate the subarray beamforming weights according to optimized correlation matrix [22]. In this paper, we focus on covariance based method for waveform designing. A convex optimization problem is selected for finding a suitable cross correlation matrix. This optimization has two constraints which are equality and inequality. This problem is solved with penalty function and barrier methods. In section 2 we formulate our problem and show how it is possible to make a beam pattern for MIMO radar. Section 3 discusses pattern synthesis and our main problem which solved by penalty function and barrier methods. In section 4 simulation results are demonstrated and performance of our novel method is presented while conclusions are brought after that.

2. Problem Formulation

Consider a collection of N transmitters which are located at known coordinate $\mathbf{x}_i = (x_{1i}, x_{2i}, x_{3i}) = (x, y, z)$ as shown in

Fig. 1. In this paper we use spherical coordinate in which system elevation θ is the angle made by $\mathbf{v}$ with the equatorial $x - y$ plane. Each T\R module is driven by a different signal with wavelength λ. Then the normalized power density $P(\theta)$ which is the summation over all transmitted waveforms in watts per steradian (W/ster) is [12]:

$$P(\theta) = \frac{1}{4\pi} \mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta)$$

(1)

In this quadratic form of normalized power density, $\mathbf{R}$ is cross correlation matrix of transmitted signals. Note that electrical angle $\psi_k(\theta)$ and direction vector are defined as [12]:

$$\psi_k(\theta) = \frac{2\pi z_k}{\lambda} \sin(\theta) \tag{2}$$

$$\mathbf{a}(\theta) = \begin{bmatrix} e^{j\psi_1(\theta)} & e^{j\psi_2(\theta)} & \cdots & e^{j\psi_N(\theta)} \end{bmatrix}^T \tag{3}$$

z_k is distance between array elements. The normalized power density $P(\theta)$ is exactly the beam that we want to synthesis. Besides, power density units are watts per steradian which ensures that

$$2\pi \int_{-\pi/2}^{\pi/2} P(\theta)\cos(\theta)d\theta = Tr(\mathbf{R}) = N \tag{4}$$
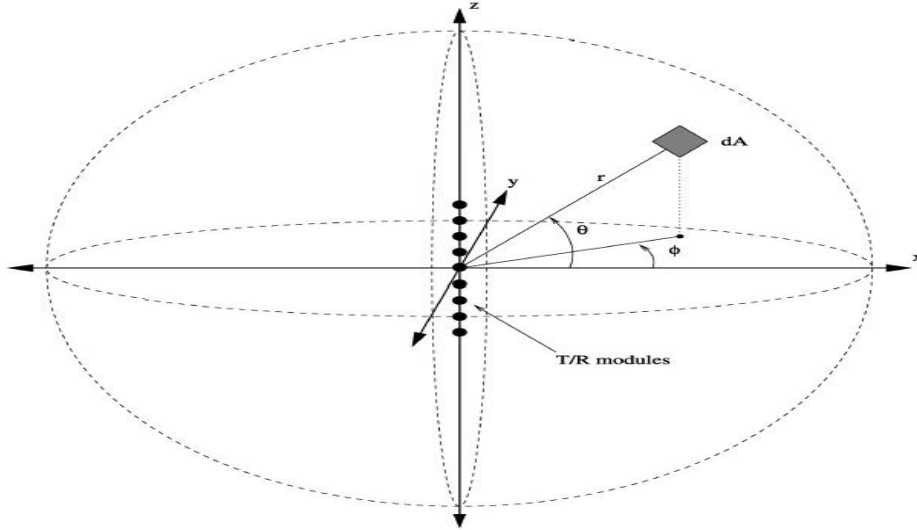

Fig. 1. T\R modulus and spherical coordinate[12]

By selection of optimum R, the desired pattern can be constructed, therefore an optimization problem should be derived such that optimum R is obtained. As stated in [12] the cost function used for this problem is :

$$J_2(\mathbf{R}) = \int_{\theta} \left| P_d(\theta) - \mathbf{a}^H(\theta)\mathbf{R}\mathbf{a}(\theta) \right|^2 \cos(\theta)d\theta \tag{5}$$

That is a weighted squared error metric and $\cos(\theta)$ is weighting function. It can be shown that optimizing (5) can be rewritten as a convex optimization problem. Using a single linear matrix inequality (LMI) $\mathbf{R}$ can be written as the weighted sum of basis matrices $\mathbf{F}_k$ and $\mathbf{G}$ [12]. Mentioned LMI is as follow:

$$\mathbf{R} = \mathbf{G} + \sum_{k=1}^{N(N-1)} x_{N+k}\mathbf{F}_{N+k} \succ 0 \tag{6}$$

Where $X \succ 0$ indicates that X is positive semi definite. $x_1, x_2, ..., x_N$ are components of coefficient vector which makes linear combination of basis matrices. Note that $\mathbf{G} = \mathbf{I}_N$. Dropping the first N basis matrices, vector space will be $\mathbf{x} = (x_1, x_2, ..., x_{N(N-1)}) \in \Re^{N(N-1)}$ and thus

$$\mathbf{R} = \mathbf{G} + \sum_{k=1}^{N(N-1)} x_k\mathbf{F}_k \succ 0 \tag{7}$$

Therefore, the problem is the following squared error constrained optimization [12] :

$$\min_{\mathbf{R}} \quad J_2(\mathbf{R}) = \int_\theta \left| P_d(\theta) - \mathbf{a}^H(\theta)\mathbf{F}(x)\mathbf{a}(\theta) \right|^2 \cos(\theta)d\theta \tag{8}$$

$$s.t \quad \mathbf{R} = \mathbf{F}(\mathbf{x}) = \mathbf{G} + \sum_{k=1}^{N(N-1)} x_k\mathbf{F}_k \succ 0$$

Barrier method has been proposed as a solution approach for this problem [12,23-25]. Now suppose we want to force some deep nulls to our desired pattern. For example, θ_0 is a desired angle. Therefore, we have

$$H(\mathbf{x}) = \mathbf{a}^H(\theta_0)\mathbf{F}(\mathbf{x})\mathbf{a}(\theta_0) \tag{9}$$

Therefore, there is a new optimization problem with equality and inequality constraints. Now we form an optimization problem as follow:

$$\min_{\mathbf{R}} \quad J_2(\mathbf{R}) = \int_\theta \left| P_d(\theta) - \mathbf{a}^H(\theta)\mathbf{F}(\mathbf{x})\mathbf{a}(\theta) \right|^2 \cos(\theta)d\theta \tag{10}$$

$$s.t \quad \mathbf{R} = \mathbf{F}(\mathbf{x}) = \mathbf{G} + \sum_{k=1}^{N(N-1)} x_k\mathbf{F}_k \succ 0$$

$$H(\mathbf{x}) = \mathbf{a}^H(\theta_0)\mathbf{F}(\mathbf{x})\mathbf{a}(\theta_0) = 0$$

Penalty function and barrier methods are employed jointly to solve this problem [23-25]. First, penalty function method is applied to convert problem to an inequality constraint one and then barrier method solves our optimization problem which had one inequality constraint.

Fig. 2 shows a simple flow chart for analytic solution of problem proposed in (10).
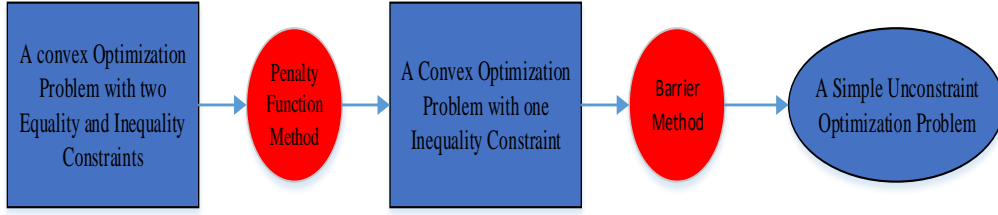
Fig. 2. Flow chart of solving problem

3. Solving problem

Methods which use penalty function, transform a constrained problem into a single unconstrained problem or into a sequence of unconstrained problems. The constraints are placed into the objective function via a penalty parameter in order to penalize any violation of the constraints. For solving the problem proposed in (10), penalty function method should be used. We should minimize a cost function like $J_2(\mathbf{x}) + \mu\alpha(\mathbf{x})$ instead of a complicated problem as (10). $\alpha(\mathbf{x})$ should be chosen so that $J_2(\mathbf{x}) + \mu\alpha(\mathbf{x})$ becomes a convex function [23]. As we have proved $J_2(\mathbf{x})$ is a convex function [12], our attention must be focused on choosing an $\alpha(\mathbf{x})$ suitable function so that cost the function remains convex. Since there are two constraints, $\alpha(\mathbf{x})$ is as follow:

$$\alpha(\mathbf{x}) = \phi(\mathbf{F}(\mathbf{x})) + \psi(H(\mathbf{x})) \tag{11}$$

Note that ϕ and ψ should have the following conditions [23]:

$$\phi(y) = 0 \quad y \le 0 \quad and \quad \phi(y) > 0 \quad y > 0 \tag{12}$$
$$\psi(y) = 0 \quad y = 0 \quad and \quad \psi(y) > 0 \quad y \ne 0$$

Generally, a proper penalty function must incur a positive penalty for infeasible points and no penalty for feasible points. Applying constraints mentioned in (12) make $J_2(\mathbf{x}) + \mu\alpha(\mathbf{x})$ a convex function. For this problem, we have chosen ϕ and ψ as follows:

$$\phi(y) = \psi(y) = \begin{cases} y^4 & y \ge 0 \\ 0 & y < 0 \end{cases} \tag{13}$$

Therefore equation (10) transforms to:

$$\min_{\mathbf{R}} \quad J_2(\mathbf{x}) + \mu(\phi(\mathbf{F}(\mathbf{x})) + \phi(H(\mathbf{x}))) \tag{14}$$

Note that, since $H(\mathbf{x})$ is a scalar so $\phi(H(\mathbf{x}))$ is scalar. $\mathbf{F}(\mathbf{x})$ is a matrix then $\phi(\mathbf{F}(\mathbf{x}))$ will be a matrix. Solving this problem by rewriting (14) results to:

$$\min_{\mathbf{R}} \quad \{J_2(\mathbf{x}) + \mu\phi(H(\mathbf{x}))\} + \mu(\phi(\mathbf{F}(\mathbf{x})) = J_3(\mathbf{x}) + (\phi(\mathbf{F}(\mathbf{x})) \tag{15}$$

Now for explaining novelty of our work, by comparing (15) with a standard optimization problem which is solved in (8), a new objective function and constraint can be defined as follows:

$$\min_{\mathbf{R}} \quad \int_{\theta} \left| P_d(\theta) - \mathbf{a}^H(\theta)\mathbf{F}(\mathbf{x})\mathbf{a}(\theta) \right|^2 \cos(\theta)d\theta + \mu\phi(H(\mathbf{x})) = J_2(\mathbf{x}) + \mu\phi(H(\mathbf{x})) \tag{16}$$

$$s.t \quad \mathbf{R} = \mathbf{F}(\mathbf{x}) = \mathbf{G} + \sum_{k=1}^{N(N-1)} x_k \mathbf{F}_k \succ 0$$

In simple expression, an optimization problem with an objective function and two constraints which are equality and inequality is transformed to a simpler optimization problem with another objective function which is inequality constraint and can be solved by barrier method easily. In other words for constructing a pattern with some specific nulls one can minimize $J_2(\mathbf{x}) + \mu\phi(H(\mathbf{x}))$ instead of $J_2(\mathbf{x})$ and obtain a rather deep null with minimum error. This helps to synthesizing a pattern which is similar to a desired pattern and we can have our special nulls in special angles. To solve (16) using barrier method, a barrier function $\varphi(\mathbf{F}(\mathbf{x}))$ is chosen that is strictly convex over the feasible region of the problem. We choose log-det function which is a well-known one [23-25]. We use here:

$$\varphi(\mathbf{F}(\mathbf{x})) = \begin{cases} \log(\det(\mathbf{F}^{-1}(\mathbf{x}))) & \mathbf{F}(\mathbf{x}) \succ 0 \\ \infty & otherwise \end{cases} \tag{17}$$

Gradient and Hessian of the barrier function can be derived as follow [24]:

$$\nabla(\varphi(\mathbf{F}(\mathbf{x})))_i = -Tr(\mathbf{F}^{-1}(\mathbf{x})\mathbf{F}_i) \tag{18}$$

$$\nabla^2(\phi(\mathbf{F}(\mathbf{x})))_{ij} = Tr(\mathbf{F}^{-1}(\mathbf{x})\mathbf{F}_i\mathbf{F}^{-1}(\mathbf{x})\mathbf{F}_j) \tag{19}$$

Also, we have

$$\nabla J_3(\mathbf{x}) = \nabla J_2(\mathbf{x}) + \mu\nabla\phi(H(\mathbf{x})) = \nabla J_2(\mathbf{x}) + \mu\phi'(H(\mathbf{x}))\nabla H(\mathbf{x}) \tag{20}$$

$$\nabla^2 J_3(\mathbf{x}) = \nabla^2 J_2(\mathbf{x}) + \mu\nabla^2\phi(H(\mathbf{x})) \tag{21}$$

$$= \nabla^2 J_2(\mathbf{x}) + \mu\phi'(H(\mathbf{x}))\nabla^2 H(\mathbf{x}) + \mu\phi''(H(\mathbf{x}))\nabla H(\mathbf{x})\nabla H(\mathbf{x})^T$$

Note that

$$\nabla \mathrm{H}(\mathbf{x}) = \begin{bmatrix} \mathbf{a}^H(\theta_0)\mathbf{F}_1\mathbf{a}(\theta_0) \\ \vdots \\ \mathbf{a}^H(\theta_0)\mathbf{F}_{N(N-1)}\mathbf{a}(\theta_0) \end{bmatrix} \tag{22}$$

and $\nabla^2 H(\mathbf{x}) = 0$. Also, gradient and hessian of $J_2(\mathbf{x})$ can be calculated as follow [24]:

$$\nabla(\mathrm{J}_2(\mathbf{x}))_i = 2\int_\theta \alpha_i(\theta)\cos(\theta)(\mathrm{P}_d(\theta) - \mathbf{a}^H(\theta)\mathbf{F}(\mathbf{x})\mathbf{a}(\theta))d\theta \tag{23}$$

$$\alpha_i(\theta) = \mathbf{a}^H(\theta)\mathbf{F}_i\mathbf{a}(\theta) \tag{24}$$

$$\nabla^2(\mathrm{J}_2(\mathbf{x}))_{ij} = 2\int_\theta \alpha_i(\theta)\alpha_j(\theta)\cos(\theta)d\theta \tag{25}$$

According to proposed derivations, we summarize steps of solving (16) .Note that the barrier function is multiplied by a constant factor $1/t$ and as t approaches to ∞, barrier term diminishes.

- Select initial point $\mathbf{x}_0$ and $t > 0$, $\mu > 1$ and $\varepsilon > 0$.
- $\mathbf{x}^*(t) = \arg\min \quad J_2(\mathbf{F}(\mathbf{x})) + \mu\phi(H(\mathbf{x})) + (1/t)\varphi(\mathbf{F}(\mathbf{x}))$.
- Set and $\mathbf{x}_{i+1} = \mathbf{x}^*(t)$.
- Repeat step 2 using $\mathbf{x}_{i+1}$ as initial point until
  $$\left| J_2(\mathbf{F}(\mathbf{x}_{i-1})) + \mu\phi(H(\mathbf{x}_{i-1})) - J_2(\mathbf{F}(\mathbf{x}_i)) - \mu\phi(H(\mathbf{x}_i)) \right| \le \varepsilon$$

In fact, the cost function which we optimize in step 2 is described in (26).We did this optimization with Newton iterations because of its good convergence properties.

$$G(\mathbf{x}) = \mathrm{J}_3(\mathbf{x}) + (1/t)\varphi(\mathbf{F}(\mathbf{x})) \tag{26}$$

Newton solutions for unconstraint optimization problem in step 2 are given by

$$\mathbf{x}_{k+1} = \mathbf{x}_k - [\nabla^2 \mathrm{G}(\mathbf{x}_k)]^{-1}\nabla \mathrm{G}(\mathbf{x}_k) \tag{27}$$

where

$$\nabla \mathrm{G}(\mathbf{x}) = \nabla \mathrm{J}_3(\mathbf{x}) + (1/t)\nabla\varphi(\mathbf{F}(\mathbf{x})) \tag{28}$$

$$\nabla^2 \mathrm{G}(\mathbf{x}) = \nabla^2 \mathrm{J}_3(\mathbf{x}) + (1/t)\nabla^2\varphi(\mathbf{F}(\mathbf{x})) \tag{29}$$

In the implementation of our method selection of t and μ values must be cared. For large values of μ, number of Newton iterations may be more than usual for unconstraint optimization problem in step 2. Also, large t yields the fixed point solution in step2. The result of barrier method optimization is a vector as $\mathbf{x} = (x_1, x_2, ..., x_N)$. By using this vector components and their corresponding basis matrices, minimum mean square signal cross correlation matrix $\mathbf{R}_{MSE}$ can be constructed which optimally generates a pattern that is close to our desired beam pattern.

4. Simulation Results

The array configuration in all simulations of this paper, is uniform linear array (ULA) of $N = 10$ sensors with $\lambda / 2$ sensor spacing.

Fig. 3 shows a desired pattern which is constructed by cross correlation matrix of signals and has a null in $\theta_0 = 44.4^\circ$.
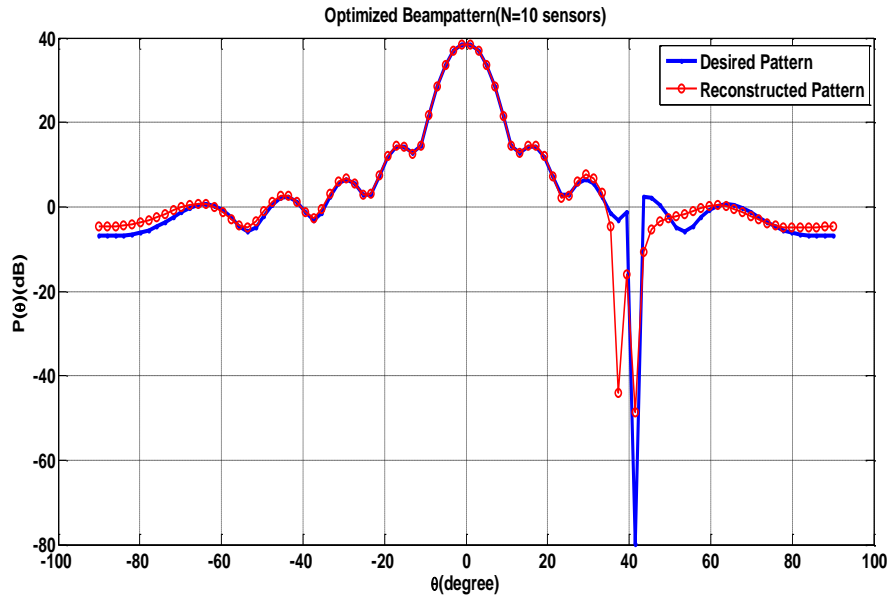

Fig. 3. Synthesized pattern for one null

Also, you can see reconstructed pattern and ability of this method for creation a deep null in this angle.

We used minimum square error criterion as a cost function for our beam pattern matching problem. This criterion has a specific property in this kind of problems.

In this problem MSE cost function causes the synthesized pattern does not converge to our desired pattern rapidly. Because of this low rate of convergence, one can see additional nulls besides desired nulls. The most important subject for us is the location of the desired null which is so important in many applications.

One of the abilities of our novel method is its capability for creation of multi nulls in a desired angle.

Fig. 4 shows that this method has no restriction for beam-pattern synthesizing and creating multi nulls. This fact is true because of the characteristics of penalty function method. If one increase number of equality constraints in this optimization, objective function will have little changes while its convexity does not change. Just new objective function will be sum of new penalty functions. This robustness helps us in many applications which we need multi nulls in multi directions.
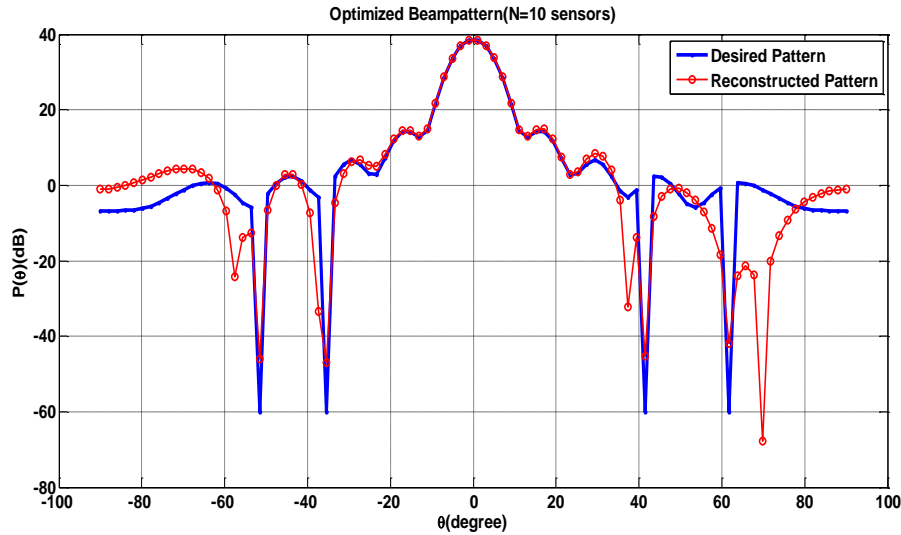

Fig. 4. Synthesized pattern for four nulls

Note that the nulls are located at $\theta_1 = 44.4^\circ$, $\theta_2 = 61.6^\circ$, $\theta_3 = -35.3^\circ$ and $\theta_4 = -51.5^\circ$. Now for showing the ability of the method, consider two nulls which are located at $\theta_1 = 29.32^\circ$ and $\theta_2 = 81.91^\circ$. Suppose two angles are changing and they are to be close to each other. Performance of reconstructed pattern can be shown in

Table 1.

Comparison Between MSE of Reconstructed Patterns Based on Proximity of Nulls

First Null Angle	Second Null Angle	MSE
29.32	81.91	0.1656
33.37	77.86	0.0909
37.41	73.82	0.0922
41.46	69.77	0.1267
45.50	65.73	0.1562
49.55	61.68	0.1204
53.59	57.64	0.1068

If we admit minimum square error of reconstructed samples as a criterion for similarity of reconstructed pattern to desired pattern, according to

Table 1 one can see reconstructed pattern has tolerable error and can provide us an appropriate pattern which is so rival for desired pattern. According to

Table 1 when nulls are so close to each other the method can construct a pattern with tolerable minimum square error. This robustness can help us in special applications which we need exact nulls in our patterns and nulls are so close to each other. For more description suppose two nulls which are located in $\theta_1 = 53.75^\circ$ and $\theta_2 = 57.38^\circ$ in Fig. 5 .
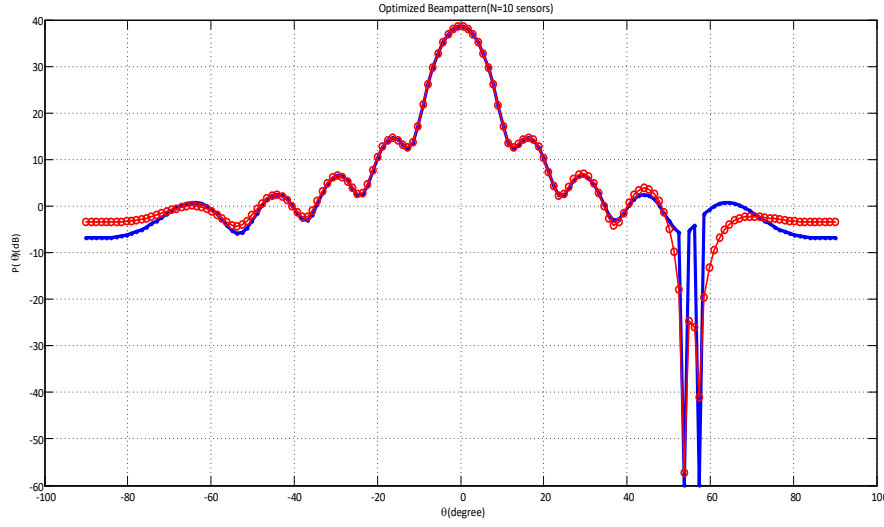
Fig. 5 Reconstructed pattern for $\theta_1 = 53.75^{\circ}$ and $\theta_2 = 57.38^{\circ}$

This Simulation verifies our proposed method ability for constructing nulls located in the neighborhood of each other.

Now consider a different kind of pattern which is constructed with proposed method in [12] and proposed in this paper. We should attract your attention to this tip that our proposed method works for every pulse shape with a tolerable error that can be an advantage for this algorithm.

Table 2 shows a comparison between three kinds of patterns. For all three categories of desired patterns our proposed method can make a better null in jammer angle and attenuation is so better than signal cross correlation method proposed in [12]. According to

Table 2 the reconstructed pattern with our proposed method has better attenuation in specific nulls than signal cross correlation method proposed in [12]. Note that despite of better attenuation in null directions we have tolerable error which is little more than signal cross correlation method in [12]. Therefore, this proposed method has a tradeoff between minimum square error and attenuation in null directions.
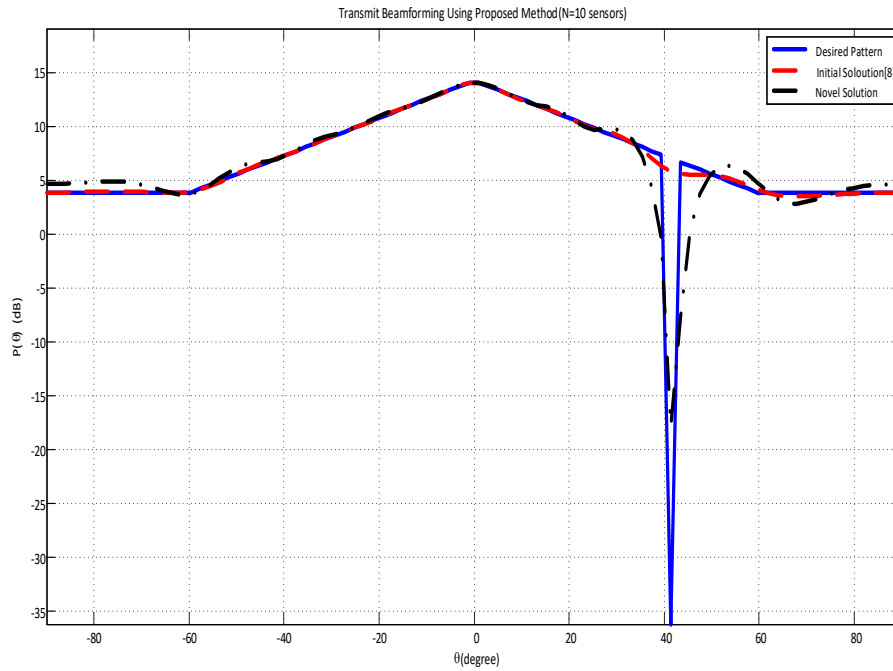
Fig. 6 Comparing Initial Solution with Our Novel Solution

We should attract your attention to this tip that the solution proposed in [12] attenuates 1dB in the null direction but our proposed method attenuates 23dB more than in the null direction. Fig. 6 shows this result. Note that MSE error for first experiment is 0.23 and for second one is 0.97.

This ability of proposed method is effective in some applications such as deceptive jamming suppression. A deceptive jammer tries to record signals and broadcast them with delays. If a MIMO radar can transmit a signal with a deep null in jammer direction it can confront against jammer's operation.

Table 2

Comparison of Signal Cross Correlation Method versus our Method

Pulse	SignalCCMethod MSE	Proposed Method MSE	Signal CC Method Attenuation(dB)	Proposed Method Attenuation(dB)
R Constructed	0.0112	0.0426	3.0344	32.3757
Rectangular	3.2263	6.0502	0.2550	30.5725
Triangular	0.2379	0.9787	1.0518	35.1049

5. Conclusion

In this paper, a transmit beamforming method for multiple input-multiple output radar has been proposed where transmitted signals can have arbitrary cross correlation matrix R. R can be chosen so that a desired beam-pattern can be constructed. Our criterion for constructing desired pattern is minimum mean square and then we applied a constraint to our problem. This approach leads to interior-point methods for a constrained convex optimization problem. We solved this optimization problem using penalty function and barrier method. Through the simulations it was shown that our proposed method can also synthesize every desired beam. One of the advantages of our method is its null depth and it can construct a pattern with tolerable error and deep null. Other capability of this method is its ability different waveforms and pulse shapes. This characteristic can be used in many applications in MIMO radar such as deceptive jamming suppression. Much work remains for this problem in choosing different cost functions.

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