CONTRIBUTIONS TO THE DETERMINING OF THE INTERSECTIONS OF PRISMATIC BODIES BY THE USE OF THE REPRESENTATION METHOD FOR THE PARALLEL VECTORS' PROJECTION

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În lucrare se determina linia poligonala de intersectie a doua prisme folosind metoda proiectiei vectorilor paraleli.

In the paper we determined the polygonal outline of intersection for two prisms, based on the properties of the method for the parallel vectors projection.

Keywords: vector, projection, graphic, prism, polygon

Introduction

The method of the "Parallel vector's projection" is a graphic method, bearing a lot of applications in various fields. This method was created by the Russian geometrician Fedorov [1]. It could also be used to solve the problems of descriptive geometry, being preferred in most of cases, for the simplicity of the graphic construction. As a principle, the method of the parallel vectors' projection takes as an essential statement that only point in the space is defined as a position towards a plane through a vector; in a particular and simplifying case it could be a segment representing the distance from that point to the plane, to which a sense is associated. Let us consider in figure 1 a point A in space and a projection plane [P]. The segment $\overline{aA}$ is the vector which determines the position of the point A in respect to the plane [P] following a random direction $\overline{L}$ the projection, $\overline{aa'}$ is proportional to the distance from the point to the plane ($\overline{aa'} = \overline{aA} \cdot ctg\, j$) and, as a consequence, it is this projection which determines the position of the point in space.

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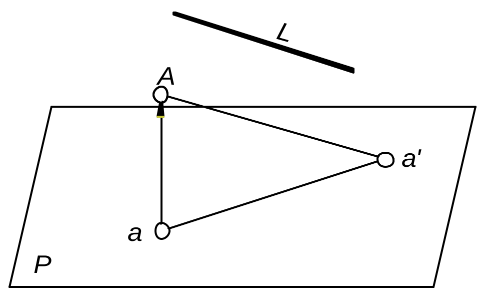

Fig. 1

The point “a” corresponds to the orthogonal projection of the point and is known as initial projection, while “a’” corresponds to the oblique projection of the point following the direction $\overline{L}$, and is known as the dependence between the vectors projection and the position of the point A in respect to the plane [P], valid for all the points, within the space that are projected on a plane [P], following a direction that would be parallel to the line $\overline{L}$.

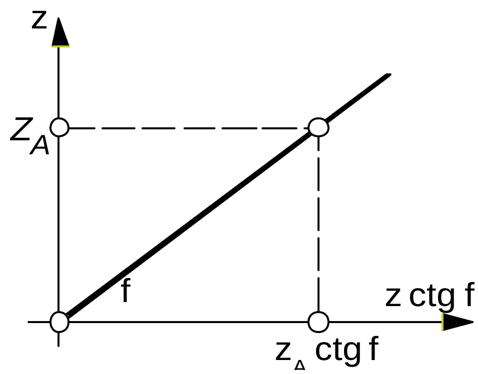

Fig. 2

In order to represent a straight line, it is enough to project two of its points, respectively to project the vectors of these points (figure 3).
By connecting the kindred projections of these points the initial end final projections of the straight line are determined. These projections are intersecting one to another into a point H, which is known as the trace of the straight line on the projection plane. The projections of the line point’s vectors will be parallel to the orthogonal projections of the direction towards which the projection is made -

$\overline{L}$ - since the plane determined by $\overline{bB}$ and $\overline{Bb'}$ is parallel to the projecting plane of the straight line $\overline{L}$. [2] In the particular case when the projecting direction is

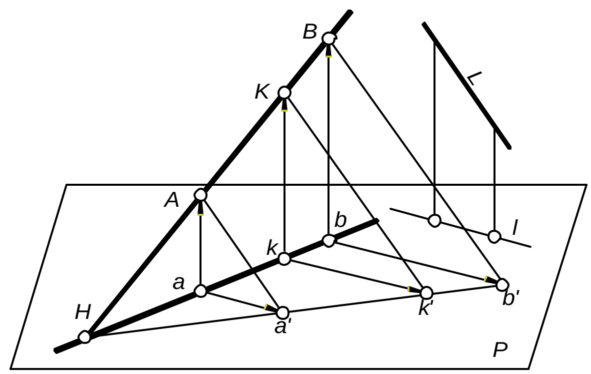

Fig. 3

parallel to the straight line it self, (figure 4), the final projection of the line will be reduced to a unique point, situated on the initial projection of the line, that also coincides to the trace of the lie.

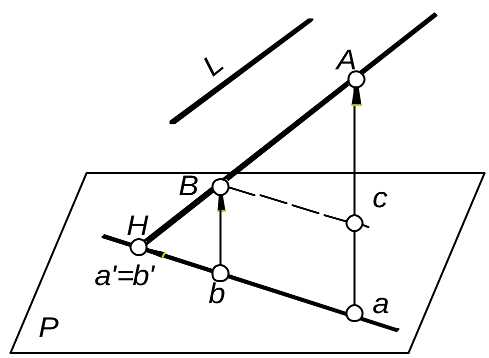

Fig. 4

The initial and final projections of the line's trace, no matter on which projecting direction, get confounded with the trace it self. In order to determine a line's trace, it is enough to know one point of the line and the angle made by the line with the projection plane, because, by adopting as a projecting direction the line it self, the projection of the known point will coincide to the trace it self, as in figure 5. [3]

When the angle is not known, two points of the line are required, the value of the angle being determined through the relation:

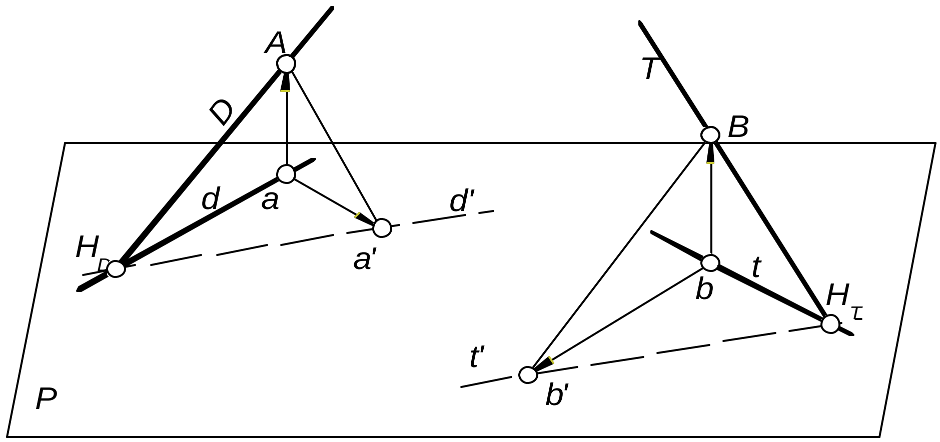

Fig. 5

$$ctg\varphi = \frac{\overline{ab}}{Z_A - Z_B}$$

Two random lines each projected according to a direction that is parallel to the other one, figure 5, have their final projections parallel one to another, and because the planes form by each of the lines and the projecting line issued from the point on the line are parallel. $\overline{D} // \overline{Bb'}$ and $\overline{Aa'} // \overline{I}$, while the intersection of these planes with the projection plane are the final projections of the lines themselves. A line situated in a random plane (figure 6), will have its trace situated on the plane's trace, no matter according to which direction it would be projected.

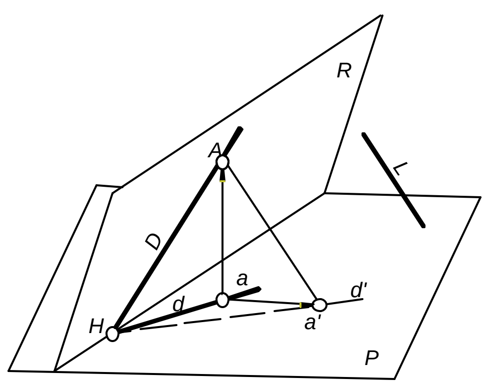

Fig. 6

A line which intersects a plane (figure 7), projected following a direction which is parallel to that plane, will have the final projection of the intersecting point situated at the meeting point of the line's final projection and the plane's trace. [4] Relying on the proprieties shown beyond, the polygonal outline of intersection for two prisms can be determined. [5],[6]

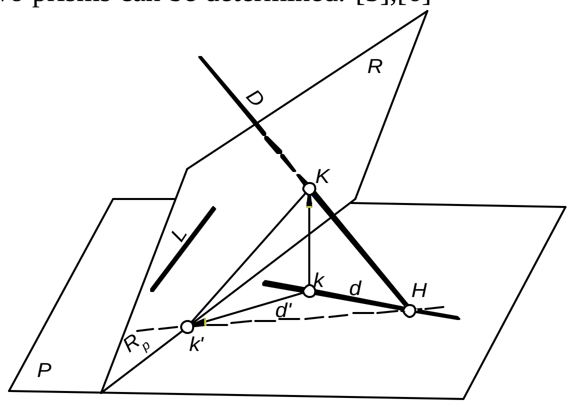

Fig. 7

Let us consider two triangular prisms at random, the edges of which are defined by the points: $\overset{\circ}{A}(170,20,20)$, $\overset{\circ}{B}(170,95,85)$, $\overset{\circ}{C}(170,110,45)$ and $\overset{\circ}{A}_1(145,35,45)$, situated on the edge $\overset{\circ}{A}$, respectively $\overset{\circ}{M}(70,20,10)$, $\overset{\circ}{N}(70,95,85)$, $\overset{\circ}{P}(70,130,65)$ and $\overset{\circ}{M}_1(85,40,40)$ situated on the edge $\overset{\circ}{M}$. In principle the problem should be viewed as the intersection between a line and a plane, determining the point where the edges of each prism intersect the faces of other prism, these point being the vertexes of the spatial polygon (or of the polygons) upon which the two prisms are intersecting.

The given prism are presented on a projection plane, following the method of the parallel vectors' projection; in the case we considered the prism were represented on the horizontal projection plane (figure 8). The initial projections of the prisms edges are determined by projecting orthogonally the positions which define them ($\overset{\circ}{A}$, $\overset{\circ}{B}$, $\overset{\circ}{C}$, $\overset{\circ}{A}_1$, respectively $\overset{\circ}{M}$, $\overset{\circ}{N}$, $\overset{\circ}{P}$, $\overset{\circ}{M}_1$). The final projections of these edges are obtained by projecting them upon two directions: $\overline{L_{A_1}}$, parallel to the edges of the prism ABC and $\overline{L_{M_1}}$, parallel to the edges of the prism MNP. By projecting upon the direction $\overline{L_A}$, the point where the edges of MNP strike the faces of $ABCA_1B_1C_1$, are determined, respectively upon $\overline{L_M}$ the points where the edges of $ABCA_1B_1C_1$ strike the faces of $MNP_1N_1P_1$. The final projection of only one edge, while for the other edges only the final projection of a point belonging

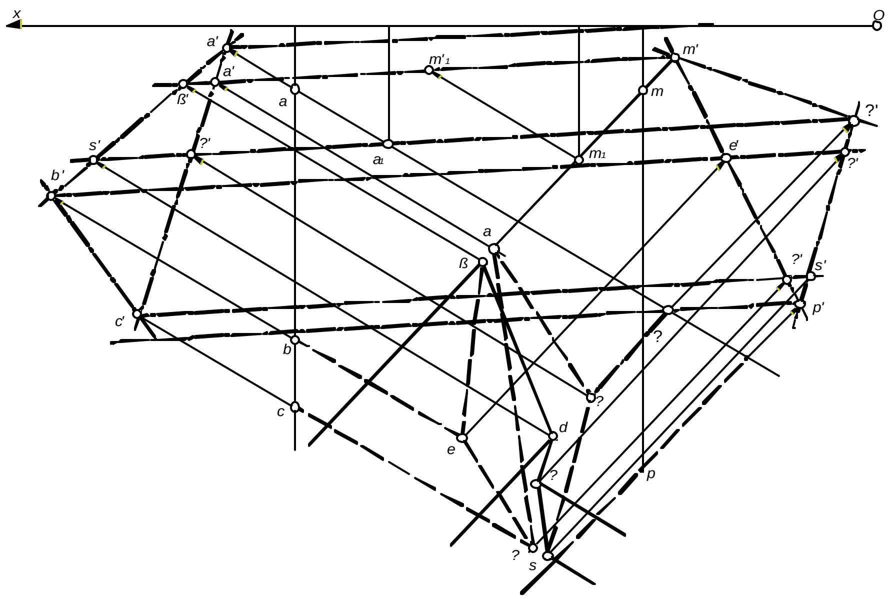

Fig. 8

to each of them will be required, through which the final projection of the edges will be drawn. In the chosen projection of the edges MM_1 was determined, by drawing the final projections of the point M_1 and of the edge's trace. The projection of M_1's vector, projected suiting $\overline{L_A}$, is parallel to the initial projections of the edges of the prism $ABCA_1B_1C_1$ and has the value:

$$\overline{mm_1} = \overline{Z_{M_1}} \cdot ctg\, j_A$$

$$ctg\, j_A = \frac{\overline{a_1 a}}{\overline{Z_{A_1}} - \overline{Z_A}}$$

the trace of the edge $\overline{MM_1}$ was determined by projecting the vector of point M suiting $\overline{L_M}$. The projection of this vector coincides to the initial projection of $\overline{MM_1}$ and has the value:

$$\overline{mm_1} = \overline{Z_M} \cdot ctg\, j_M$$

$$ctg\, j_M = \frac{\overline{m_1 m}}{\overline{Z_{M_1}} - \overline{Z_M}}$$

By connecting the final projection of M_1 suiting $\overline{L_A}$ to the final projection of M suiting $\overline{L_M}$, which confounds it self with the trace of the edge $\overline{MM_1}$, the final projection of the edge could be determined. The traces of the edge could be determined either by calculating or by using the graph from figure 9. The graph was constructed for the two projecting directions, $\overline{L_A}$ and $\overline{L_M}$, previously knowing $\overline{a_1 a}$ and $\overline{Z_{A_1}} - \overline{Z_A}$, respectively $\overline{m_1 m}$ and $\overline{Z_{M_1}} - \overline{Z_M}$. So, it is enough to know the Z quotation of any point projected suiting one of the lines $\overline{L_A}$ or $\overline{L_M}$. By transposing it to the graph's axis, we would be able to read direct by the value of the projection owned by the vector of the respective point. By connecting the traces of the edges of each prism into their natural order, the traces of the planes where the face of the prism lag were determined. The point where the edge $\overline{MM_1}$ strikes the faces of the prism ABC have their final projection situated at the meeting points of the final projection of this edge with the trace of the planes where lay the faces of the prism ABC, respectively the projections a' and ß'.

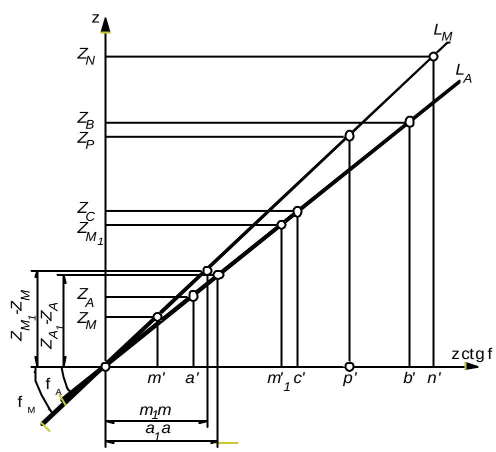

Fig. 9

The initial projection a and ß of these point are to be found at the intersection between the initial projection of the edge $\overline{MM_1}$ and parallel drawn to the initial projection of the edges of the prism ABC passing trough the final projections a' and ß'. The segments $\overline{aa'}$ and $\overline{bb'}$ are the projections of the vectors of the vectors of the respective point. The value of the vectors of these points (the quotations of these points in respect to the projection plane) is determined still by the graph from figure 9. In a similar way will be determined all the point where the edges of each prism strike the faces of the other prism and which, connected in their natural order, determine the polygon (or the polygons) of intersection. The connection order of the point could be established by applying the method of the mobile or either the method of the common point of the pairs of faces of the prism, as in Table 1.

Table 1

Prisma ABC / MNP			AB	BC	CA
			vizibila	*vizibila*	*vizibila*
			e,?,ß,d	*e,?,?,s*	*?,s,a,?*
PM	*acoperita*	*e,?,ß,a*	*e-ß*	*e-?*	?-a
			acoperita	*acoperita*	*acoperita*
NP	*vizibila*	*?,s,d,?*	*?-d*	?-s	s-?
			vizibila	*vizibila*	*acoperita*
MN	*vizibila*	*ß,a,d,?*	*ß-d*	-	a-?
			vizibila	-	*acoperita*
Poligonul e - ß - d - ? - s - ? - a - ? - e					

In order to construct the prisms' unfolding, each prism should be sectioned by a plane that would be perpendicular on the edges. Figure 1 represents a straight line $\overline{D}$ and a plane [R] normal to that line.

By projecting upon a direction $\overline{L}$ situated in the plane [R], the final projection of the given straight line finds the trace of the plane [R] in a point A where the line $\overline{D}$ strikes the plane [R]. The initial projection of the line is normal to the trace of the plane. The distance $\overline{aI}$ from the initial projection of A to the trace of the plane could be calculated through the relation:

$$aI = Z_A \cdot ctg\left(\frac{p}{2} - j\right)$$

or graphically by the unfolding of the rectangular triangle AaI on the plane [P]. in order to draw the unfolded triangle A_0aI, the segment $\overline{aA_0}$ is drawn, normal to the initial projection of the line, and from A_0 is drawn the line $\overline{A_0I}$, under the angle f .

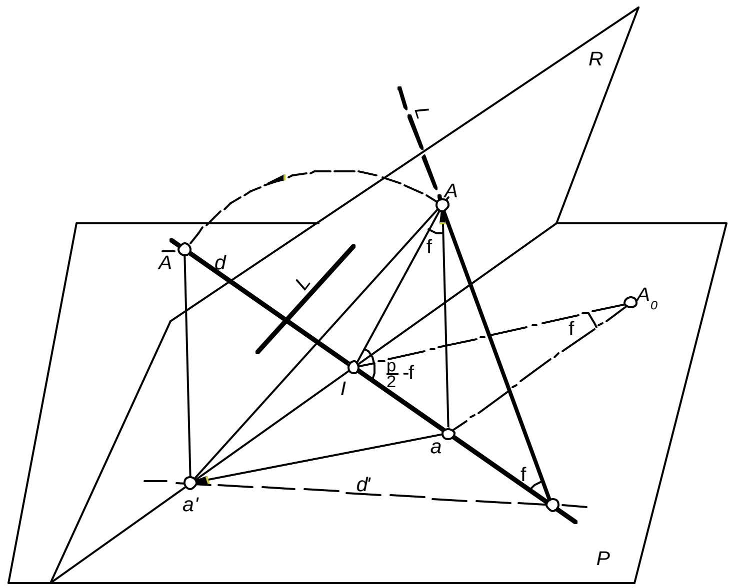

Fig. 10

In order to situate by unfolding it, the point A of the plane [P], together with the plane [R], the unfolding radius AI is required, which is the hypotenuse of the triangle AaI and which could be determined graphically from the triangle A_0I or by calculation, through the relation:

$$AI = Z_A \cdot \frac{1}{\cos j}$$

The value of the unfolding radius could be obtained by using the projecting direction $\overline{L}$, inclined at 45^0 towards the trace of the plane, a case into which $\overline{a}\overline{I} = \overline{a'I}$. After the unfolding, the unfolding radius coincides to the initial projection of the line (the segment IA). [7]

The size of a line segment situated in the plane [R] is determined by the unfolding of the plane [R], a similar procedure as for point A.

These remarks allow us to construct the unfoldings of the two prisms, ABC and MNP. [8]

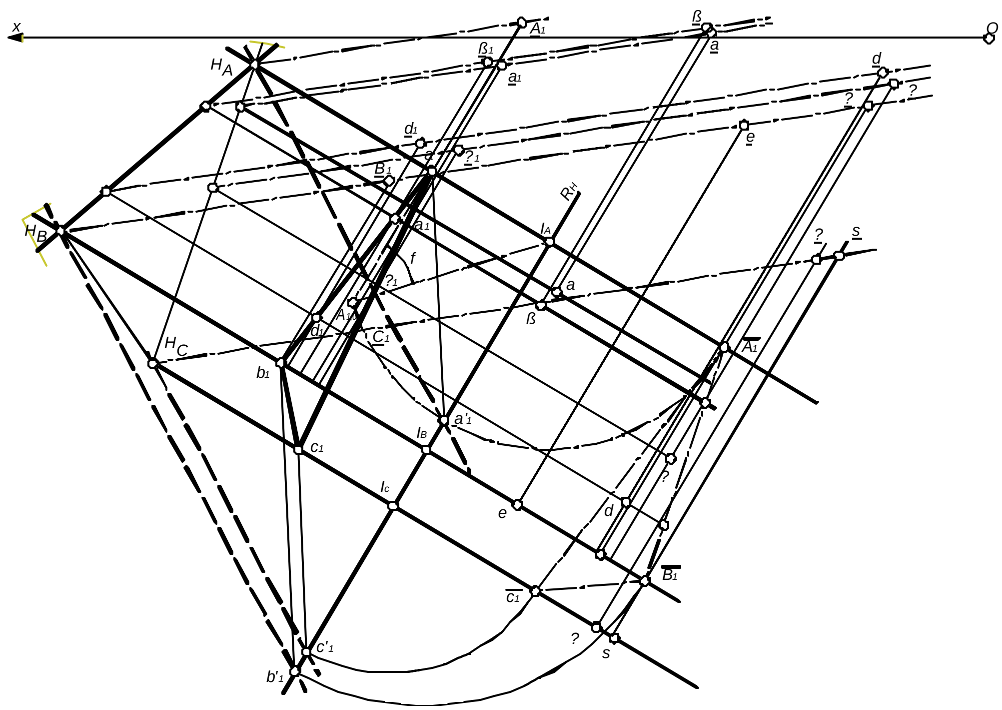

Fig. 11

In Figure 11 were represented the prism ABC and the points of the sections' polygon. The prism is sectioned by the plane [R] normal to its edges and containing the point A_1. The trace of this plane - $\overline{R_H}$ - is perpendicular to the projections of the edges and is situated initial at a distance $\overline{a_1 I_A}$ from the initial projection of point A_1, in the sense opposed to the line's trace (figure11). This distance is determined graphically, by drawing the rectangular triangle $a_1A_{10}I_A$. The segment I_AA_{10} is the unfolding radius which determines the unfolded position of A_1. The projecting direction is situated in the plane [R], under an angle of 45^0 towards the trace of this plane, so the final projection of A_1 is situated on the plane's trice, at the distance $\overline{I_A a_1'} = \overline{I_A A_{10}}$. The final projection of the AA_1 passes through a_1' and H_A, while the projection of the vector A_1 is a_1a_1'. The other edges have their final projection parallel to the final projection of the edge AA_1. The striking points between the prism's edges and the plane[R], have their final projections b_1' and c_1' at the intersection sites between the final projections of the edges and the trace of the plane [R]. The initial projections, b_1 and c_1, of these point, are sited at the intersection points between the initial projections of the edges and parallels drawn to the projection of the vector of A_1, through their final

projections, b_1' and c_1'. The segments $\overline{I_B b_1}'$ and $\overline{I_C c_1}'$ are equal to the folding radii of points B_1 and C_1, and they determine the unfolded position of B_1 and C_1.
The triangle $a_1b_1c_1$ constitutes the initial projection of the perpendicular section, while the triangle $A_1B_1C_1$represents the surface of this section. On the sides of this triangle are also situated the striking points between the plane [R] and the lines, parallel to the prism's edges, that pass trough the vertexes of the polygon representing the section. The sizes of the segments from the edges, in respect to the normal section, are determined by un folding, the edges and the lines on the plane [H], around their initial projection. The unfolded positions are determined by the trace of each edge end by a point from each respective unfolded edge. The unfolding radius of a point is its vector itself. In the chosen example, it was the point A_1 that was unfolded. The other edges and lines have their unfolded position parallel to the unfolded position parallel to the unfolded position of edge AA_1, and will pass through their traces. To determine the position of the vertexes of the intersection' polygon, as well as ones of the points of the perpendicular section, on the unfolded edges and lines was drawn perpendiculars on the initial projections of the edges from the initial projections points.
Once these elements determined, the unfolding of the prism ABC was constructed (figure 12).

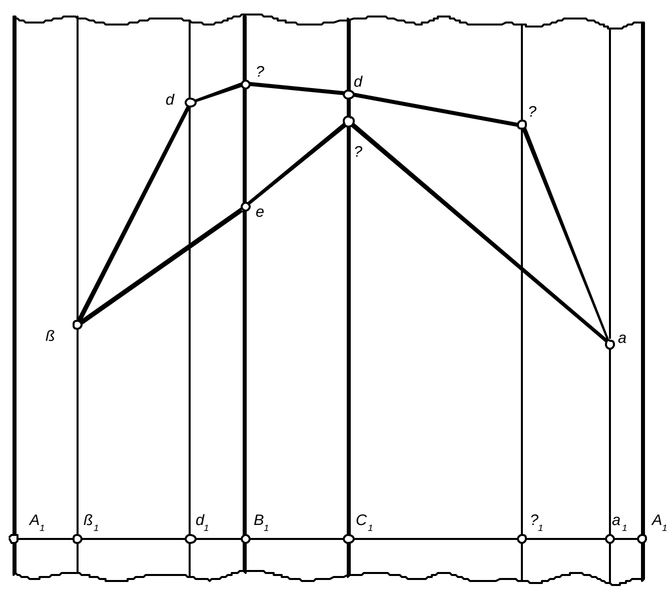

Fig. 12

By a similar procedure, the unfolding of the prism MNP is also constructed.

Conclusions

This method, by the use of the projection on a one and only plane, needs a simplified graphical construction, if compared to the method of the orthogonal projection on two projecting planes. This fact will lead to the decrease of the number of possible errors in the graphical representations.

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