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THE FOURIER IMAGES OF SOME HOMOGENOUS DISTRIBUTIONS WITH SINGULARITIES IN $D'(R^2)$ AND $D'(R^3)$ WITH APPLICATIONS IN ELASTICITY

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In aceasta lucrare se determină imaginile Fourier ale unor distribuții omogene cu singularități, din $D'(R^2)$ si $D'(R^3)$. Se arată utilitatea formulelor stabilite în studiul vibrațiilor barelor elastice precum şi la determinarea soluției generalizate a problemei planului elastic în tensiuni şi a problemei spațiului elastic în deplasări.

In this paper the Fourier Images of some homogenous distributions with singularities in $D'(R^2)$ and $D'(R^3)$ are obtained. These are useful in the study of the vibrations of the elastic rods and thin elastic plans as well as in obtaining the generalised solutions of the problems in strains of the elastic plane and of the problems in displacements of the elastic space.

Keywords: elasticity, The Fourier transforms, distributions.

1. Introduction

The distributions generated by homogenous functions are useful in the study of the problems of the elastic plane as well as in the study of the problems of the elastic space. The way of computing the derivates of such distributions as well as their Fourier Images are very important.

According to [3], [5], if $\varphi \in D(R^n)$ is a test – function from the space of infinite differentiable functions with compact supports D, and $f \in D'(R^n)$ is a distributions, then its Fourier Transform $\widehat{f}(\xi) = F[f](\xi)$ is defined by the relation: $(\widehat{f}, \widehat{\varphi}) = (2\pi)^n (f, \varphi)$, where $\widehat{\varphi}(\xi) = F[\varphi](\xi) = \int_{R^n} \varphi(x) \exp(i(\xi, x)) dx$, $(\xi, x) = \xi_1 x_1 + ... + \xi_n x_n$ the inverse Fourier Transform F^{-1} acts by the formula: $(F^{-1}(\widehat{f}), \varphi) = \frac{1}{(2\pi)^n} (\widehat{f}, \widehat{\varphi})$.

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The following formulas hold: $F[\delta](\xi)=1$, $F[1](\xi)=(2\pi)^n\delta(\xi)$, $(F^{-1}[F[f]]=f$, $F[F^{-1}[\widehat{f}]]=\widehat{f}$, $f\in D'(R^n)$, where $\delta\in D'(R^n)$ represents the Dirac distribution concentrated at the origin.

2. General Results

We use distributions generated by the homogenous functions in the study of the static problems of the elastic plane, of the elastic half plan as well as of the elastic space.

Taking into account [5], [3] we have the following result:

Proposition 2.1 Let $f:R^n\setminus\{0\}\to C$ be o homogenous functions of λ degree having (x=0), the singularity. Then its derivative in the distributions sense is computed by the formula:

$$\frac{\partial f}{\partial x_i}=\begin{cases}\dfrac{\widetilde{\partial} f}{\partial x_i} & ,\lambda>-n+1\\ \dfrac{\widetilde{\partial} f}{\partial x_i}+\partial(x)(rezf)(0) & ,\lambda=-n+1\end{cases} \tag{2.1}$$

Where
$$(rezf)(0)=\int_{S_1} f(x)\cos\alpha_i dS_1 \tag{2.2}$$

represents the residuum of the function f in the singularity x=0 corresponding to Ox_i axes.

The symbol $\dfrac{\widetilde{\partial}}{\partial x_i}$ represents the derivative in the common sense, S_1 represents the unit sphere centrated at the origin, dS_1 the area element and α_i the angle between the exterior norm at S_1 and Ox_i axes. From [5] we have:

$$\Delta\ln(x^2+y^2)=4\pi\delta(x,y)\ \ \Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2} \tag{2.3}$$

Applying Fourier Transform in $D'(R^2)$ to [2.3] we obtained:

$$F[\ln(x^2+y^2)](\xi_1,\xi_2)=-\frac{4\pi}{\xi_1^{\ 2}+\xi_2^{\ 2}}\ ,\ (\xi_1,\xi_2)\in R^2 \tag{2.4}$$

$$F[\frac{\partial}{\partial x}\ln(x^2+y^2)](\xi_1,\xi_2)=(-i\xi_1)F[\ln(x^2+y^2)](\xi_1,\xi_2)=4\pi i\frac{\xi_1}{\xi_1^{\ 2}+\xi_2^{\ 2}}$$

Which means that
$$F[\frac{x}{x^2+y^2}](\xi_1,\xi_2)=2\pi i\frac{\xi_1}{\xi_1^{\ 2}+\xi_2^{\ 2}} \tag{2.5}$$

Analogue we have: $F[\frac{y}{x^2+y^2}](\xi_1,\xi_2)=2\pi i\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}}$ (2.6)

Taking into account (2.6) we have:

$$F[(ix)\frac{y}{x^2+y^2}](\xi_1,\xi_2)=2\pi i\frac{\partial}{\partial\xi_1}(\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}}) \quad (2.7)$$

We remark that the function $\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}}$, $(\xi_1,\xi_2)\in R^2\setminus\{(0,0)\}$ is a homogenous function with the degree λ=-1=-n+1, n=2, where $\xi_1=\xi_2=0$ is the discontinuity point.

Appling [2.1] $\frac{\partial}{\partial\xi_1}(\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}})=\frac{\widetilde{\partial}}{\partial\xi_1}(\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}})+\delta(\xi_1,\xi_2)\int_{C_1}\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}}\cos\theta ds_1$ (2.8)

Where C_1 represents the unit circle centrated at the origin, $ds_1=d\theta$, $\cos\theta=\xi_1,\sin\theta=\xi_2$

Since $\int_{C_1}\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}}\cos\theta ds_1=\int_0^{2\pi}\cos\theta\sin\theta d\theta=0$, (2.8) becomes:

$$\frac{\partial}{\partial\xi_1}(\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}})=\frac{\widetilde{\partial}}{\partial\xi_1}(\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}})=-2\frac{\xi_1\xi_2}{(\xi_1^{\ 2}+\xi_2^{\ 2})^2}$$

Consequently from (2.7) we have: $F[\frac{xy}{x^2+y^2}](\xi_1,\xi_2)=-4\pi\frac{\xi_1\xi_2}{(\xi_1^{\ 2}+\xi_2^{\ 2})^2}$ (2.9)

The following relations hold:

$$F[\frac{y^2}{x^2+y^2}](\xi_1,\xi_2)=2\pi(\frac{\xi_1^{\ 2}-\xi_2^{\ 2}}{(\xi_1^{\ 2}+\xi_2^{\ 2})^2}+\pi\delta(\xi_1,\xi_2)) \quad (2.10)$$

$$F[\frac{x^2}{x^2+y^2}](\xi_1,\xi_2)=2\pi(\frac{\xi_2^{\ 2}-\xi_1^{\ 2}}{(\xi_1^{\ 2}+\xi_2^{\ 2})^2}+\pi\delta(\xi_1,\xi_2)) \quad (2.11)$$

From [2.6] we have:

$$F[(iy)\frac{y}{x^2+y^2}](\xi_1,\xi_2)=\frac{\partial}{\partial\xi_2}(2\pi i\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}})=2\pi i\frac{\partial}{\partial\xi_2}(\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}}) \quad (2.12)$$

Using (2.1) $\frac{\partial}{\partial\xi_2}(\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}})=\frac{\widetilde{\partial}}{\partial\xi_2}(\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}})+\delta(\xi_1,\xi_2)\int_{C_1}\frac{\xi_2}{\xi_1^{\ 2}+\xi_2^{\ 2}}\sin\theta ds_1=$

$$=\frac{\xi_1^{\,2}-\xi_2^{\,2}}{(\xi_1^{\,2}+\xi_2^{\,2})^2}+\delta(\xi_1,\xi_2)\int_0^{2\pi}\sin^2\theta d\theta=\frac{\xi_1^{\,2}-\xi_2^{\,2}}{(\xi_1^{\,2}+\xi_2^{\,2})^2}+\pi\delta(\xi_1,\xi_2)$$

Substituting in (2.12) we obtaining the formula (2.4)

Similarly

$$F[(ix)\frac{x}{x^2+y^2}](\xi_1,\xi_2)=\frac{\partial}{\partial\xi_1}F[\frac{x}{x^2+y^2}](\xi_1,\xi_2)=2\pi i\frac{\partial}{\partial\xi_2}(\frac{\xi_1}{\xi_1^{\,2}+\xi_2^{\,2}}) \quad (2.13)$$

From (2.10) and (2.11) we obtained

$$F[\frac{x^2}{x^2+y^2}+\frac{y^2}{x^2+y^2}](\xi_1,\xi_2)=F[1](\xi_1,\xi_2)=(2\pi)^2\delta(\xi_1,\xi_2)$$

Next we establish the formulas:

$$F[\frac{x(3y^2+x^2)}{(x^2+y^2)^2}](\xi_1,\xi_2)=F[\frac{x}{x^2+y^2}+2\frac{xy^2}{(x^2+y^2)^2}](\xi_1,\xi_2)=4\pi i(\frac{\xi_1^{\,3}}{(\xi_1^{\,2}+\xi_2^{\,2})^2}) \quad (2.14)$$

$$F[\frac{y(y^2-x^2)}{(x^2+y^2)^2}](\xi_1,\xi_2)=F[\frac{y}{x^2+y^2}-2\frac{yx^2}{(x^2+y^2)^2}](\xi_1,\xi_2)=4\pi i(\frac{\xi_1^{\,2}\xi_2}{(\xi_1^{\,2}+\xi_2^{\,2})^2}) \quad (2.15)$$

$$F[\frac{x(x^2-y^2)}{(x^2+y^2)^2}](\xi_1,\xi_2)=F[\frac{x}{x^2+y^2}-2\frac{xy^2}{(x^2+y^2)^2}](\xi_1,\xi_2)=4\pi i(\frac{\xi_2^{\,2}\xi_1}{(\xi_1^{\,2}+\xi_2^{\,2})^2}) \quad (2.16)$$

$$F[\frac{y(3x^2+y^2)}{(x^2+y^2)^2}](\xi_1,\xi_2)=F[\frac{y}{x^2+y^2}+2\frac{x^2y}{(x^2+y^2)^2}](\xi_1,\xi_2)=4\pi i(\frac{\xi_2^{\,3}}{(\xi_1^{\,2}+\xi_2^{\,2})^2}) \quad (2.17)$$

Proof:

Using the inverse Fourier Transform F^{-1} in the distributions space $D'(R^2)$ we have:

$$F^{-1}[\frac{\xi_1^{\,3}}{(\xi_1^{\,2}+\xi_2^{\,2})^2}](x,y)=F^{-1}[\frac{\xi_1(\xi_1^{\,2}+\xi_2^{\,2})-\xi_1\xi_2^{\,2}}{(\xi_1^{\,2}+\xi_2^{\,2})^2}](x,y)=$$

$$=F^{-1}[\frac{\xi_1}{\xi_1^{\,2}+\xi_2^{\,2}}](x,y)+\frac{1}{i}F^{-1}[(-\xi_2 i)\frac{\xi_1\xi_2}{(\xi_1^{\,2}+\xi_2^{\,2})^2}](x,y)$$

Taking into account (2.5) we obtained:

$$F^{-1}[\frac{\xi_1^{\,3}}{(\xi_1^{\,2}+\xi_2^{\,2})^2}](x,y)=\frac{1}{2\pi i}\frac{x}{x^2+y^2}-\frac{1}{4\pi i}\frac{\partial}{\partial y}(\frac{xy}{x^2+y^2}) \quad (2.18)$$

Since the function $\frac{xy}{x^2+y^2}$, $(x,y)\in R^2\setminus\{(0,0)\}$ is a homogenous function with the degree λ=0>-n+1, n=2, based on (2.1) we have

$$\frac{\partial}{\partial y}\frac{xy}{x^2+y^2}=\frac{\tilde{\partial}}{\partial y}\frac{xy}{x^2+y^2}=\frac{x(x^2-y^2)}{(x^2+y^2)^2}=\frac{x}{x^2+y^2}-2\frac{xy^2}{(x^2+y^2)^2}$$

Thus, (2.18) becomes:

$$F^{-1}[\frac{\xi_1{}^3}{(\xi_1{}^2+\xi_2{}^2)^2}](x,y)=\frac{1}{4\pi i}\frac{x(3y^2+x^2)}{(x^2+y^2)^2}$$

In the same manner we obtain

$$F^{-1}[\frac{\xi_1{}^2\xi_2}{(\xi_1{}^2+\xi_2{}^2)^2}](x,y)=\frac{1}{4\pi i}\frac{\tilde{\partial}}{\partial y}(\frac{xy}{x^2+y^2}).$$

Consequently we obtain

$$F^{-1}[\frac{\xi_1{}^2\xi_2}{(\xi_1{}^2+\xi_2{}^2)^2}](x,y)=\frac{1}{4\pi i}\frac{y(y^2-x^2)}{x^2+y^2}$$

Also

$$F^{-1}[\frac{\xi_1{}^2\xi_2}{(\xi_1{}^2+\xi_2{}^2)^2}](x,y)=-\frac{1}{i}F^{-1}[(-i\xi_2)\frac{\xi_1\xi_2}{(\xi_1{}^2+\xi_2{}^2)^2}](x,y)=$$

$$=-\frac{1}{i}F^{-1}\left\{F[\frac{\partial}{\partial y}(-\frac{1}{4\pi}\frac{xy}{x^2+y^2})]\right\}(x,y)=$$

$$=\frac{1}{4\pi i}\frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

Based on (2.15) we have:

$$F^{-1}[\frac{\xi_2{}^3}{(\xi_1{}^2+\xi_2{}^2)^2}](x,y)=F^{-1}[\frac{\xi_2}{\xi_1{}^2+\xi_2{}^2}](x,y)-F^{-1}[\frac{\xi_2\xi_1{}^2}{(\xi_1{}^2+\xi_2{}^2)^2}](x,y)=$$

$$=\frac{1}{4\pi i}(\frac{y}{x^2+y^2}+2\frac{x^2y}{(x^2+y^2)^2})=\frac{1}{4\pi i}\frac{y(3x^2+y^2)}{(x^2+y^2)^2}$$

We remark that the Fourier Images permit the determination of the generalised solution in $D'(R^2)$ of the problem of infinite elastic plan.

3. The problem in strains of the infinite elastic plane

Let $X(x,y), Y(x,y) \in D'(R^2)$ be the components of the mass forces which act on the homogeneous elastic plane, written in distributions from $D'(R^2)$.

We denote by $(\sigma) = \begin{pmatrix} \sigma_{xx}(x,y) & \sigma_{xy}(x,y) \\ \sigma_{yx}(x,y) & \sigma_{yy}(x,y) \end{pmatrix} \in (D'(R^2))^{2\times 2}$ the symmetrical tensor of strain of second degree $\sigma_{xy}(x,y) = \sigma_{yx}(x,y)$.

The problem in strains of the elastic plane in distributions consists in finding the tensor $(\sigma) \in (D'(R^2))^{2\times 2}$ that satisfies the balance equation:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + X = 0, \quad \frac{\partial \sigma_{xyx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + Y = 0 \tag{2.19}$$

as well as the compatibility equation:

$$\Delta(\sigma_{xx} + \sigma_{yy}) = -\frac{1}{1-\mu}\left(\frac{\partial X}{\partial x} + \frac{\partial Y}{\partial y}\right), \quad \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \tag{2.20}$$

where $\mu \in (0,1)$ represents the Poissons coefficient.

The solution of this problem has the following form:

$$\sigma_{xx}(x,y) = u_1(x,y) * X(x,y) + u_2(x,y) * Y(x,y) \tag{2.21}$$

$$\sigma_{xy}(x,y) = v_1(x,y) * X(x,y) + v_2(x,y) * Y(x,y)$$

$$\sigma_{yy}(x,y) = w_1(x,y) * X(x,y) + w_2(x,y) * Y(x,y)$$

where "$*$" represents the convolution product in $D'(R^2)$ with respect to the variables $(x,y) \in R^2$.

In order to determine $u_i, v_i, w_i \in D'(R^2), i = 1,2,3,$ we consider the cases:

1. $X = \delta(x,y), \quad Y = 0$
2. $X = 0, \quad Y = \delta(x,y)$ where $\delta(x,y) \in D'(R^2)$ represents the Dirac distribution concentrated at the origin of 0xy axes.

Applying the Fourier Transform in $D'(R^2)$ to (2.19) and (2.20) we determine.

$$u_1(x,y) = -\frac{1}{4\pi}\left[\frac{2x}{x^2+y^2} + \frac{1}{1-\mu}\frac{x(x^2-y^2)}{(x^2+y^2)^2}\right]$$

$$v_1(x,y)=\frac{1}{4\pi}\left[\frac{-2y}{x^2+y^2}+\frac{1}{1-\mu}\frac{y(y^2-x^2)}{(x^2+y^2)^2}\right]$$
$$w_1(x,y)=\frac{1}{4\pi}\left[\frac{2x}{x^2+y^2}-\frac{1}{1-\mu}\frac{x(3y^2+x^2)}{(x^2+y^2)^2}\right]$$

$$u_2(x,y)=\frac{1}{4\pi}\left[\frac{2y}{x^2+y^2}-\frac{1}{1-\mu}\frac{y(3x^2+y^2))}{(x^2+y^2)^2}\right] \tag{2.22}$$

$$v_2(x,y)=\frac{1}{4\pi}\left[-\frac{2x}{x^2+y^2}+\frac{1}{1-\mu}\frac{x(x^2-y^2))}{(x^2+y^2)^2}\right]$$
$$w_2(x,y)=-\frac{1}{4\pi}\left[\frac{2y}{x^2+y^2}+\frac{1}{1-\mu}\frac{y(y^2-x^2))}{(x^2+y^2)^2}\right]$$

For the existence of the convolution product, we consider $X,Y\in E'(R^2)$ distributions with compact support.

4. The Fourier Images of some homogenous distributions with singularities in $D'(R^3)$

Let $\Delta=\frac{\partial^2}{\partial x^2{}_1}+\frac{\partial^2}{\partial x_2{}^2}+\frac{\partial^2}{\partial x^2{}_3}$ be the Laplace operator from $D'(R^3)$.

Then from [3], [5], the following relation holds:

$$\Delta\frac{1}{r}=-4\pi\delta(x),\quad r=\|x\| \quad ,x\in R^3 \tag{2.23}$$

Let us consider the functions:

$$f_{j,k}(x)=\frac{x_j x_k}{r^2},\quad j,k=1,2,3,\quad r\neq 0 \tag{2.24}$$

$$g_j(x)=\frac{x_j}{r},\quad h(x)=\frac{1}{r}$$

These functions are homogeneous with degrees 0,0,-1 respectively, and $r=0$ singularity.

One can use (2.1) in order to compute the derivatives of these functions and we will compute the Fourier Images of (2.24).

Thus we have:

$$F\left[\frac{1}{r}\right](\xi)=\frac{4\pi}{\|\xi\|^2},\quad \xi=(\xi_1,\xi_2,\xi_3)\in R^3 \tag{2.25}$$

$$F\left[\frac{x_jx_k}{r^3}\right](\xi)=-\frac{8\pi\xi_j\xi_k}{\|\xi\|^4},\quad j,k=1,2,3,\quad j\neq k \tag{2.26}$$

$$F\left[\frac{1}{r}\right](\xi)-F\left[\frac{{x_j}^2}{r^3}\right](\xi)=\frac{8\pi\xi^2}{\|\xi\|^4}, \tag{2.27}$$

$$F\left[\frac{x_j}{r}\right](\xi)=\frac{8\pi i\xi_j}{\|\xi\|^4},\quad j=1,2,3 \tag{2.28}$$

Proof:

Applying to (2.23) the Fourier Transform with respect to the variables $(x_1,x_2,x_3)\in R^3$, we obtain:

$F\left[\Delta\frac{1}{r}\right](\xi)=-4\pi,$ thus $-\|\xi\|^2F\left[\frac{1}{r}\right](\xi)=-4\pi$, it means (2.25).

From (2.25) results:

$\frac{\partial}{\partial x_j}F\left[\frac{1}{r}\right](\xi)=iF\left[\frac{x_j}{r}\right](\xi),$ and $-8\pi\frac{\xi_j}{\|\xi\|^4}=iF\left[\frac{x_j}{r}\right](\xi)$, then (2.28).

In order to prove (2.26), we have:

$$F\left[\frac{\partial}{\partial x_k}(\frac{x_j}{r})\right](\xi)=(-i\xi_k)F\left[\frac{x_j}{r}\right](\xi) \tag{2.29}$$

Since degree of $(\frac{x_j}{r})$>-3+1, from (2.1) we have:

$$\frac{\partial}{\partial x_k}(\frac{x_j}{r})=\frac{\tilde{\partial}}{\partial x_k}(\frac{x_j}{r})=-\frac{x_jx_k}{r^3},\quad j\neq k.$$

Using (2.29) and (2.28) we obtain (2.26).

Similarly we have

$F\left[\frac{\partial}{\partial x_j}(\frac{x_j}{r})\right](\xi)=F\left[\frac{\tilde{\partial}}{\partial x_j}(\frac{x_j}{r})\right](\xi)(-i\xi_j)F\left[\frac{x_j}{r}\right](\xi)$ and

$F\left[\frac{1}{r}\right](\xi) - F\left[\frac{x_j^{\ 2}}{r^3}\right](\xi) = (-i\xi_j)\frac{8\pi i\xi_j}{\|\xi\|^4} = \frac{8\pi\xi_j^{\ 2}}{\|\xi\|^4}$ which means that

$F\left[\frac{x_j^{\ 2}}{r^3}\right](\xi) = \frac{4\pi}{\|\xi\|^4}\left(\|\xi\|^2 - 2\xi_j^{\ 2}\right)$

5. The generalised solution of the problem in displacements of elastic space

Let us consider $u_i(x), X_i(x) \in D'(R^3), \quad i = 1,2,3, \quad x \in R^3$, the components of the displacements and the projections of the volume forces that act on the homogeneous elastic space.

The problem of the elastic space on the action of the volume forces $X_i(x) \in D'(R^3)$, consists in the determination of the distributions $u_i(x) \in D'(R^3)$, that should be the solutions of the Lame's equations:

$$\mu\Delta u_i + (\lambda + \mu)\frac{\partial e}{\partial x_i} + X_i = 0, \quad i = 1,2,3 \qquad (2.30)$$

And satisfies the conditions of regularity: $\lim_{\|x\|\to\infty} u_i(x) = 0 \quad (i = 1,2,3)$ (2.31)

In (2.30), $e = \sum_{i=1}^{3}\frac{\partial u_i}{\partial x_i}$, represents net volume strain, λ and μ are the Lame's elastic constants, and Δ represents Laplace's operator.

The solution of the problem [6] can be written in the following manner:

$$u_i(x) = E_{i1}(x) * X_1(x) + E_{i2}(x) * X_2(x) + E_{i3}(x) * X_3(x), \quad i = 1,2,3 \qquad (2.32)$$

where "*"represents the convolution product with respect to the variables $x = (x_1, x_2, x_3)$.

In order to determine the distributions $E_{ij} \in D'(R^3), \quad i, j = 1,2,3$ we consider the following cases:

1. $X_1 = \delta(x), \quad X_2 = X_3 = 0$
2. $X_2 = \delta(x), \quad X_1 = X_3 = 0$
3. $X_3 = \delta(x), \quad X_2 = X_1 = 0$

Applying the Fourier Transform to (2.30) and (2.32) we determine the Fourier Images $\hat{E}_{ij} = F[E_{ij}](\xi)$.

Applying the inverse Fourier Transform F^{-1} and taking into account the established Fourier Images already established, we obtain:

$$E_{ij}(x) = \frac{\partial_{ij}}{4\pi\mu}\frac{1}{r} + \frac{1}{8\pi\mu(\lambda+2\mu)}\frac{x_i x_j}{r^3} \quad (i, j = 1,2,3) \tag{2.33}$$

where $\delta_{ij} = \begin{cases} 0, & i \neq j \\ 1, & i = j \end{cases}$ represents the Kronecker's symbol.

Thus, the solution in displacements (2.32) of the problem of the elastic space is determined.

For the existence of the convolution product from (2.32), the volume forces X_i are distributions with compact supports, which means $X_i \in \mathrm{E}'(R^3)$.

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