TWO LIQUID TANKS CONTROL

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Acest articol prezintă modul de conducere a unor rezervoare cu lichid, din punctul de vedere al controlului automat; motivul studiului este necesitatea unei strategii de conducere automată, pentru două rezervoare cu lichid. Problema tratată în acest articol este controlul nivelului de lichid în fiecare rezervor, cu ajutorul unui regulator PD [2].

Intâlnim o problema foarte importantă, dată de perturbația realizată din intersectarea celor două rezervoare, şi scurgerea lichidului din cel de-al doilea rezervor; acestă perturbație are un efect negativ în sistemul nostru. Folosim pentru ambele rezervoare două valve care sunt controlate de două motoare de CC pentru ajustarea fluxului de lichid de la intrarea fiecărui rezervor. Din câte se cunosc, astfel de specificații de performanță nu pot fi realizate cu metodele clasice existente [1].

This paper deals with liquid tanks management faced from an automatic control point of view; the motivation for the study is the need for an automated management strategy for two liquid tanks. The problem that is addressed in this paper is the control of liquid level on each tank by using the PD controller [2].

We encountered with an important problem that is the resulting disturbance from the intersection between these two tanks and the second tank drain, this disturbance has a negative effect in our system. We use in both tanks two valves that are controlled by two DC motors for adjusting the inflow of the liquid for each tank. As far as we know, such performance specifications are unattainable with the old existing methods [1].

Keywords: Liquid level, PD controller, DC motor, Valve, Liquid flow.

1. Introduction

The Ziegler-Nichols Closed Loop method is one of the most common methods used to tune control loop. It was first introduced in a paper published in 1942 by J.G. Ziegler and N.B. Nichols. The closed loop method is useful for most control loops process. It determines the gain at which a loop with proportional only control will oscillate, and then derives the controller gain, reset, and

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derivative values from the gain at which the oscillations are sustained and the period of oscillation at that gain [9].
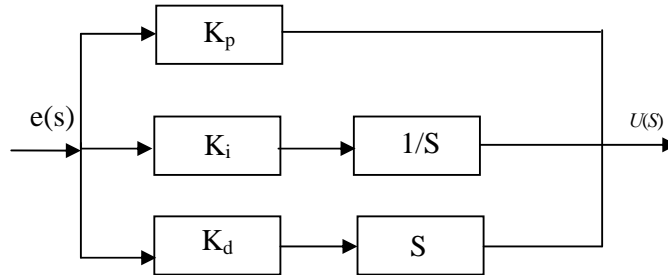

Fig. 1. PID controller

In Fig. 1 a PID controller design is illustrated. The controller input is the error signal and the controller output is the actuator signal. We achieve particular cases like PI and PD by eliminating respectively the derivative branch or the integrated branch. The proportional coefficient gain (K_p) will have the effect of reducing the rise time but it will never eliminate it. The derivative coefficient gain (K_d) will have the effect of increasing the stability of the system by reducing the overshoot, and by improving the transient response. The effects of each coefficient K_p and K_d on a closed-loop system are summarized in the table shown below [6][9].

Table 1

Two controller gains effects

CL RESPONSE	OVERSHOOT	RISE TIME	SETTLING TIME	S-S ERROR
K_p	Increase	Decrease	Small Change	Decrease
K_d	Decrease	Small Change	Decrease	Small Change

2. The DC motor control

The direct current (DC) motor is one of the first machines devised to convert
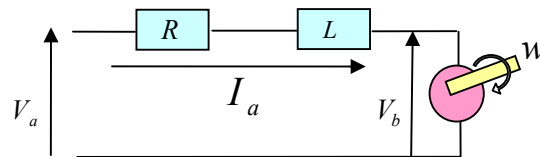

Fig. 2. Basic Motor Model

electrical power into mechanical power and its origins can be traced to the disk-type machines conceived and tested by Michael Faraday.

V_a: Applied voltage;
V_b: Induced back *emf* voltage;
I_a: Motor current;
T: Motor output torque;
L: Armature winding inductance;
W: Motor output speed;
R: Armature resistance;

The electrical relation between these variables is given by

$$F_2(f) = F_1(f) \otimes F_s(t) \tag{1}$$

The dynamic equation of a motor is given by

$$F_s(w) = \frac{2\pi}{T} t_0 \sum_{-\infty}^{\infty} \frac{\sin \frac{n\pi t_0}{T}}{\frac{n\pi t_0}{T}} \delta\left(W - \frac{2\pi n}{T}\right) \tag{2}$$

And its corresponding transfer function is

$$G_m = \frac{Y(s)}{V_a(s)} = \frac{K_T}{[(Ls+R)(js+b) + K_d.K_T]} \tag{3}$$

From what, we can deduct the two poles:

$$p_{1,2} = \frac{(Lb+Rj)}{2LJ} \pm \frac{1}{2LJ} \sqrt{4LJ(Rb + K_d.K_T)} \tag{4}$$

3. Mathematical equations and state space

Here we have some equations that the two tanks system can be based on; we find the following relation between the capacitance c_1, liquid level h_1 and the flow Q_1 [3].

Tank$_1$:

$$c_1 \frac{d(h_1)}{d(t)} = (Q_1 - q_1) \Rightarrow c_1 d(h_1) = (Q_1 - q_1)d(t) \tag{5}$$

The direction of flow q_1 can be either positive or negative depending on the tanks levels. The positive direction flow is defined by the liquid flow from tank$_1$ to tank$_2$ to be positive, and then in terms of the deviation quantities, here it is:

$R_1 = \dfrac{h_1 - h_2}{q_1}$ is the fluid resistance of the valve$_1$, which results in:

$$c_1 \frac{d(h_1)}{d(t)} = (Q_1 - \frac{h_1 - h_2}{R_1})$$ (6)

It is the same for the second tank.

Tank$_2$:

$$c_2 \frac{d(h_2)}{d(t)} = (Q_2 + q_1 - q_2)$$ (7)

The fluid resistance of the valve$_2$ is: $R_2 = \dfrac{h_2}{q_2}$, the equation (3) can be written as:

$$c_2 \frac{d(h_2)}{d(t)} = (Q_2 + \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2})$$ (8)

Taking Laplace transformations for the two final equations (6), (7), and solving for $H_1(s)$, $H_2(s)$ we find:

$$c_1 s H_1(s) = (Q_1(s) - \frac{h_1(s) - h_2(s)}{R_1})$$ (9)

$$H_1(s) = Q_1(s) \frac{R_1}{R_1 c_1 + 1} + \frac{h_2(s)}{R_1 c_1 + 1}$$ (10)

$$c_2 s H_2(s) = (Q_2(s) + \frac{h_1(s) - h_2(s)}{R_1} - \frac{h_2(s)}{R_2})$$ (11)

$$H_2(s) = Q_2(s) \frac{R_1 R_2}{c_2 R_1 R_2 + R_1 + R_2} + h_1(s) \frac{R_2}{c_2 R_1 R_2 + R_1 + R_2}$$ (12)

Define state variable:

$x_1 = h_1$

$x_2 = h_2$

Input variable

$u_1 = Q_1$

$u_2 = Q_2$

The equations (6) and (7) can be written as

$$x_1^{\cdot} = -\frac{1}{c_1 R_1} x_1 + \frac{1}{c_1 R_1} x_2 + \frac{1}{c_1} u_1$$

$$x_2^{\cdot} = \frac{1}{c_2 R_1} x_1 - \frac{R_1 + R_2}{c_1 R_1 R_2} x_2 + \frac{1}{c_2} u_2$$

Output variables

$y_1 = x_1 = h_1$

$y_2 = x_2 = h_2$

We know that the simple equation of any   state space is:

$x^.(s) = AX(s) + BX(s)$

$y(s) = CX(s) + DX(s)$

In the form of the standard vector-matrix representation, we have

$$\begin{bmatrix} x^._1 \\ x^._2 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{c_1 R_1} & \dfrac{1}{c_1 R_1} \\ \dfrac{1}{c_2 R_1} & -\dfrac{R_1 + R_2}{c_2 R_1 R_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{c_1} & 0 \\ 0 & \dfrac{1}{c_2} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

(13)

Which is the state equation, and

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

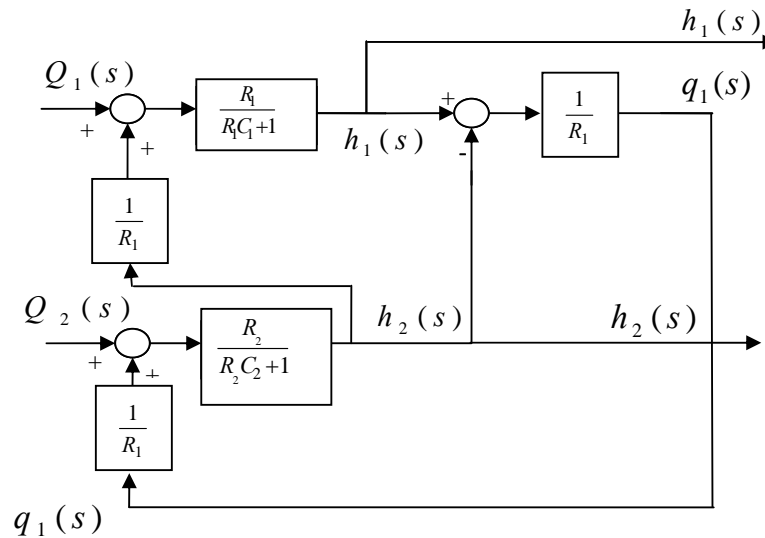(14)

This is the output equation.

Fig. 3. The plan

The plan in Fig. 3 explains the effect of each tank on the other; it is based on the mathematical equations mentioned above and on the state space represented by the two equations (13), (14). We have found the relations between

the level and the inflow rate in each tank. We can introduce them in the general electronic diagram of our system. Our system is mapped in Fig. 4 as having three stages that are the control, the actuator and the plant. The simulation was built in MATLAB, it contains two PD controllers, each one drives the connected DC motor by giving to it the actuator signal; the output of the DC motors and the grouped valves are the plant's inputs.
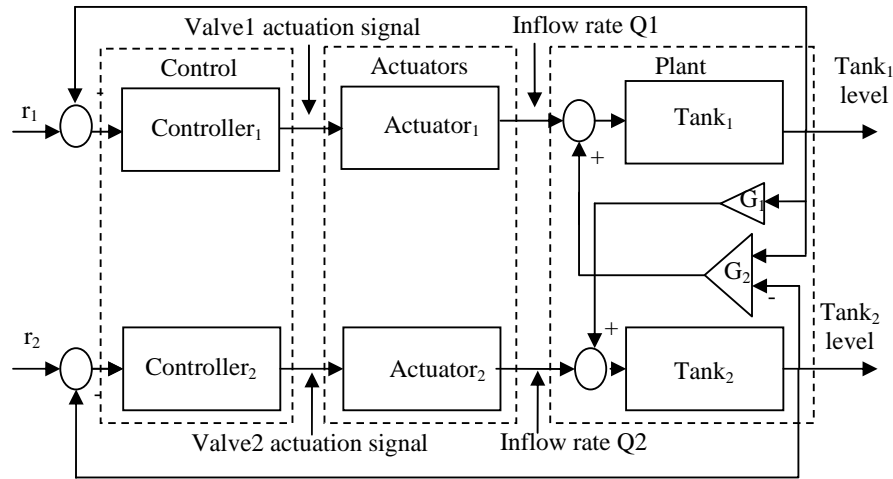

Fig.4. Bloc diagram of two tanks level control system

4. Experiment and simulation

4.1. Experiment

This experiment consists of two tanks with orifices, level sensor at the bottom of each one (see Fig. 5). The two tanks have the same diameters and can be fitted with different diameter outflow orifices, the valves infeed both tanks with different quantity of liquid, these valves are controlled by two DC motors. The outflow of the $tank_1$ infeeds the $tank_2$.

The outflow of $Tank_2$ is emptied out [8]. The liquid levels are adjusted by the two controlled valves, supposing that the two orifices have the same values. It means that this system has two outputs controlled by two inputs under two disturbance signals q_1, q_2. [4] [5].
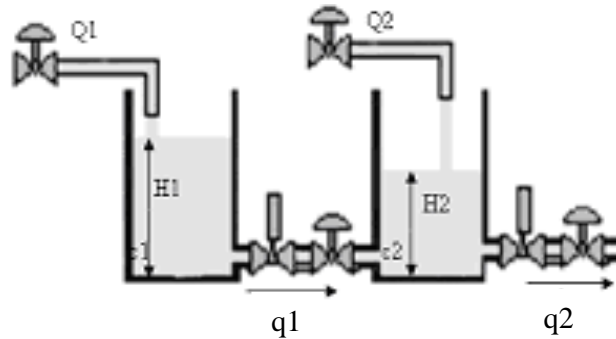
Fig. 5. Two liquid tanks control system

Where:

$Q_1 (m^3 / s)$: The steady state flow rate 1;

$Q_2 (m^3 / s)$: The steady state flow rate 2;

$c_1 (m^2)$: The capacitance of the tank$_1$;

$c_2 (m^2)$: The capacitance of the tank$_2$;

$h_1 (m)$: The level of the tank$_1$;

$h_2 (m)$: The level of the tank$_2$;

$q_1 (m)(m^3 / s)$: The valve$_2$ outflow;

$q_2 (m)(m^3 / s)$: The valve$_2$ outflow;

4.2. Simulation

Giving data simulation
For DC motor
        Gain [0.5]
        Time constant [0.1]
For tank$_1$, tank$_2$
        Gain [1]
        Time constant [50]

Then we will simulate this system with two PD controllers that are tuned according to the theory of ZIEGLER NICHOLS. The point of "tuning" a PD loop is to adjust on how aggressively the controller reacts to errors between the measured process variable and the desired setpoint. If the controlled process happens to be relatively sluggish, the PD algorithm can be configured to take immediate and dramatic actions whenever a random disturbance changes the process variable or an operator changes the setpoint [7].

(A)$K_1 = 30$ (B) $K_1 = 50$ (C) $K_1 = 40$

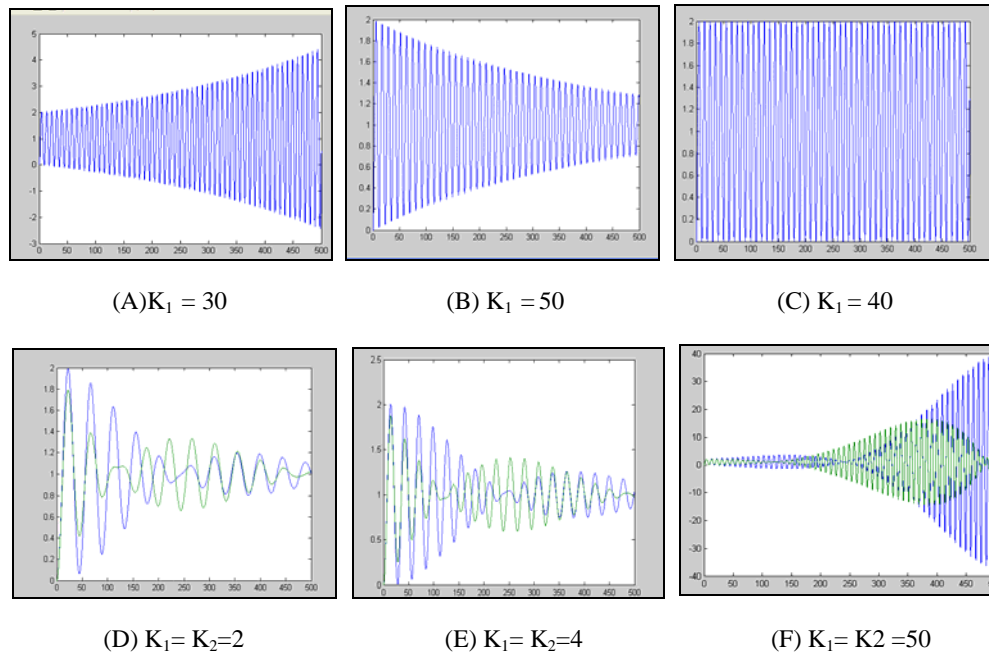(D) $K_1 = K_2 = 2$ (E) $K_1 = K_2 = 4$ (F) $K_1 = K_2 = 50$

Fig. 6. Tuning gains

Fig. 6A shows the increasing loop oscillation when the gain is 30 Fig. 6B shows the decreasing loop oscillation when the proportional coefficient gain is equal to 50. And in Fig. 6C is illustrated the undiminished loop oscillation when the gain is 40.

Then we try to process this PD controller by adding the second PD controller to study the tuning in both controllers, we remark that, in Figs. 5D and 5E, it is illustrated the decreasing system oscillation, where Fig. 5F shows the increasing oscillation of the loop. It is clear; the tuning of the two PD parameters K_p and K_d is an important issue. Experimentally we find the suitable values of these parameters because the system has two loop and two disturbance inputs. We have chosen the proportional gain that is $K_p = 4$ and the derivative gain $K_d = 24$ for $tank_1$ loop and $K_p = 2$ and $K_p = 22$ for $tank_2$ loop.

Fig. 7 illustrates the step responses of our system in different cases; these step signals illustrate the control response on many levels. We observe that the rising time and the overshoot point have suitable values. These two tanks system is robust and stable by keeping the liquid levels wherever we need to be.
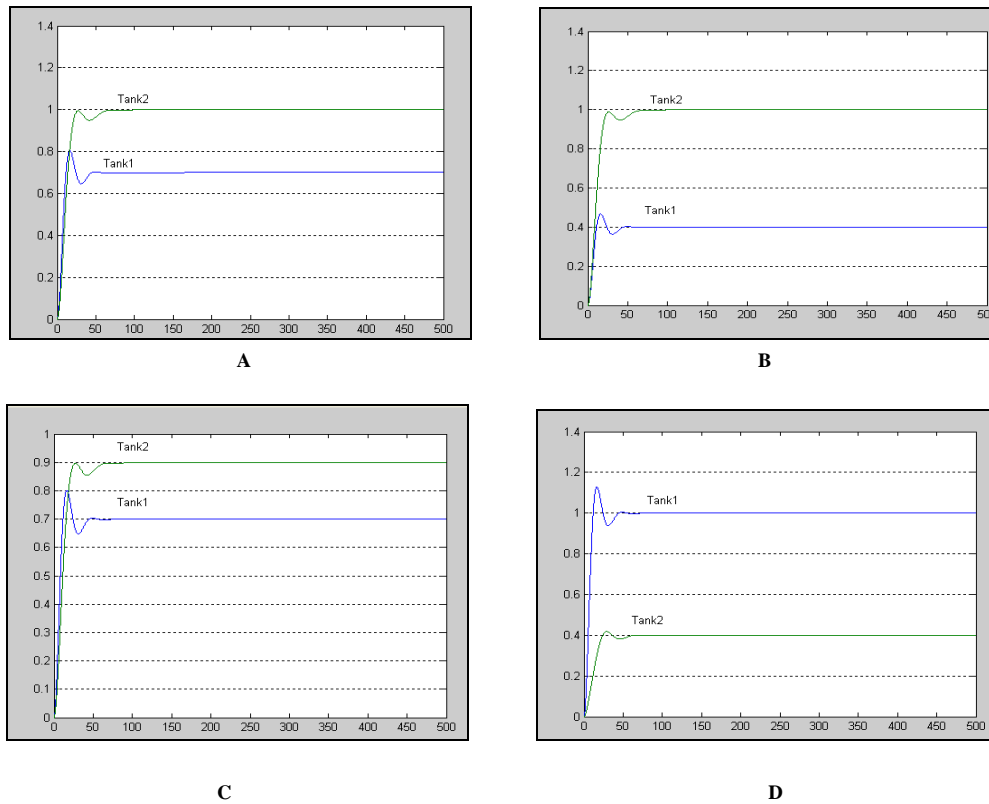
Fig. 7. Step response with different levels.

Fig. 7A shows the control ability to keep the levels of liquid where $H_1 = 0.7$ presents the altitude of the liquid in the $tank_1$ and $H_2 = 1$ presents the altitude of the liquid in the $tank_2$. Fig. 7B shows the stabilization of the two liquid levels to have these two values $H_1 = 0.4$ for $tank_1$ and $H_2 = 1$ for $tank_2$.

In the Fig. 7C two curves representing $tank_1$ and $tank_2$ are illustrated which are the liquid levels in the case of $H_1 = 0.7$ and $H_2 = 0.9$. Also, the result obtained in Fig. 7D proves the ability of our control system to set the two levels at the desired values $H_1 = 1$ and $H_2 = 0.4$. So in these four cases of control we tried to vary the both inputs to see what the output will be, and to know if our simulation is able to control well two liquid tanks system, to stabilize the liquid level in these two thanks by minimizing the error signal which is resulting from the difference between the reference and feedback signals.

5. Conclusions

Control of liquid inventory in many industrial processes, and specifically in petrochemical plants, is an important basic problem. In this paper, we present a simple means of controlling liquid levels. The PD controller is a robust controller giving us the possibility to adjust two deferent liquid levels, in spite of the two disturbance signals which are the drain and the intersection between the tanks. Wherever the liquid levels our controller achieves its goal, making its task with high performance and the shortest time.

R E F E R E N C E S

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