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APPLICATION OF THE $(\frac{G'}{G})$-EXPANSION METHOD FOR (2+1)-DIMENSIONAL BOUSSINESQ EQUATION AND KADOMTSEV-PETVIASHVILI EQUATION

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By using the $(\frac{G'}{G})$-expansion method proposed recently, we derive the exact traveling wave solutions of two nonlinear evolution equations in this paper. As a result, the traveling wave solutions with three arbitrary functions are obtained including hyperbolic function solutions, trigonometric function solutions and rational solutions. The computation for the method can be fulfilled by the general mathematical software. So the method appears to gain an advantage over the traditional method.

Keywords: $(\frac{G'}{G})$-expansion method, Traveling wave solutions, (2+1) dimensional Boussinesq equation, Kadomtsev-Petviashvili equation
MSC2000: 35Q 51, 35Q 53.

1. Introduction

During the past four decades or so searching for explicit solutions of nonlinear evolution equations by using various different methods have been the main goal for many researchers, and many powerful methods for constructing exact solutions of nonlinear evolution equations have been established and developed such as the inverse scattering transform, the Backlund/ Darboux transform, the tanh-function expansion and its various extension, the Jacobi elliptic function expansion, the homogeneous balance method, the sine-cosine method, the rank analysis method, the exp-function expansion method and so on [1-17], but there is no unified method that can be used to deal with all types of nonlinear evolution equations.

In [18], Mingliang Wang proposed a new method called $(\frac{G'}{G})$-expansion method. The main ideas of the proposed method are that the traveling wave solutions of a nonlinear evolution equation can be expressed by a polynomial in $(\frac{G'}{G})$, where $G = G(\xi)$ satisfies a second order LODE, $G' = \frac{dG(\xi)}{d\xi}, \xi = \xi(x, t)$

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or $\xi = \xi(x, y, t)$, and the degree of the polynomial can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in a given nonlinear evolution equation. Then the coefficients of the polynomial can be obtained by solving a set of algebraic equations. Based on its simplicity and validity, this method has soon been applied to get the exact solution of many nonlinear equations by several researchers [19-22].

In this paper, we will consider the (2+1) dimensional Boussinesq equation [23]

$$u_{tt} - u_{xx} - u_{yy} - (u^2)_{xx} - u_{xxxx} = 0$$

and the Kadomtsev-Petviashvili equation [24]

$$(u_t + 6uu_x + u_{xxx})_x + 3\sigma u_{yy} = 0$$

We will derive the traveling wave solutions of the two equations by using the $(\frac{G'}{G})$-expansion method.

The rest of the paper is organized as follows. In Section 2, we describe the $(\frac{G'}{G})$-expansion method for finding traveling wave solutions of nonlinear evolution equations, and give the main steps of the method here. In the subsequent sections, we illustrate the method in detail with the celebrated the (2+1) dimensional Boussinesq and the Kadomtsev-Petviashvili equations. In the last Section, the features of the $(\frac{G'}{G})$-expansion method are briefly summarized.

2. Description of the $(\frac{G'}{G})$-expansion method

In this section we will describe the $(\frac{G'}{G})$-expansion method for finding out the traveling wave solutions of nonlinear evolution equations.

Suppose that a nonlinear equation, say in three independent variables $x,\ y$ and t , is given by

$$P(u, u_t, u_x, u_y, u_{tt}, u_{xt}, u_{yt}, u_{xx}, u_{yy}, ...) = 0 \tag{1}$$

where $u = u(x, y, t)$ is an unknown function, P is a polynomial in $u = u(x, y, t)$ and its various partial derivatives, in which the highest order derivatives and nonlinear terms are involved. In the following we give the main steps of the $(\frac{G'}{G})$-expansion method.

Step 1. Combining the independent variables $x,\ y$ and t into one variable $\xi = \xi(x,\ y,\ t)$, we suppose that

$$u(x,\ y,\ t) = u(\xi), \quad \xi = \xi(x,\ y,\ t) \tag{2}$$

the traveling wave variable (2) permits us reducing Eq. (1) to an ODE for $u = u(\xi)$

$$P(u,\ u', u'',\ ...) = 0. \tag{3}$$

Step 2. Suppose that the solution of (3) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \alpha_m (\frac{G'}{G})^m + \ ... \tag{4}$$

where $G = G(\xi)$ satisfies the second order LODE in the form

$$G'' + \lambda G' + \mu G = 0 \tag{5}$$

$\alpha_m, ..., \lambda$ and μ are constants to be determined later, $\alpha_m \neq 0$. The unwritten part in (4) is also a polynomial in $(\frac{G'}{G})$, the degree of which is generally equal to or less than $m-1$. The positive integer m can be determined by considering the homogeneous balance between the highest order derivatives and nonlinear terms appearing in (3).

Step 3. Substituting (4) into (3) and using second order LODE (5), collecting all terms with the same order of $(\frac{G'}{G})$ together, the left-hand side of Eq. (3) is converted into another polynomial in $(\frac{G'}{G})$. Equating each coefficient of this polynomial to zero, yields a set of algebraic equations for $\alpha_m, ..., \lambda$ and μ.

Step 4. Assuming that the constants $\alpha_m, ..., \lambda$ and μ can be obtained by solving the algebraic equations in Step 3, since the general solutions of the second order LODE (5) have been well known for us, substituting $\alpha_m, ...$ and the general solutions of Eq.(5) into (4) we can obtain the traveling wave solutions of the nonlinear evolution equation (1).

In the subsequent sections we will illustrate the proposed method in detail by applying it to various nonlinear evolution equations.

3. (2+1) dimensional Boussinesq Equation

In this section, we will consider the (2+1) dimensional Boussinesq equation:

$$u_{tt} - u_{xx} - u_{yy} - (u^2)_{xx} - u_{xxxx} = 0 \tag{6}$$

In order to obtain the traveling wave solutions of Eq.(6), we suppose that

$$u(x,\ y,\ t) = u(\xi),\ \xi = kx + ly + mt + d \tag{7}$$

$k,\ l,\ m,\ d$ are constants that to be determined later.

By using (7), (6) can be converted into an ODE

$$(m^2 - k^2 - l^2)u'' - 2k^2(u'^2 + uu'') - k^4 u^{(4)} = 0 \tag{8}$$

Integrating the ODE (8) with respect to ξ once, we obtain

$$(m^2 - k^2 - l^2)u' - 2k^2(uu') - k^4 u''' = g \tag{9}$$

where g is the integration constant that can be determined later.

Suppose that the solution of (9) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i (\frac{G'}{G})^i \tag{10}$$

where a_i are constants, $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \tag{11}$$

where λ and μ are constants.

Balancing the order of uu' and u''' in (9), we have $m + m + 1 = m + 3 \Rightarrow m = 2$. So Eq.(10) can be rewritten as

$$u(\xi) = a_2 (\frac{G'}{G})^2 + a_1 (\frac{G'}{G}) + a_0, \ a_2 \neq 0 \tag{12}$$

$a_2, \ a_1, \ a_0$ are constants to be determined later. Then it follows

$$u'(\xi) = -2a_2(\frac{G'}{G})^3 + (-a_1 - 2a_2\lambda)(\frac{G'}{G})^2 + (-a_1\lambda - 2a_2\mu)(\frac{G'}{G}) - a_1\mu$$

$$u''(\xi) = 6a_2(\frac{G'}{G})^4 + (2a_1 + 10a_2\lambda)(\frac{G'}{G})^3 + (8a_2\mu + 3a_1\lambda + 4a_2\lambda^2)(\frac{G'}{G})^2$$
$$+(6a_2\lambda\mu + 2a_1\mu + a_1\lambda^2)(\frac{G'}{G}) + 2a_2\mu^2 + a_1\lambda\mu$$

$$u'''(\xi) = -24a_2(\frac{G'}{G})^5 + (-54a_2\lambda - 6a_1)(\frac{G'}{G})^4 + (-12a_1\lambda - 38a_2\lambda^2 - 40a_2\mu)(\frac{G'}{G})^3$$
$$+(-52a_2\lambda\mu - 7a_1\lambda^2 - 8a_2\lambda^3 - 8a_1\mu)(\frac{G'}{G})^2 + (-14a_2\lambda^2\mu - a_1\lambda^3$$
$$-16a_2\mu^2 - 8a_1\lambda\mu)(\frac{G'}{G}) - a_1\lambda^2\mu - 2a_1\mu^2 - 6a_2\lambda\mu^2$$

Substituting Eq.(12) into (9) and collecting all terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0: \ -m^2a_1\mu + 2k^4a_1\mu^2 + k^2a_1\mu + 6k^4a_2\lambda\mu^2 + l^2a_1\mu + k^4a_1\lambda^2\mu - g$$
$$+2k^2a_0a_1\mu = 0$$

$$(\frac{G'}{G})^1: \ 4k^2a_0a_2\mu + 8k^4a_1\lambda\mu + 14k^4a_2\lambda^2\mu + 2l^2a_2\mu + 2k^2a_1^2\mu + k^2a_1\lambda$$
$$+2k^2a_0a_1\lambda + l^2a_1\lambda - 2m^2a_2\mu + 16k^4a_2\mu^2 + 2k^2a_2\mu - m^2a_1\lambda + k^4a_1\lambda^3 = 0$$

$$(\frac{G'}{G})^2: \ l^2a_1 + 2l^2a_2\lambda - m^2a_1 + 4k^2a_0a_2\lambda + 2k^2a_2\lambda + 52k^4a_2\lambda\mu + k^2a_1$$
$$+2k^2a_0a_1 + 7k^4a_1\lambda^2 + 8k^4a_1\mu - 2m^2a_2\lambda + 8k^4a_2\lambda^3 + 2k^2a_1^2\lambda$$

$$+6k^2a_1a_2\mu = 0$$

$$(\frac{G'}{G})^3: \; 2k^2a_2 + 12k^4a_1\lambda + 4k^2a_2^2\mu - 2m^2a_2 + 40k^4a_2\mu + 2k^2a_1^2 + 2l^2a_2$$
$$+6k^2a_1a_2\lambda + 38k^4a_2\lambda^2 + 4k^2a_0a_2 = 0$$

$$(\frac{G'}{G})^4: \; 54k^4a_2\lambda + 4k^2a_2^2\lambda + 6k^4a_1 + 6k^2a_1a_2 = 0$$

$$(\frac{G'}{G})^5: \; 24k^4a_2 + 4k^2a_2^2 = 0$$

Solving the algebraic equations above, yields:

$$a_2 = -6k^2, a_1 = -6k^2\lambda, a_0 = -\frac{1}{2}\frac{k^2 + k^4\lambda^2 + 8k^4\mu - m^2 + l^2}{k^2}$$
$$k = k, l = l, m = m, d = d \qquad g = 0 \tag{13}$$

where k, l, m, d are arbitrary constants.

Substituting (13) into (12), we get that

$$u(\xi) = -6k^2(\frac{G'}{G})^2 - 6k^2\lambda(\frac{G'}{G}) - \frac{1}{2}\frac{k^2 + k^4\lambda^2 + 8k^4\mu - m^2 + l^2}{k^2}$$
$$\xi = kx + ly + mt + d \tag{14}$$

where k, l, m, d are arbitrary constants.

Substituting the general solutions of Eq.(11) into (14), we can obtain the traveling wave solutions of (3.1) as follows:

Case (a): when $\lambda^2 - 4\mu > 0$

$$u_1(\xi) = \frac{3}{2}k^2\lambda^2 - \frac{3}{2}k^2(\lambda^2 - 4\mu)\left(\frac{C_1\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2$$
$$-\frac{1}{2}\frac{k^2 + k^4\lambda^2 + 8k^4\mu - m^2 + l^2}{k^2}$$

where $\xi = kx + ly + mt + d$, k, l, m, d, C_1, C_2 are arbitrary constants. In particular, if $C_1 = 1,\ C_2 = 0,\ \mu = 0,\ \lambda = 1,\ k = l = m = d = 1$, then we have

$$u(x, y, t) = \frac{1}{2} - \frac{3}{2}[\tanh\frac{1}{2}(x + y + t + 1)]^2.$$

Case (b): when $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = \frac{3}{2}k^2\lambda^2 - \frac{3}{2}k^2(4\mu-\lambda^2)\left(\frac{-C_1\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2$$
$$-\frac{1}{2}\frac{k^2+k^4\lambda^2+8k^4\mu-m^2+l^2}{k^2}$$

where $\xi = kx + ly + mt + d$, k, l, m, d, C_1, C_2 are arbitrary constants. In particular, if $C_1 = 1,\ C_2 = 0,\ \mu = 0,\ \lambda = 1,\ k = l = m = d = 1$, then

$$u(x, y, t) = 6[\tan(x + y + t + 1)]^2 - \frac{9}{2}.$$

Case (c): when $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{3}{2}k^2\lambda^2 - \frac{6k^2C_2^2}{(C_1 + C_2\xi)^2} - \frac{1}{2}\frac{k^2+k^4\lambda^2+8k^4\mu-m^2+l^2}{k^2}$$

where $\xi = kx + ly + mt + d$, k, l, m, d, C_1, C_2 are arbitrary constants. In particular, if $C_1 = C_2 = 1,\ \mu = 1,\ \lambda = 2,\ k = l = m = d = 1$, then we have

$$u(x, y, t) = -\frac{1}{2} - \frac{6}{(x + y + t + 2)^2}.$$

4. **Kadomtsev-Petviashvili Equation**

In this section, we will consider the Kadomtsev-Petviashvili equation

$$(u_t + 6uu_x + u_{xxx})_x + 3\sigma u_{yy} = 0 \tag{15}$$

Suppose that

$$u(x,\ y,\ t) = u(\xi),\ \xi = x + y - ct \tag{16}$$

c is a constant that to be determined later.

By (16), Eq.(15) can be converted into an ODE

$$-cu'' + 6(u')^2 + 6uu'' + u^{(4)} + 3\sigma u'' = 0 \tag{17}$$

Integrating the ODE (17) once, we obtain

$$(-c + 3\sigma)u' + u^{(3)} + 6uu' = g \tag{18}$$

where g is the integration constant that can be determined later.

Suppose that the solution of (18) can be expressed by a polynomial in $(\frac{G'}{G})$ as follows:

$$u(\xi) = \sum_{i=0}^{m} a_i(\frac{G'}{G})^i \tag{19}$$

where a_i are constants, and $G = G(\xi)$ satisfies the second order LODE in the form:

$$G'' + \lambda G' + \mu G = 0 \tag{20}$$

where λ and μ are constants.

Balancing the order of uu' and u''' in Eq.(18), we have $m+m+1=m+3 \Rightarrow m=2$. So Eq.(19) can be rewritten as

$$u(\xi)=a_2(\frac{G'}{G})^2+a_1(\frac{G'}{G})+a_0,\ a_2\neq 0 \tag{21}$$

$a_2,\ a_1,\ a_0$ are constants to be determined later.

Substituting Eq.(21) into (18) and collecting all terms with the same power of $(\frac{G'}{G})$ together, equating each coefficient to zero, yields a set of simultaneous algebraic equations as follows:

$$(\frac{G'}{G})^0:\ -6a_2\lambda\mu^2-g-3\sigma a_1\mu+ca_1\mu-2a_1\mu^2-6a_0a_1\mu-a_1\lambda^2\mu=0$$

$$(\frac{G'}{G})^1:\ -3\sigma a_1\lambda-8a_1\lambda\mu+2ca_2\mu-6a_0a_1\lambda-6a_1^2\mu-14a_2\lambda^2\mu-12a_0a_2\mu$$
$$+ca_1\lambda-16a_2\mu^2-6\sigma a_2\mu-a_1\lambda^3=0$$

$$(\frac{G'}{G})^2:\ -18a_1a_2\mu-6\sigma a_2\lambda-12a_0a_2\lambda-7a_1\lambda^2-3\sigma a_1-8a_1\mu-6a_0a_1$$
$$-6a_1^2\lambda-8a_2\lambda^3-52a_2\lambda\mu+ca_1+2ca_2\lambda=0$$

$$(\frac{G'}{G})^3:\ -6a_1^2+2ca_2-12a_2^2\mu-12a_1\lambda-40a_2\mu-38a_2\lambda^2-18a_1a_2\lambda$$
$$-12a_0a_2-6\sigma a_2=0$$

$$(\frac{G'}{G})^4:\ -54a_2\lambda-18a_1a_2-12a_2^2\lambda-6a_1=0$$

$$(\frac{G'}{G})^5:\ -12a_2^2-24a_2=0$$

Solving the algebraic equations above, yields:

$$a_2=-2,\ a_1=-2\lambda,\ a_0=a_0,\ c=8\mu+3\sigma+6a_0,\ g=0 \tag{22}$$

where a_0,λ,μ are arbitrary constants.

Substituting (22) into (21), we get that

$$u(\xi)=-2(\frac{G'}{G})^2-2\lambda(\frac{G'}{G})+a_0$$

$$\xi=x+y-(8\mu+3\sigma+6a_0)t \tag{23}$$

Substituting the general solutions of (20) into (23), we can obtain the traveling wave solutions of (15) as follows:

Case (a): when $\lambda^2-4\mu>0$

$$u_1(\xi) = \frac{1}{2}\lambda^2 - \frac{1}{2}(\lambda^2 - 4\mu)\left(\frac{C_1 \sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}{C_1\cosh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi + C_2\sinh\frac{1}{2}\sqrt{\lambda^2-4\mu}\xi}\right)^2 + a_0$$

where $\xi = x + y - (8\mu + 3\sigma + 6a_0)t$, a_0, C_1, C_2 are arbitrary constants.

Case (b): when $\lambda^2 - 4\mu < 0$

$$u_2(\xi) = \frac{1}{2}\lambda^2 - \frac{1}{2}(4\mu - \lambda^2)\left(\frac{-C_1 \sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}{C_1\cos\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi + C_2\sin\frac{1}{2}\sqrt{4\mu-\lambda^2}\xi}\right)^2 + a_0$$

where $\xi = x + y - (8\mu + 3\sigma + 6a_0)t$, a_0, C_1, C_2 are arbitrary constants.

Case (c): when $\lambda^2 - 4\mu = 0$

$$u_3(\xi) = \frac{1}{2}\lambda^2 - \frac{2C_2^2}{(C_1 + C_2\xi)^2} + a_0$$

where $\xi = x + y - (8\mu + 3\sigma + 6a_0)t$, a_0, C_1, C_2 are arbitrary constants.

5. **Conclusions**

In this paper we have seen that the traveling wave solutions of the (2+1) dimensional Boussinesq equation and the Kadomtsev-Petviashvili equation are successfully found out by using the $(\frac{G'}{G})$-expansion method. Now we briefly summarize the method in the following.

Firstly, we assume the solution of the ODE can be expressed by an m-th degree polynomial in $(\frac{G'}{G})$ by using the traveling wave variable as well as integration, where $G = G(\xi)$ is the general solutions of a second order LODE, and the positive integer m is determined by the homogeneous balance between the highest order partial derivatives and nonlinear terms appearing in the reduced ODE. Then the coefficients of the polynomial can be obtained by solving a set of simultaneous algebraic equations resulted from the process of using the method.

Secondly, it is important to solve the algebraic equations resulted. We can use the MATHEMATICA or MAPLE to find out a useful solution of the algebraic equations.

In all, the $(\frac{G'}{G})$-expansion method has its own advantages: direct, concise, elementary, and it can be used for many other nonlinear evolution equations.

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