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A NEW APPROACH ON CHUA'S CIRCUIT ANALYSIS

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This paper considers a new approach of Chua's circuit analysis by adding a circuit element of great interest in current research papers, namely the memristor. The traditional nonlinear element is not completely replaced by a memristive circuit element but proposes to improve the existing models by using a HP memristor alongside diodes and operational amplifiers.

Keywords: Chua's circuit, chaos, HP memristor, double scroll atractor

1. Introduction

With the emergence of the theory of chaos and fractal geometry, the chaotic behavior of the surrounding physical structures such as informational, economic, social and political has become more and more studied over the last 20 years. Among the few electrical structures with chaotic behavior induced intentionally by the human factor, one can mention the Chua circuit, named after the Japanese engineer Leon Chua, a member of the Department of Electrical and Computer Engineering at the University of Berkeley, California, who also discovered the existence of memristors. The Chua circuit is an oscillator with a complex behavior, characterized by state bifurcations and a tendency towards chaotic behavior. It consists of a coil, two capacitors, a resistor and a nonlinear element known as the "Chua diode" which makes it unable to be implemented through a network containing only standard elements such as resistors, inductors, and capacitors.

Memristive chaotic circuits are not new as there are currently numerous research in this area, the simplest model of a chaotic oscillator being realized by connecting a memristor in series with a capacitor and a coil. All these works have in common the fact that different memristive models are integrated in one way or another in Chua's chaotic oscillator circuit because it is a very simple circuit and contains a non-linear element, which is exactly the definition of a memristor. This paper presents the simulation of a chaotic circuit based on the traditional Chua oscillator model but combining the use of a traditional nonlinear resistor with the addition of a memristor made in HP laboratories. An HP memristor is a thin film

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made of T iO2. It is doped with oxygen vacancies and bounded between two platinum plates that act as metal contacts. In this paper, the nonlinear element is not completely replaced by a memristor but proposes to improve the existing models by adding an HP memristor and it has been found that the obtained circuit has a chaotic behavior whose output signal can be used to generate random sequences.

2. Chua's circuit: Modeling chaos

Chaos represents a stretching and folding mechanism caracterised by nearby trajectories of a dynamical system that are repeatedly pulled apart exponentially and folded back toghether. In order to exhibit chaos an autonomous circuit consisting of resistors, capacitors and inductors must contain at least 3 storage elements, an active local resistance and a nonlinear resistor. Also, the output of any circuit based on Chua's oscillator should meet two conditions to be recognized as chaotic:

- Circuit variables (current and voltage) measured at any circuit node should be chaotic and random. In other words, their time-varying graph looks like a noisy signal.
- Chaotic attractors (the voltage variation graph on one of the capacitors relative to the voltage across the other capacitor/the coil current) should show or approach the bifurcation phenomenon. [1]

The active resistor supplies energy to separate trajectories, the nonlinearity provides folding, and the three-dimensional state space permits persistent stretching and folding in a bounded region without violating the non-crossing property of trajectories.

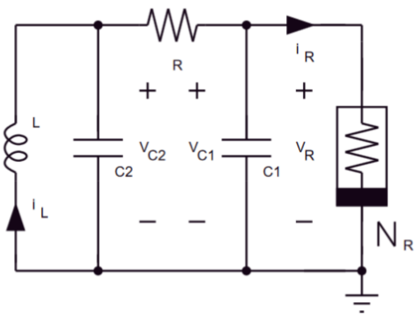


Fig. 1. Chua's chaotic oscillator.

Chua's circuit shown in Fig.1 is the simplest electronic circuit that satisfies these criteria and exhibits a rich variety of bifurcations and chaos. It consists of an inductance coil L, two capacitors with capacities C1 and C2, a linear resistor R and a non-linear element N_R. [2]·All the elements of Chua's circuit are standard electronic components that can be easily purchased in real life except N_R, an element that confers the non-linearity property shown in that graph I-V (Fig.2), a characteristic obtained by connecting several negative slope segments.

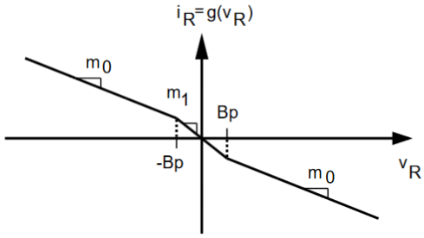


Fig. 2. The I-V characteristic of the non-linear element N_R

Due to this particularity, the nonlinear element cannot be implemented using only standard circuit elements such as resistors, inductors and capacitors. There are several techniques that have been used in the practice of circuits, it can be synthesized using an operational amplifier with two bipolar transistors, two diodes and resistors or two operational amplifiers and six resistors.

Applying Kirchhoff's laws to Chua’s circuit, one can study its dynamic behavior, which can be modeled by a system of three differential equations of voltages applied to the branches of the capacitors C1 and C2 and the electric current in the inductor L1, respectively.

$$C_1 \frac{dv_{C_1}}{dt} = G(v_{C_2} - v_{C_1}) - g(v_{C_1})$$
$$C_2 \frac{dv_{C_2}}{dt} = G(v_{C_1} - v_{C_2}) + i_L$$
$$L \frac{di_L}{dt} = -v_{C_2}$$

where G represents the inverse of the resistance R and g(v) is a piecewise linear function that shape the characteristic of the negative resistor N_R and is defined by the following equation: [3]

$$g(v_R) = m_0 v_R + \frac{1}{2}(m_1 - m_2)[|v_R + B_p| - |v_R - B_p|]$$

This is a system of three-order equations due to the three reactive elements, thus having three degrees of freedom. In order to highlight the complex behavior of the system, one can study any relationship between the three state variables, and in this paper the relationship between i_L and v_{C2} is analyzed. As a result of the simulations made in the researches published so far, it has been found that this circuit has an attractor with a special shape called "Double Scroll Atractor". His experimental confirmation was carried out by Zhong and Ayrom, after which Chua's circuit was studied extensively to obtain the bifurcation phenomenon and haotic attractors, thus displaying different models obtained experimentally but also mathematically demonstrated.

3. Implementation of Chua's circuit in LTSpice

In this paper, we started in the first case from the LTSpice implementation using an operational amplifier with two bipolar transistors illustrated in Figure 3. The circuit is powered by two voltage sources of +9V and -9V respectively, and the non-linear negative resistance N_R is modeled by the right half of the circuit by means of the U1 operational amplifier and the R1, R2, R3 resistors. The D1, D2 diodes together with resistors R7 and R8 are used to generate the positive half of the non-linear feature I-V.

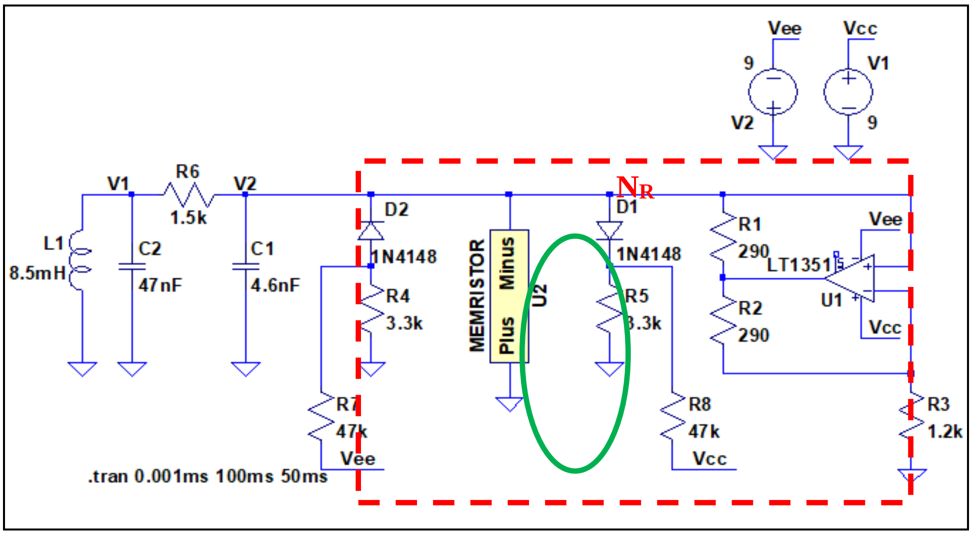


Fig. 3. Chua's memristive oscillator circuit topology implemented in LTSpice

Table 1

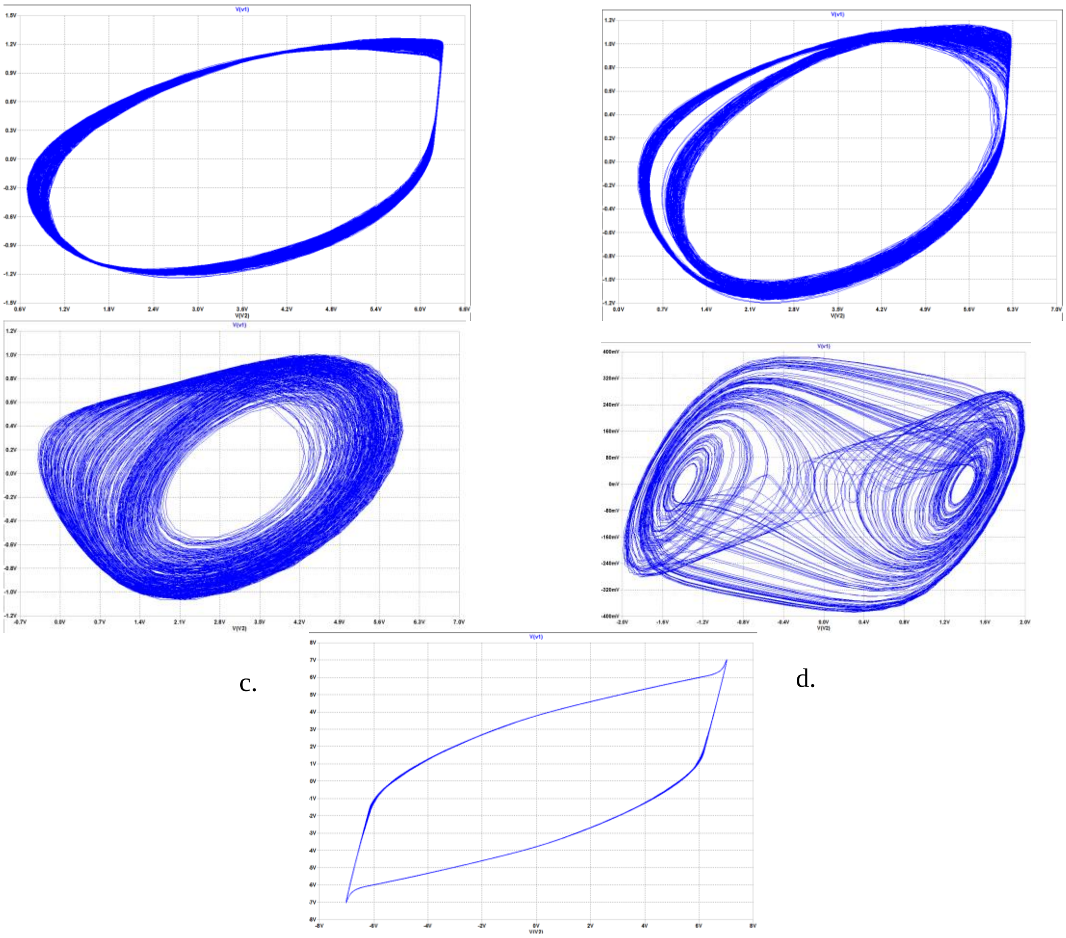
Parameters and values for the LTSpice implementation of Chua's circuit from Fig. 3

Element	Description	Value	Tolerance
D_1	Diode 1N4148		
D_2	Diode1N4148		
U_1	Op Amp LT1351		
C_1	Capacitor	10 nF	±5%
C_2	Capacitor	100 nF	±5%
R_1	Resistor	290 Ω	±5%
R_2	Resistor	290 Ω	±5%
R_3	Resistor	1.2 kΩ	±5%
R_4	Resistor	3.3 kΩ	±5%
R_5	Resistor	3.3 kΩ	±5%
R6	Potentiometer	2 kΩ	±5%
R7	Resistor	47 kΩ	±5%
R8	Resistor	47 kΩ	±5%

L1	Inductor	18 mH	±10%
Memristor	HP Memristor	Ron=50k Roff=100K Rinit=80K D=10N uv=10F p=1	

The innovative part brought to the existing topology is the memristive element highlighted in the green circle, a HP memristor modeled as a subcircuit in LTSpice by the means of a netlist file which defines the mathematical model and the initial values used for all parameters, according to the model proposed by Zdeněk BIOLEK. [4]

The control of the circuit is realized through the conductance of the resistor R6 and as its value increases, the phenomenon of doubling the period is observed as illustrated in sequences from Fig. 4. By changing the resistance R6 in the value range [0-2kΩ] it was found that for the value of 1.6KΩ the circuit behaves as a periodic oscillator, sustained by Chua's attractor or "Double Scroll Attractor" shown in Fig.4.a. By reducing the variable resistor R6 from 2kΩ to 1.4kΩ Chua’s circuit exhibits a sequence of period doubling bifurcations to a spiral Chua’s attractor, a double scroll Chua’s attractor and a boundary crisis, as illustrated in Fig. 4. Decreasing the value of the control resistor R6 from 2k kΩ to 1.87 kΩ, a period bifurcation occurs (Fig. 4.b) because a trajectory takes approximately twice the time to complete this closed orbit as to complete the preceding period-one orbit from Fig.4.a. Decreasing the resistance R6 still further to 1.83 kΩ produces a cascade of period-doubling bifurcations until an orbit of infinite period is reached, beyond which we have chaos, this is a spiral Chua’s chaotic attractor from Fig.4.c. Because we chose a nonlinear resistor with a symmetric nonlinearity (see Fig.5), every attractor has a mirror image. As the coupling resistance is decreased further to 1.6 kΩ, the spiral Chua’s attractor “collides” with its mirror image and the two merge to form a single compound attractor called a double-scroll Chua’s chaotic attractor (Fig. 4.d). Eventually, for a sufficiently small value of the resistance R of 1.4 kΩ the unstable saddle trajectory collides with the double-scroll Chua’s attractor and a blue sky catastrophe called boundary crisis occurs (Fig. 4.d). After this, all trajectories become unbounded.[5] The I-V curve of the non-linear element assembly was obtained using the .DC analysis in the LTspice program used and is illustrated in Fig. 5, it can be seen that it is piecewise-linear characteristic, symmetric about the origin and consisting of three straight-line segments, with negative slope, connected to each other.



c.

d.

e.

Fig. 4. Typical bifurcation sequences encountered in Chua's circuit shown in Fig.3 (horizontal axis V2, vertical axis V1): (a) R6 = 2 kΩ period-1, (b) R6 = 1.87 kΩ period-2, (c) R6 = 1.83 kΩ spiral Chua’s attractor, (d) R6 = 1.6 kΩ double-scroll Chua’s attractor, (e) R6 = 1.4 kΩ corresponding to boundary crisis.

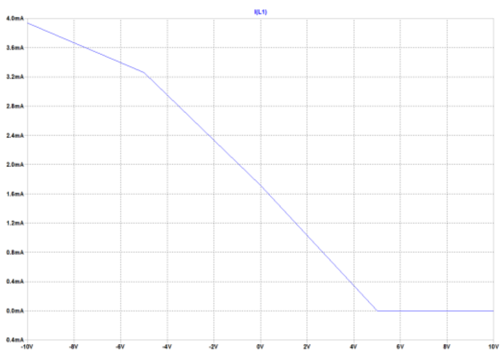


Fig. 5. Three segment piecewise-linear I-V characteristic for the nonlinear circuit element N_R.

Another commonly implementation of the nonlinear resistor N_R with the I-V characteristic described previously in Fig. 2 is realized by connecting two negative resistance converters in parallel as shown in Fig.5. The op amp subcircuit consisting of A_1, A_2, R_1-R_6 functions as a negative resistance converter N_R, the circuit is also powered by two voltage sources of +9V and -9V respectively, a complete list of components and their properties is presented in Table 2.

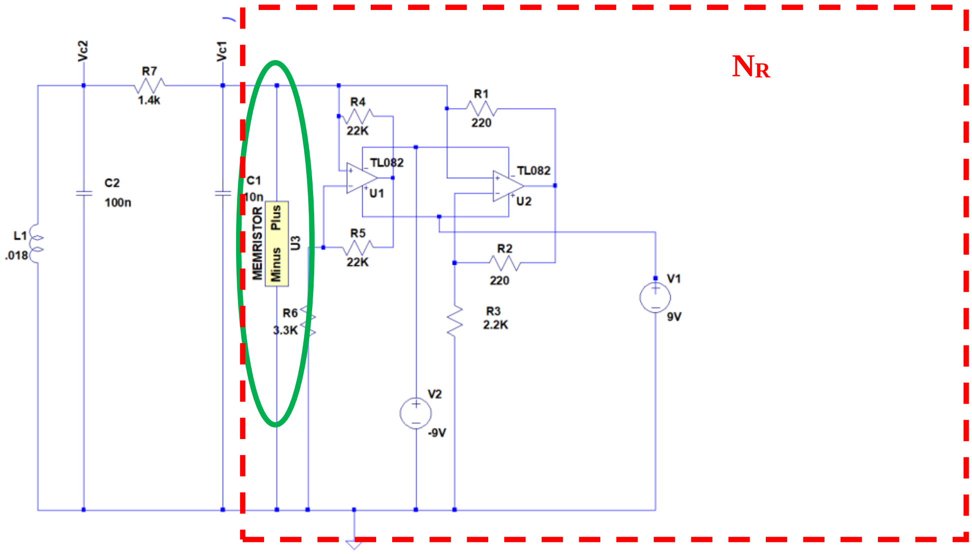


Fig. 6. Chua's memristive oscillator circuit topology implemented in LTSpice

Once more, the original part brought to the existing topology is the HP memristor highlighted in the green circle, modeled as a subcircuit in LTSpice and with the same initial parameters as in the previous model presented.

Parameters and values for the LTSpice implementation of Chua's circuit from Fig. 6

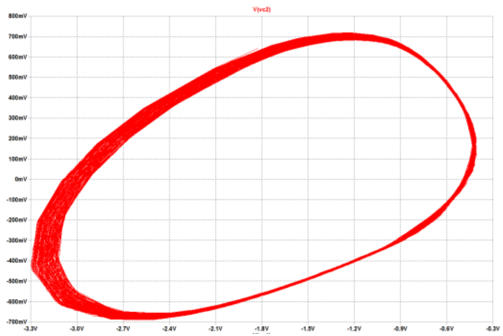
Element	Description	Value	Tolerance
U_1	Op Amp TL082		
R_4	Resistor	22 kΩ	±5%
R_5	Resistor	22 kΩ	±5%
R_6	Resistor	3.3 kΩ	±5%
U_2	Op Amp TL082		
R_1	Resistor	220 Ω	±5%
R_2	Resistor	220 Ω	±5%
R_3	Resistor	2.2 k Ω	±5%
C_1	Capacitor	10 nF	±5%
R7	Potentiometer	2 kΩ	±5%
C_2	Capacitor	100 nF	±5%
L1	Inductor	18 mH	±10%
Memristor	HP Memristor	Ron=100 Roff=100K Rinit=1K D=10N uv=1F p=1	

The circuit's dynamics is studied in the same manner presented earlier, by changing the resistor's R7 value range from 0 to 2kΩ. As expected, while its resistance increases, the phenomenon of period doubling is obtained leading to chaotic behavior as illustrated in sequences from Fig. 7.

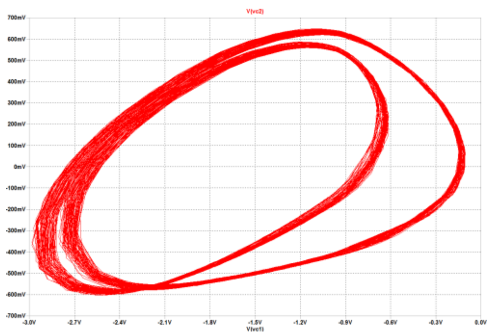
While reducing the variable resistor R6 from 2kΩ to 1.4kΩ Chua's circuit exhibits a cascade of period doubling bifurcations (Fig.7.a) to period–two (Fig. 7b), period-four (Fig.7.c) and so on untill a spiral Chua's attractor is obtained (Fig.7.d). By changing the resistance R7 in the value range [0-2kΩ] it was found that for the same value of 1.6kΩ the circuit behaves as a periodic oscillator, sustained by Chua's attractor or "Double Scroll Attractor" shown in Fig.7.e. For a sufficiently low resistance R7 value of 1.4kΩ the blue sky catastrophe occurs (Fig. 7.f).Another characteristic of the circuit is the unexpected disappereance of the chaotic attractor for a small period of time, when a periodic solution can be observed, followed by an immediate reappearance of the initial attractor.

Another interesting phenomenon observed is that such manner of changing R values causes as well a size modification for each attractor obtained, hence the period one orbit is large, period–two is smaller, period-four even smaller and at the end the double scroll decreases noticeably befores it vanishes altogether. The simulation results confirm that period doubling is a phenomena usualy described by nonlinear dynamical systems and period doubling cascades are a common way to achieving chaos.

The I-V characteristic of the op amp based nonlinear resistor (Fig.8) differs from three segment piecewise-linear one defined by Chua in Fig.2 for it has four segments.It can also be observed that the I-V curve has a greater number of slopes than the one presented in Fig.5 proving that the implemention of Chua's circuit with operational amplifiers it is has a greater non-linearity than the one with diodes. The LTSpice simulation results confirm that the new memristive circuits proposed for the impelemtation of Chus's oscillator can maintain and enhance the chaotic behavior.



a.



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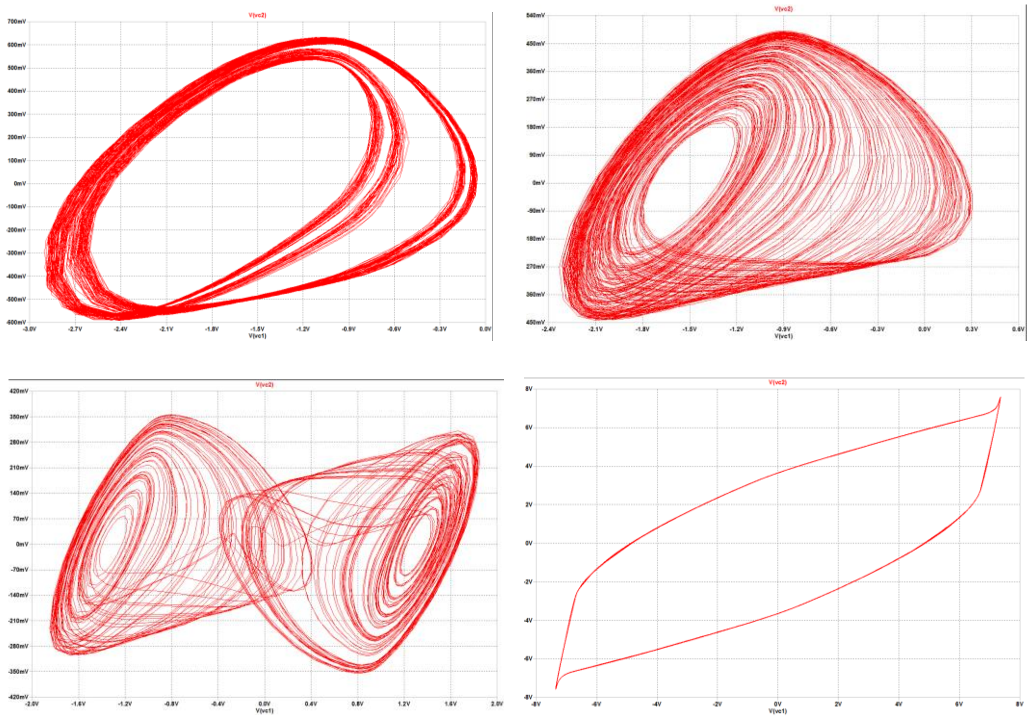

e. f.

Fig. 7. Typical bifurcation sequences encountered in Chua's circuit shown in Fig.6 (horizontal axis V_2, vertical axis V_1):(a) R6 = 2kΩ period-1, (b) R6 = 1.95kΩ period-2, (c) R6 = 1.83kΩ period-4, (d) R6 = 1.72 kΩ spiral Chua’s attractor, (e) R6= 1.6 kΩ double-scroll Chua’s attractor, (f) R6 = 1.4 kΩ corresponding to boundary crisis.

The output signals are random as expected, leading therefore to a double scroll attractor specific to chaos phenomenon. The advantage of the modified version of Chua’s oscillator introduced in this paper is related to the power consummation as memristors require less voltage.

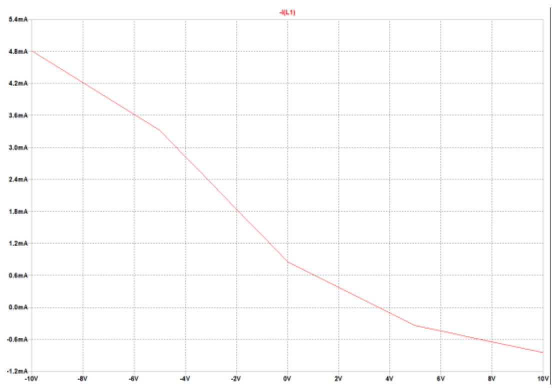

Fig.8 .I-V carcateristic for N_R

The original circuit had a total power consumtion of 49.1 mW which decreased to 44.33 mW in the memristive circuit, hence the proposed circuit with an HP memristor is 9.53 per cent more efficient than the original circuit. The main disadvantage is that the circuit is non-reliable for commercial even though the researchers of memristive devices are widely increasing in the last years. It is hoped that the analysis presented in this paper will conduct to enhanced and efficient further studies of fully integrated low-voltage and low-power chaotic circuits.

6. Conclusions

This paper presents a new approach on the implemention of a random numeric sequence generator by using an HP memristor integrated into the well-known Chua circuit. The proposed circuit combines the classical implementations available so far where the nonlinear element was modeled by diodes with an innovative approach by adding a memristor, a new type of nonlinear circuit element that is still in the research stage nowadays. Both implementations proposed in this paper are able to generate a random output signal and show a chaotic character supported by the attractor patterns obtained and the phenomenon of period doubling. In conclusion, the analysis and numerical simulations incuired indicate that the complex dynamics of the circuit are heavily dependent on the initial state of the memristor, except for the circuit parameters. Hence, the memory of initial state of the memristor plays an important role on the appearance of complicated transient transition dynamics.

R E F E R E N C E S

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