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ADOMIAN DECOMPOSITION METHOD FOR DETECTION OF CHAOS IN THE RUCKLIGE SYSTEM

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The Rucklidge system is an important model for the two-dimensional magneto-convection. The convenient Adomian decomposition method is adopted to propose a numerical formula. The chaotic behaviors are discussed accurately. The result reveals the method is reliable for chaos detection and provides a tool to investigate dynamics numerically.

Key words: Adomian decomposition method; Analytical method; Chaos
MSC: 34A45; 65P20

1. Introduction

Chaos frequently appears everywhere in fluid, quantum mechanics, control engineering systems, biology and chemistry [1-7]. The chaotic behaviors in the two-dimensional magneto-convection were firstly reported in [8] by the following Rucklidge system

$$\begin{cases} \dfrac{dx}{dt} = -ax + by - yz, \\ \dfrac{dy}{dt} = x, \\ \dfrac{dz}{dt} = -z + y^2, \end{cases} \tag{1}$$

where $(x, y, z) \in \mathbb{R}^3, (a,b) \in \mathbb{R}^2$. It is a quadratic and Lorenz-like model.The integrability [9], the limit cycles [10], the chaos synchronization [11] and the feedback controller [12] et al., have been discussed very recently.

Many researchers have made much effort to analytical and numerical methods for solving the chaotic systems. For example, the variational iteration method (VIM) [13] and the Adomian decomposition method (ADM) [14]

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are the most often used analytical ones. The obtained solutions are global approximate ones and only efficient near the initial point. The analytical solutions result in truncated errors. Due to the high sensitivity of chaos, the analytical methods are not suitable to directly investigate chaotic systems. The numerical methods are proved to be efficient to detect chaotic behaviors such as the Euler method, the predict-corrector method and the finite difference method [15] but involve in complicated numerical compuation and stability analysis. This paper applies the convenient ADM [16-18] to numerically solve the Rucklidge system and detects the chaos.

Consider a general differential equation

$$\frac{du}{dt}+R[u]+N[u]=0 \tag{2}$$

subjected to the initial condition $u(0)=C$ and C is a constant. The term $R[u]$ is a linear operator and $N[u]$ is a nonlinear one. The ADM can decompose the nonlinear terms of $N[u]$ into linear ones and accelerate the convergences of the analytical solution

$$\begin{cases} u_{n+1}=-\int_0^t R[u_n]d\tau-\int_0^t A_n d\tau, 0\le n \\ u_0=C, \end{cases} \tag{3}$$

where $u=\sum_{i=0}^{\infty}u_n$ and $\sum_{i=0}^{n}A_i$ is the approximate representation of the $N[u]$.

In the classical ADM [14], the A_n is calculated by

$$A_n=\frac{1}{n!}\frac{\partial^n}{\partial\lambda^n}(N[\sum_{k=0}^{\infty}u_k\lambda^k])_{\lambda=0}. \tag{4}$$

In this paper, we adopt the novel and the convenient method [16-18] and give the chaotic solution of the Rucklidge system. Let's revisit the basics firstly.

2 Numerical schemes based on the Adomian decomposition method

2.1 Preliminaries

Duan [17] very recently proposed a convenient way to calculate the Adomian polynomials as

$$A_n=\frac{1}{n}\sum_{k=0}^{n-1}(k+1)u_{k+1}\frac{dA_{n-1-k}}{du_0} \tag{5}$$

as well as the case of the m-variable

$$A_n = \frac{1}{n}\sum_{i=1}^{m}\sum_{k=0}^{n-1}(k+1)u_{i,k+1}\frac{\partial A_{n-1-k}}{\partial u_{i,0}}. \tag{6}$$

For the single variable case, $N[u]=f(u)$, we only list the first three as

$$\begin{cases} A_1 = u_1 \dfrac{df(u_0)}{du_0}, \\ A_2 = \dfrac{1}{2}u_1^{\,2}\dfrac{d^2 f(u_0)}{du_0^{\,2}} + u_2\dfrac{df(u_0)}{du_0}, \\ A_3 = \dfrac{1}{6}u_1^{\,3}\dfrac{d^3 f(u_0)}{du_0^{\,3}} + u_2u_1\dfrac{d^2 f(u_0)}{du_0^{\,2}} + u_3\dfrac{df(u_0)}{du_0}. \end{cases} \tag{7}$$

Although the same result can be obtained by (4), one spend less time to calculate and the higher order numerical solution becomes possible for both differential and fractional differential equations. Now we use the general steps in [19] as

Step I：Expand $u(t)$ as the Taylor series

$$u = \sum_{i=0}^{\infty} c_i (t-t_0)^i \tag{8}$$

and u_n is denoted as $\sum_{i=0}^{n} c_i (t-t_0)^i$.

Use the iteration formula

$$\begin{cases} c_{n+1} = -R[c_n] - A_n[c_0, c_1, ..., c_n], 0 \le n, \\ c_0 = C \end{cases} \tag{9}$$

to obtain $(n+1)$-th order approximate solution u_{n+1} which can be re-written as

$$u_{n+1} = \varphi(t, t_0, C) \tag{10}$$

Step II : Set the length size and the node number to h and N . Substitute $t=t_1$ into (9) and one can obtain the value u_1^*

$$u_1^* = \varphi(t_1, t_0, C) \tag{11}$$

Step III：Replace t_0 and C with t_1 and u_1^* in (9). We can obtain

$$u_{n+1} = \varphi(t, t_1, u_1^*) \tag{12}$$

Similarly, one can have the value u_2^*

$$u_2^* = \varphi(t_2, t_1, u_1^*) \tag{13}$$

As a result, we can obtain all the information of the numerical solutions

$$u_0^*, u_1^*, ..., u_N^* \tag{14}$$

where u_0^* is the initial value

$$u_0^*(0) = C \tag{15}$$

Using the Logistic equation as an example

$$\frac{du}{dt} = 4u(1-u), u(0) = 0.3, \tag{16}$$

Let $h = 0.1$ and $N = 1000$. We check the above algorithm's validness and see the numerical formula's high accuracy in comparison with the exact solution

$$u=\frac{3}{3+7e^{-4t}}.$$

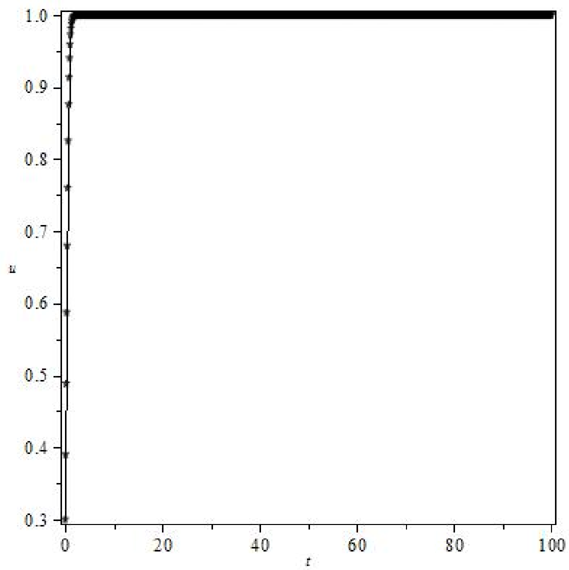

Fig 1 "*" Numerical solution and "_" exact solution of the Logistic equation

2. 2 Numerical solutions of the Rucklidge system

For the three variable case of the Rucklidge system, we can set

$$\begin{cases} x = \sum_{i=0}^{\infty} c_{i,1}(t-t_0)^i, \\ y = \sum_{i=0}^{\infty} c_{i,2}(t-t_0)^i, \\ z = \sum_{i=0}^{\infty} c_{i,3}(t-t_0)^i. \end{cases} \tag{17}$$

we only need to handle the nonlinear terms yz and y^2 as

$$\begin{cases} A_{1,0} = c_{0,2}c_{0,3}, \\ A_{1,1} = c_{1,2}c_{0,3} + c_{0,2}c_{1,3}, \\ A_{1,2} = c_{1,2}c_{1,3} + c_{2,2}c_{0,3} + c_{0,2}c_{2,3}, \\ \quad \vdots \end{cases} \tag{18}$$

and

$$\begin{cases} A_{3,0} = c_{0,2}^2, \\ A_{3,1} = 2c_{1,2}c_{0,2}, \\ A_{3,2} = c_{1,2}^2 + 2c_{0,2}c_{2,2}, \\ \quad \vdots \end{cases} \tag{19}$$

Here $A_{1,i}$ and $A_{3,i}$ are the Adomian polynomials of yz and y^2, respectively.

Consider $(a,b) = (2, 6.7)$ in the Rucklidge system. Set the length size $h = 0.05$ and the node number $N = 10000$. We can obtain the chaotic solutions $x(ih), y(ih)$ and $z(ih)$ which are shown in Figs. 2-4. We note that the solutions are very similar as the one in Lorenz system but have a more accurate description of convection in the parameter regime. The Rucklidge attractor is given in Fig. 5.

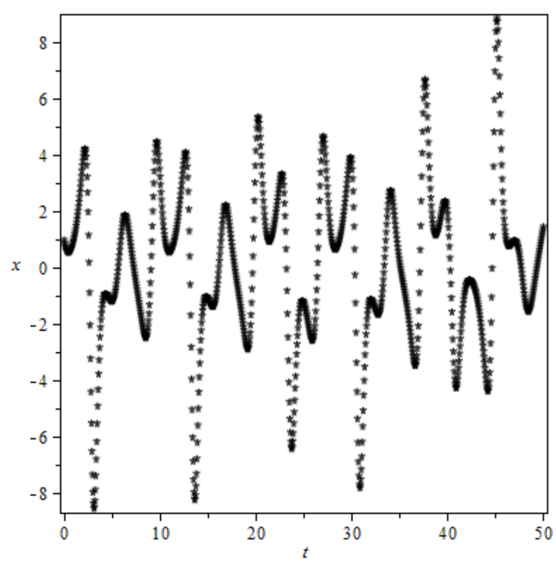

Fig 2 "*" Numerical solution x of the Rucklidge system

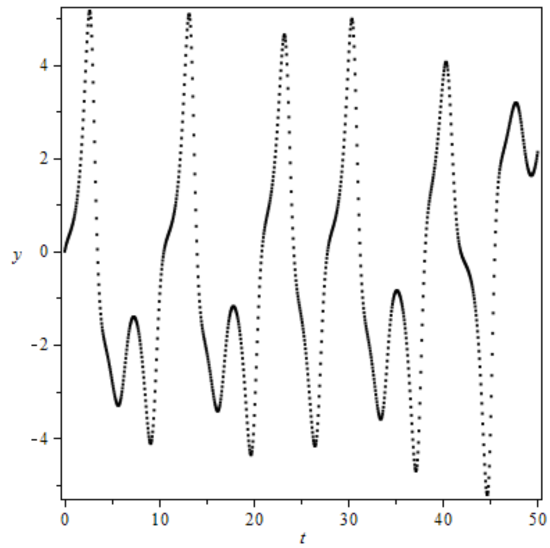

Fig 3 "." Numerical solution y of the Rucklidge system

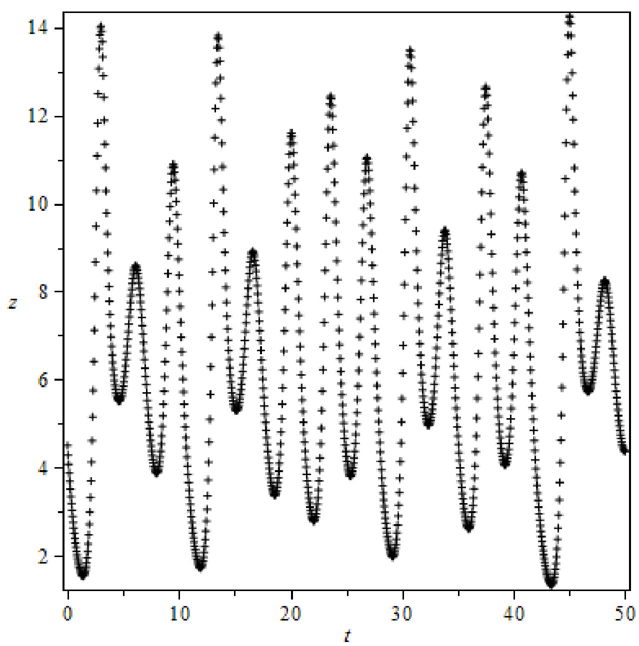

Fig 4 "+" Numerical solution z of the Rucklidge system

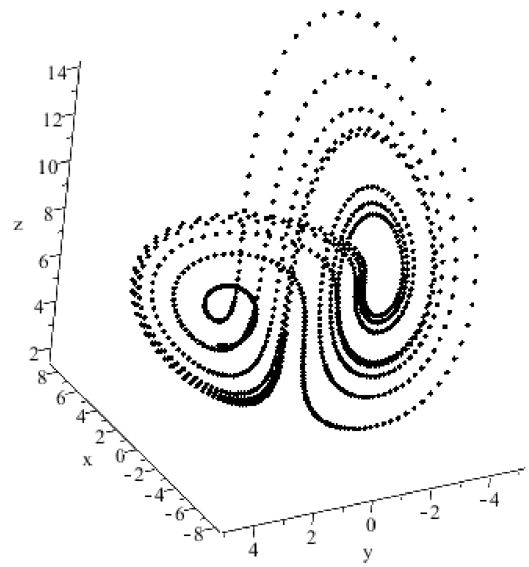

Fig 5 The Rucklidge attractor in the phase space

5. Conclusions

This study adopts the convenient ADM to give numerical schemes for chaotic solution of the famous Rucklidge system and the attractor is numerically shown. The scheme holds the traits of the analytical and the numerical methods. It's easy to use without any tedious numerical calculation. Duan's Adomian polynomial is a new one and can efficiently compute for higher truncated order even $n = 50$ or more while the classical ADM only for about $n = 10$. Particularly, the result shows that the new method improves the efficiency and saves the computation time. The error and convergence analysis is the same as that given in [20-22]. The numerical solution given in this way depicts the nonlinear dynamics more accurately and the numerical method provides an alternative method to investigate the chaotic systems.

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