# FLIGHT DYNAMICS FOR A BWB UAV

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*The purpose of this paper is to evaluate the dynamic behavior of a model plane using the formulas from Etkin [1]. The model was adapted for a blended wing-body (BWB) UAV with additions from Nelson [2] and Cook [4]. In order to have a complete dynamic model, the other references were used. Dynamic evaluation, in our case, resumed to the evaluation of step response at 1 degree elevator deflection.*

**Keywords**: dynamic model, plane polar, lift and drag coefficient, transfer functions, Laplace transform.

## 1. Introduction

The plane to which we refer is a flying wing configuration. An important element of the geometry is that the wing is not exactly distinguished from the fuselage and because of this reason the UAV is considered a blended wing-body UAV (BWB), the UAV having the loft between the wing and the fuselage quite generously. In the figure Fig. 1. the general geometry of the plane is outlined.

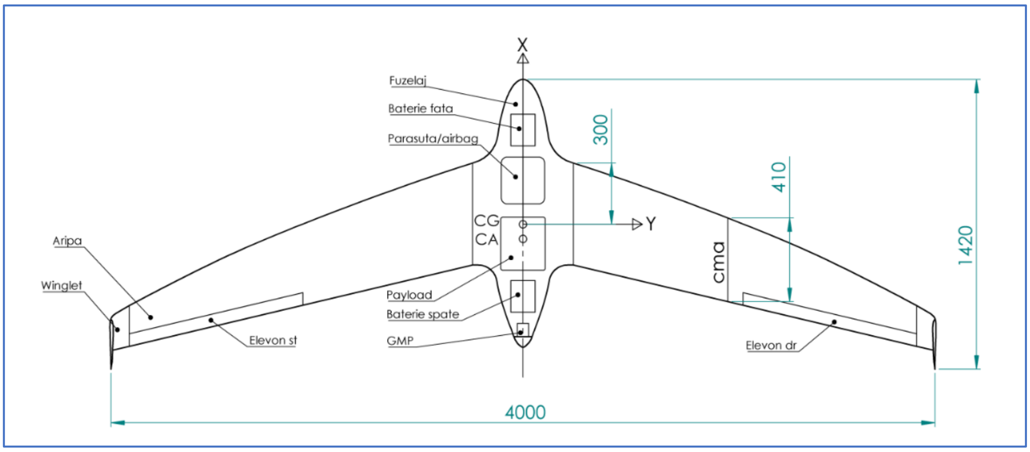


Fig. 1. Top view UAV-BWB

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## 2. Aerodynamic properties

The aerodynamic coefficients for this model are expressed in the reference system of the plane illustrated in Fig. 1. These coefficients are generally based on angular speeds (p, q, r), flight incidence and side-slip angle (α, β), geometry changes meaning elevon deflection (δe) and positioning of the gravity center. In our case we have only one value for the gravity center, 300 mm referenced from the front of the wing embed with the fuselage.

The longitudinal and lateral movement equations linearized and decoupled are shown in Fig. 2 and Fig. 3.

$$\begin{bmatrix} \Delta\dot{u} \\ \dot{w} \\ \dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{X_u}{m} & \frac{X_w}{m} & 0 & -g\cos\theta_o \\ \frac{Z_u}{m-Z_{\dot{w}}} & \frac{Z_w}{m-Z_{\dot{w}}} & \frac{Z_q+mu_o}{m-Z_{\dot{w}}} & \frac{-mg\sin\theta_o}{m-Z_{\dot{w}}} \\ \frac{1}{I_y}\left[M_u+\frac{M_{\dot{w}}Z_u}{(m-Z_{\dot{w}})}\right] & \frac{1}{I_y}\left[M_w+\frac{M_{\dot{w}}Z_w}{(m-Z_{\dot{w}})}\right] & \frac{1}{I_y}\left[M_q+\frac{M_{\dot{w}}(Z_q+mu_o)}{(m-Z_{\dot{w}})}\right] & -\frac{M_{\dot{w}}\,mg\sin\theta_o}{I_y(m-Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ w \\ q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta X_c}{m} \\ \frac{\Delta Z_c}{m-Z_{\dot{w}}} \\ \frac{\Delta M_c}{I_y}+\frac{M_{\dot{w}}}{I_y}\frac{\Delta Z_c}{(m-Z_{\dot{w}})} \\ 0 \end{bmatrix}$$

$$\Delta\dot{x}_E = \Delta u\cos\theta_o + w\sin\theta_o - u_o\,\Delta\theta\sin\theta_o$$

$$\Delta\dot{z}_E = -\Delta u\sin\theta_o + w\cos\theta_o - u_o\,\Delta\theta\cos\theta_o$$

Fig. 2. Motion equations on the longitudinal channel, Etkin [1]

$$\begin{bmatrix} \dot{v} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{Y_v}{m} & \frac{Y_p}{m} & \left(\frac{Y_r}{m}-u_o\right) & g\cos\theta_0 \\ \left(\frac{L_v}{I'_x}+I'_{zx}N_v\right) & \left(\frac{L_p}{I'_x}+I'_{zx}N_p\right) & \left(\frac{L_r}{I'_x}+I'_{zx}N_r\right) & 0 \\ \left(I'_{zx}L_v+\frac{N_v}{I'_z}\right) & \left(I'_{zx}L_p+\frac{N_p}{I'_z}\right) & \left(I'_{zx}L_r+\frac{N_r}{I'_z}\right) & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix} \begin{bmatrix} v \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} \frac{\Delta Y_c}{m} \\ \frac{\Delta L_c}{I'_x}+I'_{zx}N_c \\ I'_{zx}\Delta L_c+\frac{\Delta N_c}{I'_z} \\ 0 \end{bmatrix}$$

$$\dot{\psi} = r\sec\theta_o$$

$$\Delta\dot{y}_E = u_o\psi\cos\theta_o + v$$

$$I'_x = (I_xI_z - I_{zx}^2)/I_z$$

$$I'_z = (I_xI_z - I_{zx}^2)/I_x$$

$$I'_{zx} = I_{zx}/(I_xI_z - I_{xz}^2)$$

Fig. 3. Lateral-directional Motion equations, Etkin [1]

Each of the forces and moments used in the above equations can be obtained by their partial derivation according to the illustrated variables. However, because we apply our relationships on a flying wing, we can express the force coefficient after the X-axis, Cx, depending on the Z-axis force coefficient, Cz (plane polar).

## 3. Airplane dynamics

For the aerodynamic study it is considered the reference movement (horizontal unaccelerated rectilinear flight, elevon as an elevator with deflection of -6 degrees) characterized by the values of the parameters from Table 1 and *Table 2*, considered as the input data for the reference movement. The aerodynamic coefficients *Cx*, *Cz* and *Cm* are estimated with XFLR, an open source software started by Mark Drela and specially created for low Reynolds numbers. Forces and moments illustrated in Fig. 2 and Fig. 3 can also be estimated with XFLR but in this case I have used the formulas from Etkin [1] and from other references applied to a BWB UAV.

*Table 1.*

**Basic input data**

| Parameter | Symbol | Value | U.M. |
|---|---|---|---|
| Wing area | S | 1,569 | $m^2$ |
| Wing-span | b | 4 | m |
| Average aerodynamic chord | Cma | 0,41 | m |
| Mass | m | 10,517 | kg |
| Flight Speed (TAS) | $u_0$ | 18 | m/s |
| Air density | ρ | 1,225 | $Kg/m^3$ |
| Values for reference movement (at equilibrium) | | | |
| Angle of incident | α | 3,2 | grade |
| Lateral sliding angle | β | 0 | grade |
| Elevon deflection (aileron) | $\delta_a$ | 0 | grade |
| Elevon deflection (elevator) | $\delta_e$ | -6 | grade |
| Lift coefficient | Cz | 0,329567 | - |
| Drag coefficient | Cx | 0,014142 | - |
| Lateral force coefficient | Cy | 0 | - |

*Table 2*

**Inertia moments**

| Mass and inertia moments | UAV-BWB | M.U. |
|---|---|---|
| BWB mass | 10,517 | $Kg$ |
| Inertia moment referenced to $X$ axis, $I_{xx}$ | 4,102192 | $Kg \cdot m^2$ |
| Inertia moment referenced to $Y$ axis, $I_{yy}$ | 1,353545 | $Kg \cdot m^2$ |
| Inertia moment referenced to $Z$ axis, $I_{zz}$ | 5,417290 | $Kg \cdot m^2$ |

### 3.1 Longitudinal dynamics

Longitudinal dynamic can be written as a linear system (1):

$$\boldsymbol{E}\cdot\dot{\boldsymbol{X}}=\boldsymbol{A}\cdot\boldsymbol{X}+\boldsymbol{B}\cdot\delta e \tag{1}$$

In the equation (1), because the derivatives $X_{\dot{w}}, Z_{\dot{w}}, M_{\dot{w}}, M_{\theta}$ are considered null, the matrix **E** becomes unity matrix. The influence of flight altitude has been neglected (air density variation with altitude) for longitudinal dynamics because the influence is negligible. The eigenvalues of the stability matrix shall be determined in MATLAB with command eig(**A**), the following results were obtained: $eig(\boldsymbol{A})=(-4.01795\pm 6{,}86701i;\ -0.033906\pm 0.758788i)$. The system is stable if the eigenvalues have the real part negative.

*Table 3*

**Eigenvalues – longitudinal dynamics**

| Mode | Values |
|---|---|
| Short period mode | $\lambda_{1/2}-4.01795\pm 6{,}86701i$ |
| Phugoid mode | $\lambda_{3/4}-0.033906\pm 0.758788i$ |

The stability matrix **A**:

$$\boldsymbol{A}=\begin{bmatrix} \frac{X_u}{m_{av}} & \frac{Z_u}{m_{av}-Z_{wp}} & 0 & -g\cdot\cos(\theta_0) \\ \frac{X_w}{m_{av}} & \frac{Z_w}{m_{av}-Z_{wp}} & \frac{Z_q+m_{av}u_0}{m_{av}-Z_{\dot{w}}} & \frac{-m_{av}\cdot g\cdot\sin(\theta_0)}{m_{av}-Z_{\dot{w}}} \\ \frac{1}{I_y}\left[M_u+\frac{M_{\dot{w}}Z_u}{m_{av}-Z_{\dot{w}}}\right] & \frac{1}{I_y}\left[M_w+\frac{M_{\dot{w}}Z_w}{m_{av}-Z_{\dot{w}}}\right] & \frac{1}{I_y}\left[M_q+\frac{M_{\dot{w}}(Z_q+m_{av}u_0)}{m_{av}-Z_{\dot{w}}}\right] & \frac{M_{\dot{w}}m_{av}\cdot g\cdot\sin(\theta_0)}{I_y(m_{av}-Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{2}$$

Control matrix **B**:

$$\boldsymbol{B}=\begin{bmatrix} \frac{X_{\delta e}}{m} & \frac{X_{\delta p}}{m} \\ \frac{Z_{\delta e}}{m_{av}-Z_{wp}} & \frac{Z_{\delta p}}{m_{av}-Z_{wp}} \\ \frac{M_{\delta e}}{I_y}+\frac{M_{wp}Z_{\delta e}}{I_y(m_{av}-Z_{wp})} & \frac{M_{\delta p}}{I_y}+\frac{M_{wp}Z_{\delta p}}{I_y(m_{av}-Z_{wp})} \\ 0 & 0 \end{bmatrix} \tag{3}$$

Looking at the eigenvalues obtained we can conclude that we don't have unstable modes on longitudinal channel. The real parts of the eigenvalues are negative.

Transfer functions, longitudinal channel

The transfer functions on the longitudinal channel G(s) are obtained by applying the Laplace transformation to the equation $\dot{\boldsymbol{X}}=\boldsymbol{A}\cdot\boldsymbol{X}+\boldsymbol{B}\cdot\boldsymbol{U}$.

$$\Delta_u(s) = \begin{vmatrix} B_{0,0} & A_{0,1} & A_{0,2} & A_{0,3} \\ B_{1,0} & A_{1,1}-s & A_{1,2} & A_{1,3} \\ B_{2,0} & A_{2,1} & A_{2,2}-s & A_{2,3} \\ B_{3,0} & A_{3,1} & A_{3,2} & A_{3,3}-s \end{vmatrix} \rightarrow$$
$$\rightarrow 5{,}43138944 \cdot s^2 - 259{,}85005113 \cdot s - 2530{,}21939750$$
$$\Delta_w(s) = \begin{vmatrix} A_{0,0}-s & B_{0,0} & A_{0,2} & A_{0,3} \\ A_{1,0} & B_{1,0} & A_{1,2} & A_{1,3} \\ A_{2,0} & B_{2,0} & A_{2,2}-s & A_{2,3} \\ A_{3,0} & B_{3,0} & A_{3,2} & A_{3,3}-s \end{vmatrix} \rightarrow$$
$$\rightarrow 19{,}96442274 \cdot s^3 + 835{,}80952663 \cdot s^2 + 58{,}52247097 \cdot s - 169{,}07214419$$
$$\Delta_q(s) = \begin{vmatrix} A_{0,0}-s & A_{0,1} & B_{0,0} & A_{0,3} \\ A_{1,0} & A_{1,1}-s & B_{1,0} & A_{1,3} \\ A_{2,0} & A_{2,1} & B_{2,0} & A_{2,3} \\ A_{3,0} & A_{3,1} & B_{3,0} & A_{3,3}-s \end{vmatrix} \rightarrow$$
$$\rightarrow 49{,}64528678 \cdot s^3 + 261{,}49251631 \cdot s^2 + 13{,}40558154 \cdot s$$
$$\Delta_\theta(s) = \begin{vmatrix} A_{0,0}-s & A_{0,1} & A_{0,2} & B_{0,0} \\ A_{1,0} & A_{1,1}-s & A_{1,2} & B_{1,0} \\ A_{2,0} & A_{2,1} & A_{2,2}-s & B_{2,0} \\ A_{3,0} & A_{3,1} & A_{3,2} & B_{3,0} \end{vmatrix} \rightarrow$$
$$\rightarrow 49{,}64528678 \cdot s^3 + 261{,}49251631 \cdot s^2 + 13{,}40558154 \cdot s$$
$$\Delta(s) = |\boldsymbol{A} - \boldsymbol{I} \cdot \lambda| =$$
$$= s^4 + 8{,}19184072 \cdot s^3 + 66{,}01781558 \cdot s^2 + 9{,}34048478 \cdot s + 2530{,}21939750$$

$$G_{u\delta e} = -\frac{\Delta u(s)}{\Delta(s)}$$
$$G_{w\delta e} = -\frac{\Delta w(s)}{\Delta(s)}$$
$$G_{q\delta e} = -\frac{\Delta q(s)}{\Delta(s)} \tag{4}$$
$$G_{\theta\delta e} = -\frac{\Delta \theta(s)}{\Delta(s)}$$

In order to illustrate the time response of the system (t = 0:100) at a 1-degree elevator deflection ($\delta e = 1\text{deg}$), the reversed Laplace transform is calculated and the time evolution of disturbances can be found in (5).

$$u(t) = invLaplace(G_{u\delta e}) \cdot \delta e$$
$$w(t) = invLaplace(G_{w\delta e}) \cdot \delta e$$
$$q(t) = invLaplace(G_{q\delta e}) \cdot \delta e \tag{5}$$
$$\theta(t) = invLaplace(G_{\theta\delta e}) \cdot \delta e$$

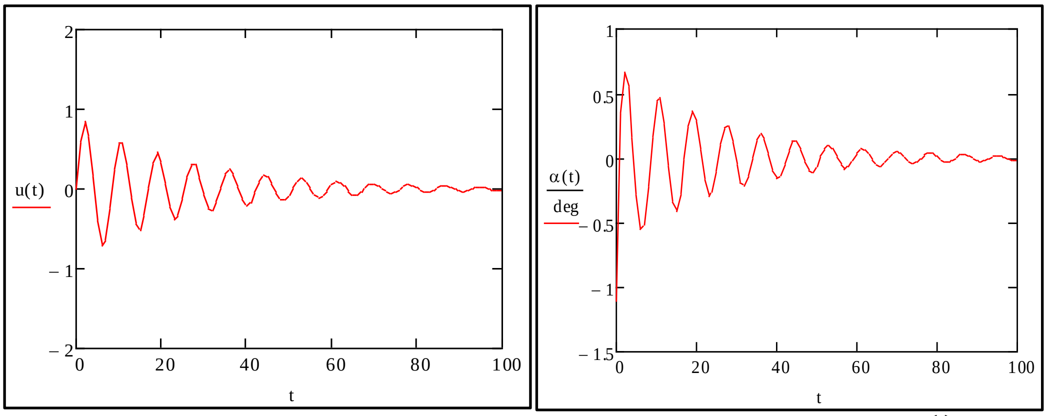


Fig. 4. Step response at 1-degree deflection for disturbances u(t) and $\alpha(t) = \frac{w(t)}{u_0}$

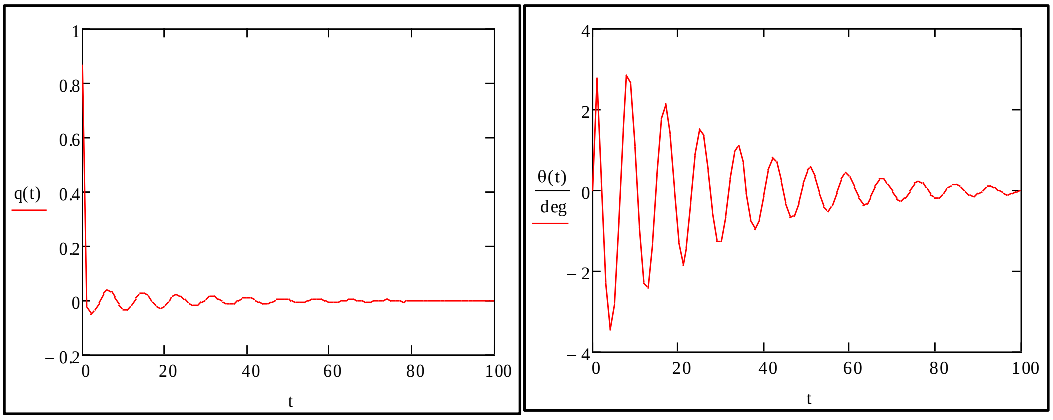


Fig. 5. Step response at 1-degree deflection for disturbances q (angular speed – Y axis) and θ

## 3.2 Lateral-directional dynamics

The lateral-directional dynamics of an airplane can be written (6):

$$\dot{\boldsymbol{X}} = \boldsymbol{A} \cdot \boldsymbol{X} + \boldsymbol{B} \cdot \boldsymbol{U} \tag{6}$$

where matrix **B** is the input matrix, in our case we have only elevons (ailerons). The matrix **A** is represented by the formulas:

$$\boldsymbol{A} = \begin{bmatrix} \dfrac{Y_v}{m_{av}} & \dfrac{Y_p}{m_{av}} & \dfrac{Y_r}{m_{av}} - u_0 & g\cos(\theta_0) \\ \dfrac{L_v}{\grave{I}_x} + \grave{I}_{zx}N_v & \dfrac{L_p}{\grave{I}_x} + \grave{I}_{zx}N_p & \dfrac{L_r}{\grave{I}_x} + \grave{I}_{zx}N_r & 0 \\ \grave{I}_{zx}L_v + \dfrac{N_v}{\grave{I}_z} & \grave{I}_{zx}L_p + \dfrac{N_p}{\grave{I}_z} & \grave{I}_{zx}L_r + \dfrac{N_r}{\grave{I}_z} & 0 \\ 0 & 1 & \tan(\theta_0) & 0 \end{bmatrix}$$

and matrix $\boldsymbol{B} = \begin{bmatrix} \frac{Y_{\delta a}}{m_{av}} \\ \frac{L_{\delta a}}{\grave{I}_x} + \grave{I}_{zx}N_{\delta a} \\ \grave{I}_{zx}L_{\delta a} + \frac{N_{\delta a}}{\grave{I}_z} \\ 0 \end{bmatrix}$

The eigenvalues of the stability matrix **A** are: $eig(\boldsymbol{A}) = [-0{,}077349 \pm 1{,}907326i \mid -0{,}172556 \mid -0{,}0001460334]$

*Table 4*

**Eigenvalues – lateral dynamics**

| Mode | Value |
|---|---|
| Spiral mode | $\lambda_1 = -0{,}0001460334$ |
| Rolling mode | $\lambda_2 = -0{,}172556$ |
| Dutch roll mode (Side oscillation mode) | $\lambda_3 = -0{,}077349 \pm 1{,}907326i$ |

Looking at the values obtained we can conclude that we don't have unstable modes. The real parts of the eigenvalues are negative.

Transfer functions, lateral-directional channel:

$$\Delta_v(s) = \begin{vmatrix} B_{0,0} & A_{0,1} & A_{0,2} & A_{0,3} \\ B_{1,0} & A_{1,1} - s & A_{1,2} & A_{1,3} \\ B_{2,0} & A_{2,1} & A_{2,2} - s & A_{2,3} \\ B_{3,0} & A_{3,1} & A_{3,2} & A_{3,3} - s \end{vmatrix} \rightarrow$$

$$\rightarrow 21{,}450664 \cdot s^2 - 279{,}547332 \cdot s - 26{,}681181$$

$$\Delta_p(s) = \begin{vmatrix} A_{0,0} - s & B_{0,0} & A_{0,2} & A_{0,3} \\ A_{1,0} & B_{1,0} & A_{1,2} & A_{1,3} \\ A_{2,0} & B_{2,0} & A_{2,2} - s & A_{2,3} \\ A_{3,0} & B_{3,0} & A_{3,2} & A_{3,3} - s \end{vmatrix} \rightarrow$$

$$\rightarrow -29{,}730626 \cdot s^3 - 9{,}774926 \cdot s^2 = 110{,}569858 \cdot s$$

$$\Delta_r(s) = \begin{vmatrix} A_{0,0}-s & A_{0,1} & B_{0,0} & A_{0,3} \\ A_{1,0} & A_{1,1}-s & B_{1,0} & A_{1,3} \\ A_{2,0} & A_{2,1} & B_{2,0} & A_{2,3} \\ A_{3,0} & A_{3,1} & B_{3,0} & A_{3,3}-s \end{vmatrix} \rightarrow$$
$$\rightarrow -29{,}730626 \cdot s^3 \pm 7{,}721414 \cdot s^2 + 0{,}051922 \cdot s - 78{,}991921$$

$$\Delta_\phi(s) = \begin{vmatrix} A_{0,0}-s & A_{0,1} & A_{0,2} & B_{0,0} \\ A_{1,0} & A_{1,1}-s & A_{1,2} & B_{1,0} \\ A_{2,0} & A_{2,1} & A_{2,2}-s & B_{2,0} \\ A_{3,0} & A_{3,1} & A_{3,2} & B_{3,0} \end{vmatrix} \rightarrow$$
$$\rightarrow -29{,}730626 \cdot s^2 - 10{,}621794 \cdot s - 144{,}986485$$
$$\Delta(s) = |\boldsymbol{A} - \boldsymbol{I} \cdot s| =$$
$$= s^4 + 0{,}327631 \cdot s^3 + 3{,}669839 \cdot s^2 + 0{,}629165 \cdot s + 2{,}186745$$

$$\begin{aligned} G_{v\delta a} &= -\frac{\Delta_v(s)}{\Delta(s)} \\ G_{p\delta a} &= -\frac{\Delta_p(s)}{\Delta(s)} \\ G_{r\delta a} &= -\frac{\Delta_r(s)}{\Delta(s)} \\ G_{\phi\delta a} &= -\frac{\Delta\phi(s)}{\Delta(s)} \end{aligned} \tag{7}$$

Similar to the longitudinal channel, in order to calculate the time response of the system (t= 0:50) at 1-degree aileron deflection ($\delta a = 1\text{deg}$) the reversed Laplace transform is used.

$$\begin{aligned} v(t) &= invLaplace(G_{v\delta a}) \cdot \delta a \\ p(t) &= invLaplace(G_{p\delta a}) \cdot \delta a \\ r(t) &= invLaplace(G_{r\delta a}) \cdot \delta a \\ \phi(t) &= invLaplace(G_{\phi\delta a}) \cdot \delta a \end{aligned} \tag{8}$$

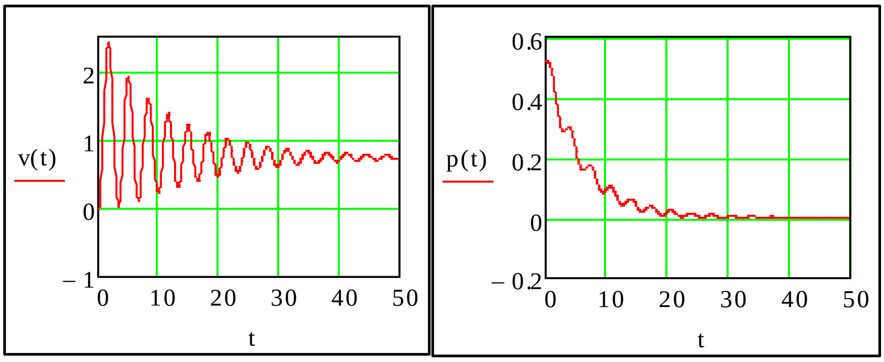


Fig. 6. Step response at 1-degree deflection for disturbances v (lateral speed) and p (angular speed – X axis)

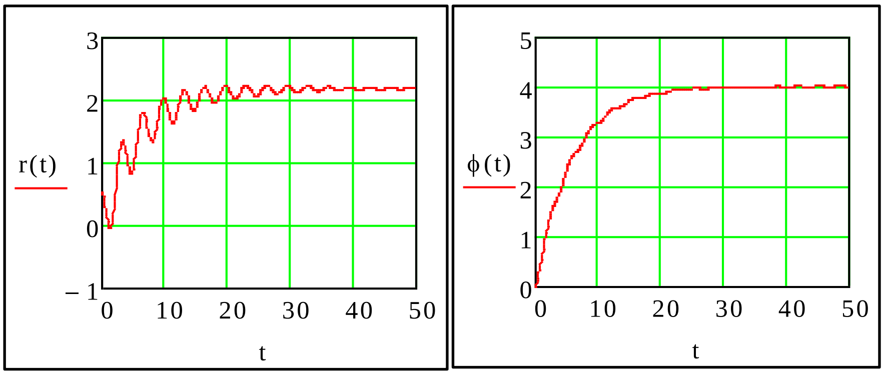


Fig. 7. Step response at 1-degree deflection for disturbances r (angular speed - Z axis) and ψ

## 6. Conclusions

Following the aerodynamic calculations resulted that the BWB UAV is stable on both longitudinal and lateral-directional channel. Following a disturbance of 1 degree of elevon deflection the parameters of the system return to its original state. The mathematical model obtained can be used to estimate the aerodynamic behavior of a model plane at low speeds (Mach number is considered 0) thus resulting in low Reynolds numbers. The reference movement was considered the one with the elevon as an elevator at -6 degrees deflection. This mathematical model can be used for other UAVs of the same configuration, BWB flying wing. For any other plane configuration, it’s necessary to review the formulas because other approximations are required.

## R E F E R E N C E S

[1]. B. *Etkin and L. D. Reid*, Dynamics of flight Stability and Control, Third Edition, John Wiley & Sons, 1996.
[2]. *R. C. Nelson*, Flight Stability and automatic control, McGraw-Hill Book Company, 1989.
[3]. *P. Parvu*, "Mecanica zborului aeronavelor" 2019. [Online]. Available: https://aero.curs.pub.ro/2018/course/view.php?id=123.
[4]. *M. V. COOK*, Flight Dynamics Principles Third Edition, 2nd ed., Elsevier Ltd., 1997.
[5]. *D. Raymer*, Aircraft design: A Conceptual Approach, Washington, D.C.: American Institute of Aeronautics and Austronautics, Inc., 1989.