U.P.B. Sci. Bull., Series D, Vol. 74, Iss. 2, 2012 ISSN 1454-2358

MATHEMATICAL MODEL FOR THE STUDY OF THE LATERAL OSCILLATIONS OF THE RAILWAY VEHICLE

Ioan SEBESAN[1], Dan BAIASU[2]

Articolul prezintă un model matematic pentru studiul serpuirii unui vagon de calatori.Modelul include mişcările de clătinare, ruliu şi serpuire ale principalelor elemente constitutive : osiile montate, boghiurile şi cutia. Sistemul de ecuații este scris aplicând metode energetice. Se consideră neliniaritățile induse de profilul neregulat al căii de rulare. Forțele de contact roata sina sunt exprimate utilizând coeficienții de pseudoalunecare stabiliți conform teoriei lineare a lui Kalker.Sistemul de ecuații este rezolvat prin metode numerice.Se determină răspunsul sistemului – vagon de călători pe o linie în aliniament şi palier, viteza critică şi influența caracteristicilor constructive ale vagonului asupra performanțelor acestuia.

The article presents a mathematical model to study a passenger coach hunting motion comprising the lateral displacement, rolling and yawing oscillations for the main constitutive elements: axles, bogies and body. The equation system is written applying energetic methods. The non-linearities determined by the irregular profile of the tracks are considered. The wheel – rail contact forces are expressed using the creepage coefficients established according to Kalker's linear theory. The equations system is solved through numeric methods. The response of the system – passenger coach on tangent track, the critical speed and the influence of the constructive characteristics on its performances are determined.

Keywords: passenger coach, hunting, mathematical model, critical speed, coach' construction

1. Introduction

The lateral railway vehicle dynamics represent a study area of great interest in the actual context where more and more railway administrations implement the high speed trains, which prove to be efficient, economic and ecological transportation means.

Trains circulating with speeds higher than 160 km/h generate vibrations in the vehicle body that induce significant operation problems: running instability, poor ride quality and track wear. From this point of view, an adequate design of the railway vehicles' suspensions holds an important role in maintaining the comfort and safety parameters of trains' operation.

[1] Prof., Dept. of Railway Vehicles, University POLITEHNICA of Bucharest, Romania,e-mail: ioan_sebesan@yahoo.com

[2] PhD. Student, Atelierele CFR Grivita SA, România, e-mail: dan.baiasu@grivita.ro

Kinematic theoretical studies of the rolling gear’s elements lateral and yawing motions [1], [2] have highlighted that the oscillation frequency increases proportionally to the circulation speed. The speed value where the amplitude of the oscillations grows and the vehicle movement becomes unstable is called critical speed. Starting off with this approach, various studies on the railway vehicle's lateral stability have showed the existence of two sources of instability for the railway vehicle:

- the bogie instability, induced by the axles' movement instability ;
- the instability of the body, which appears when, in the low frequency domain, the vehicle body has the tendency of moving along with the bogie.

The dynamic behavior study of the railway vehicle has two directions:

- the dynamic response of the system: simulation of dynamic behavior due to external stimuli, determination of the concentrated mass accelerations and speeds, and implicitly the forces that act upon the vehicle;
- the dynamic stability: the study over the system's stability in various operation conditions.

The mathematical modelling of the rail or of the railway vehicle is frequently used for study or in order to observe the rail's and railway vehicle's interaction with the tracks. The dynamic interaction between the vehicle and the tracks varies depending on the operation conditions, geography, the wheel and rail treads and the weather conditions.

Obtaining a mathematical model for the study of the dynamic behavior in the case of the railway vehicle implies the latter being formed out of rigid bodies inter-connected through weightless suspension elements. A rigid body – be it vehicle or vehicle component – has 6 degrees of freedom corresponding to the movements along the three shifting directions (longitudinal, lateral and vertical) and the rotations around these axes (rolling, pitching and yawing).

Usually, in the case of railway vehicles, the mathematical models account for the body case, the bogies and the wheelsets as rigid bodies. The equations system describing the movement of the mathematical model could thus have 42 quadratic coupled equations. Solving such a system represents a sometimes inconclusive undertaking regarding the vehicle's behavior. According to [1], [3], [4] for small amplitude movements, there is a relatively small connection between the vehicle's oscillations on a vertical and transversal directions, this is why some of the models presented in literature don't take into account the vertical oscillations in the study of movement on lateral direction or the horizontal oscillations for the study of vehicle vertical displacement.

Starting with the 60's, numerous authors have dedicated studies to the lateral oscillations phenomenon (the hunting motion): Wickens (1965), Law and Coperrider (1974), Garg and Dukkipatti (1984), Sebesan (1995), Ahmadian and

Yang (1998), He and McPhee (2002), Fan and Wu (2006), Messouci (2009), Wang and Liao (2009), Zboinski and Dusza (2011) and others.

The mathematical models used in the literature for the study of vehicles, differ depending on the number of degrees of freedom taken into account, the vehicle type, the linear or non-linear treatment of the wheel – rail contact phenomenon, of the forces appearing at the wheel – rail contact, as well as the irregularities of the tracks. The complexity of these models has evolved in proportion as the calculus technique has become more evolved allowing the finding of solutions for more and more complex sets of differential equations using up to 38 degrees of freedom and taking into consideration more and more non-linear aspects of the vehicle – rail interaction.

In paper [5] the authors demonstrated that the critical speed obtained with the help of a model with 6 degrees of freedom is bigger than the one obtained with a system having 10 degrees of freedom; as a consequence it is shown that the precision of the vehicle design increases along with the number of degrees of freedom considered in the calculus.

A large amount of the mechanical models built until now – [4], [5],[6], [11], [12] concerning especially the vibrations of the mounted axle, considering that these determine the vibration regime in the whole vehicle, are of interest because they allow the study of the non-linearities specific to the processes generated by the rolling of the mounted axle on the tracks or the assessment of the importance of various constructive parameters of the vehicle, but cannot represent the phenomena that take place at the level of the case – bogies connection. Moreover, few of the mechanical models presented in the literature are validated through dynamic tests [8], [9], [10], [13].

This article presents a mathematical model of a passenger coach built to simulate the response from the oscillating system to the irregularities of the tracks and establishes the critical speed of the coach. Simulation of the vehicle's response for various values of its constructive parameters facilitates the study of optimization possibilities for the coach's performance.

2. Mechanical model of the passenger coach

The specific construction characteristics of the vehicle were considered in order to elaborate the mechanical model. The dynamic response of the coach on bogies to the tracks' irregularities on a horizontal plan depends strongly on the configuration of the tracks which represent inputs for the vehicle as an oscillating system, and also depends on its circulation speed. An accurate design of the axle suspension can reduce the high frequency vibrations generated by the tracks, by the reduced conicity of the rolling tread and by the wheel – rail contact forces. The central suspension must absorb the vibrations transmitted by the bogies to the coach case in order to maintain the vehicle in an optimum position during circulation, so that it ensures the passengers comfort. Both suspensions also contain dissipative elements that attenuate the vibrations generated by the vehicle's movement.

A reliable model of the vehicle must include both suspension levels, allowing the highlighting of the dynamic characteristics of the vehicle's movement and the study of the relative displacements appearing between the components of the model. The model must facilitate the calculation of the suspension in order to optimize the coach performance. The model can also be used to study the vehicle instability due to the tracks' irregularities and to the inherent auto-induced instability.

The coach case center of mass is located at the h_{cc} height from the separation plane between the box and the bogie's frame – the transversal plane that equally divides the bogie's central suspension's coil spring. This plane is located at a distance h_{cb} from the bogie's mass center.

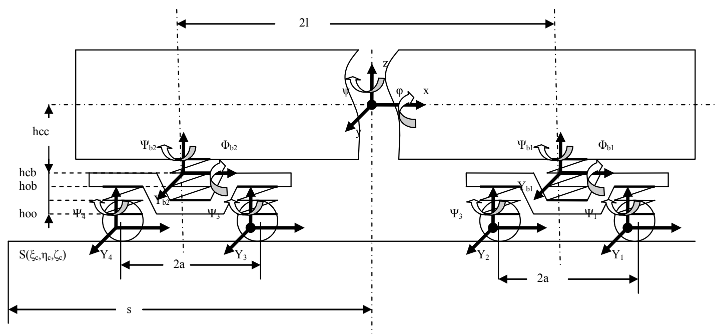

Fig. 1 Mechanical model for passenger coach's hunting (lateral view)

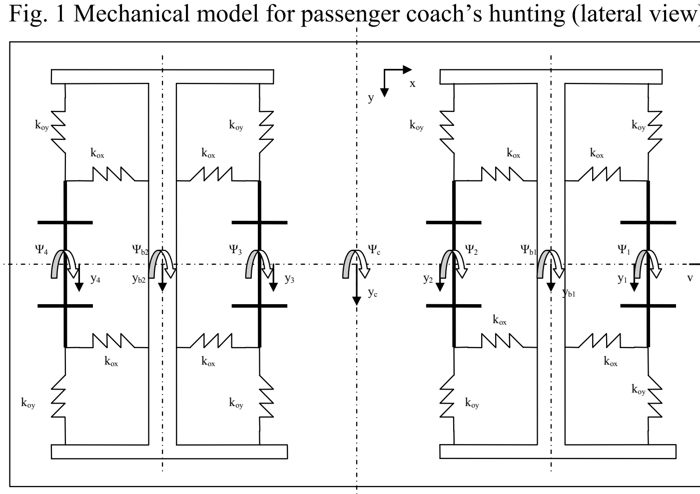

Fig. 2 Mechanical model for passenger coach's hunting (upper view)

It is assumed that all the elastic and damping elements have linear characteristics: the elastic force is directly proportional with the coil spring deformation and the damping force is directly proportional with the dampers' deformation speed. In order to deduce the mechanical model movement equations, a preliminary establishment of the reference systems and coordinates describing the movement of the concentrated mass inside the model was necessary. The mechanical model contains the following elements:

- the coach case;
- the bogies b_j , j=1,2;
- the wheelsets o_i , i=1...4;
- O_c, O_{bj}, O_i – the centers of mass for the mechanical model elements;
- x_c, y_c, z_c, ψ_c, φ_c, θ_c – the displacements, respectively rotations, of the coach case during movement;
- x_{bj}, y_{bj}, z_{bj}, ψ_{bj}, φ_{bj}, θ_{bj} – the displacements, respectively rotations, of the bogies;
- x_{oi}, y_{oi}, z_{oi}, ψ_{oi}, φ_{oi}, θ_{oi} – the displacements, respectively rotations, of the mounted axles;
- h_c – distance between O_c and O_{bj} ;
- h_b – distance between O_{bj} and O_i .

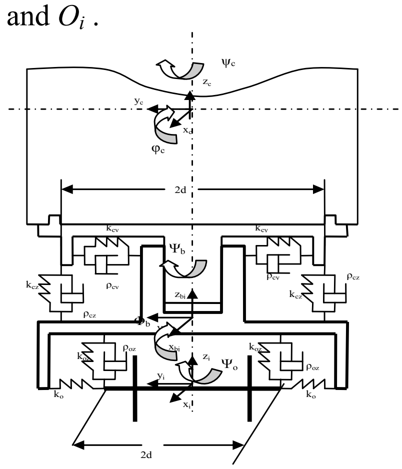

Fig. 3 Mechanical model for passenger coach's hunting (transversal view)

Considering the coach as a system of rigid bodies interconnected through suspension elements, under conditions of geometrical, elastic and inertial symmetry, with identical wheel and rail patterns, the equilibrium position of the coach coincides with its median position in relation to the tracks. The yawing motions of the coach around its equilibrium position were considered to be of relatively small amplitudes, without moving in all the available space in the vehicle slot guide. In this case, the rolling surfaces' contact angles are small, the radii of curvature for the rolling treads remain unchanged and the expression for the centering gravitational force can be linearized. Conicity has been considered as having an equal constant value with the rolling surfaces' effective conicity. The small contact angles create the premises for neglecting the contact forces' vertical components in relation with the wheel loads which can be considered equal to the normal contact forces.

Adopting the hypothesis of the small oscillations implies the existence of transversal accelerations in the small amplitude vehicle which signifies that the load transfers between the wheels of the same axle can be neglected. At the same time, the axle's vertical accelerations at small frequencies characteristic to yawing can be neglected, the axle load can be considered as being constant so that the wheel load is also considered as being constant.

The mechanical model's geometrical and elastic symmetry facilitates the decoupling of the lateral movements from the vertical ones.

In order to study the 4 axles vehicle's lateral oscillations, considering that we are using the simplifying hypotheses previously presented, the mechanical model considers the following degrees of freedom: y_c, ψ_c, φ_c, y_{bj}, ψ_{bj}, φ_{bj}, y_i, ψ_i, where j=1,2 represent the bogies and i=1 – 4 the wheelsets.

Hence, a system results, with 17 degrees of freedom corresponding to the concentrated mass movements that make up the mechanical model.

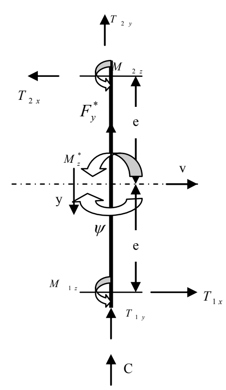

Fig. 4 Forces and moment acting on the wheelset

According to Kalker's theory [7], both the creep tangential forces T_x and T_y and the creep moment M_z that acts in the contact point wheel – rail can be written as:

$$\begin{aligned} T_x &= \chi_x \nu_x Q \\ T_y &= \chi_y \nu_y Q + \chi_s r_0 (\omega_s / v) Q \\ M_z &= -\chi_s r_0 \nu_y Q + \chi_z r_0^2 (\omega_s / v) Q \end{aligned} \tag{1}$$

where the spin creepage is given by the expressions:

$$\omega_{se} = \omega_y \sin\gamma_e \text{ - corresponding to wheel 1}$$
$$\omega_{si} = \omega_y \sin\gamma_i \text{ - corresponding to wheel 2} \quad (2)$$

where $\omega_y = v/r$ represents the angular speed transversal component in the wheel – rail contact point. According to [1], approximate values are indicated for the creepage coefficients: $\chi_x \approx \chi_y = \chi = \frac{300}{\sqrt[3]{Q}} ... \frac{400}{\sqrt[3]{Q}}$ (for Q expressed in tons), which depend on the ratio of the contact ellipse axes.

For the spin coefficient χ_s, the literature recommends a value of 0.83 because it is almost independent in respect to the ratio of the contact ellipse axes.

The χ_z coefficient for a circular contact surface is $\chi_z = 0.0043\sqrt[3]{Q}$ and for a contact surface whose axis length in the running direction is twice, respectively 0.5 times the length of the other axis is $\chi_z = 0.0014\sqrt[3]{Q}$ and, respectively, $\chi_z = 0.0134\sqrt[3]{Q}$.

The χ_z coefficient has a reduced influence over the yawing motion and can be neglected.

The creepages in the contact points of the two axle wheels have the expressions:

$$\begin{aligned} \nu_{1x} &= -\nu_{2x} = -[(\gamma/r_0)y + (e/v)\dot{\psi}] \\ \nu_{1y} &= \nu_{2y} = \dot{y}/v - \psi \\ \omega_{1s} &= -(v/r_0)\gamma_1 \\ \omega_{2s} &= (v/r_0)\gamma_2 \end{aligned} \quad (3)$$

In the contact points, the forces and the moments will have expressions given by the following relations:

$$\begin{aligned} T_{1x} &= -\chi Q(\frac{\gamma}{r_0}y + \frac{e}{v}\dot{\psi}); T_{2x} = \chi Q(\frac{\gamma}{r_0}y + \frac{e}{v}\dot{\psi}) \\ T_{1y} &= \chi Q(\frac{\dot{y}}{v} - \psi) - \chi_s Q\gamma_1; T_{2y} = \chi Q(\frac{\dot{y}}{v} - \psi) + \chi_s Q\gamma_2 \\ M_{1z} &= M_{2z} = -\chi_s Qr(\frac{\dot{y}}{v} - \psi) \end{aligned} \quad (4)$$

The centering force : $C = Q(\gamma_1 - \gamma_2) = c_g y$ (5)

A fixed reference system is considered – $O\xi\eta\zeta$ originating in the mounted axles' plan, on the tracks axis, at a distance s from the center of gravity O_c of the coach (fig. 3).

In order to determine the relative displacements of the mechanical model's elements, one has to compare their coordinates in relation to the fixed reference system. The relative displacements of the case in relation to the bogies can be determined at the level of the central suspension and the bogies' relative displacements in relation to the mounted axles can be determined at the level of axle suspension. It is also necessary to establish the coordinates for the points located in the center of the suspension in relation to each of the adjacent element in the mechanical model.

The central suspension's strokes on the three axes are:

$$\begin{aligned} \Delta x_c &= -(\psi_c - \psi_{bj})(\pm d_s) \\ \Delta y_c &= y_c + h_{cc}\varphi_c + (-1)^{j+1} l\psi_c - y_{bj} + h_{cb}\varphi_{bj} \\ \Delta z_c &= (\varphi_c - \varphi_{bj})(\pm d_s) \end{aligned} \tag{6}$$

The axle's suspension strokes on the three axes are:

$$\begin{aligned} \Delta x_o &= -(\psi_{bj} - \psi_i)(\pm d_o) \\ \Delta y_o &= y_{bj} + (-1)^{i+1} a\psi_{bj} + h_{ob}\varphi_{bj} - y_i \\ \Delta z_o &= (\pm d_o)\varphi_{bj} \end{aligned} \tag{7}$$

3. The mathematical model

Lagrange's equation method may be applied as follows, in order to establish the movement equations:

$$\frac{d}{dt}\left[\frac{\partial(E-V)}{\partial \dot{q}_k}\right] - \frac{\partial(E-V)}{\partial q_k} + \frac{\partial D}{\partial \dot{q}_k} = Q_k \tag{8}$$

where, q_k - generalized coordinate, $\dot{q}_k$ - generalized speed, E – kinetic energy, V – potential energy, D – energy dissipation function, Q_k – generalized force corresponding to the generalized coordinate q_k.

The oscillating system's kinetic energy, potential energy and the energy dissipation function have the expressions:

$$\begin{aligned} E &= \frac{1}{2}m_c\dot{y}_c^2 + \frac{1}{2}I_{cz}\psi_c^2 + \frac{1}{2}I_{cx}\varphi_c^2 + \frac{1}{2}m_b\sum_{j=1}^{2}\dot{y}_{bj}^2 + \frac{1}{2}I_{bz}\sum_{j=1}^{2}\dot{\psi}_{bj}^2 + \frac{1}{2}I_{bx}\sum_{j=1}^{2}\varphi_{bj} + \frac{1}{2}m_o\sum_{i=1}^{4}\dot{y}_i^2 + \frac{1}{2}I_{oz}\sum_{i=1}^{4}\dot{\psi}_i^2 \\ V &= k_{cy}\sum_{j=1}^{2}(y_c + h_{cc}\varphi_c + (-1)^{j+1}l\psi_c - y_{bj} + h_{cb}\varphi_{bj})^2 + k_{cx}\sum_{j=1}^{2}\left[(\psi_c - \psi_{bj})(\pm d_c)\right]^2 + k_{cz}\sum_{j=1}^{2}\left[(\varphi_c - \varphi_{bj})(\pm d_c)\right]^2 + \\ &+ k_{oy}\sum_{j=1}^{2}\sum_{i=1}^{4}(y_{bj} + (-1)^{i+1}a\psi_{bj} - h_{ob}\varphi_{bj} - y_i)^2 + k_{ox}\sum_{j=1}^{2}\sum_{i=1}^{4}\left[(\psi_{bj} - \psi_i)(\pm d_o)\right]^2 + k_{oz}\sum_{j=1}^{2}\sum_{i=1}^{4}\left[\varphi_{bj}(\pm d_o)\right]^2 \\ D &= \rho_{cy}\sum_{j=1}^{2}(\dot{y}_c + h_{cc}\dot{\varphi}_c + (-1)^{j+1}l\dot{\psi}_c - \dot{y}_{bj} + h_{cb}\dot{\varphi}_{bj})^2 + \rho_{cx}\sum_{j=1}^{2}\left[(\dot{\psi}_c - \dot{\psi}_{bj})(\pm d_c)\right]^2 + \rho_{cz}\sum_{j=1}^{2}\left[(\dot{\varphi}_c - \dot{\varphi}_{bj})(\pm d_c)\right]^2 + \\ &+ \rho_{oz}\sum_{j=1}^{2}\sum_{i=1}^{4}\left[\dot{\varphi}_{bj}(\pm d_o)\right]^2 \end{aligned} \tag{9}$$

According to the contact forces (4), the generalized forces corresponding to the generalized coordinates y_i si ψ_i, have the following expressions:

$$
\begin{aligned}
Q_{yi} &= -2\chi Q\left(\frac{\dot{y}_i}{v} - \psi_i\right) - c_g(1-\chi_s)(y_i - \eta_i) \\
Q_{\psi i} &= -2\chi Qe\left[\frac{\gamma}{r_0}(y_i - \eta_i) + \frac{e}{v}\dot{\psi}_i\right] + 2\chi_s Q r_0\left(\frac{\dot{y}_i}{v} - \psi_i\right)
\end{aligned}
\tag{10}
$$

where, χ - the creepage coefficient, v – the coach's circulation speed, Q – wheel load, γ - effective conicity of the tread, r_0 – the wheel tread radius, χ_s - spin creepage coefficient, c_g – gravitational stiffness, η_i - track deviations on transversal direction.

The mathematical model considers the aspects of the non-linearities introduced by the irregularities of the tracks. According to [1], [4], the expression of the alignment deviations is possible in a sinusoidal form:

- $\eta_{1,2} = \eta_0 \cos[2\pi(vt + l \pm a)/L]$ for the trailing bogie axles;
- $\eta_{3,4} = \eta_0 \cos[2\pi(vt - l \pm a)/L]$ for the driven bogie axles. (11)

Applying Lagrange's equations we obtain the movement equations for the coach case, bogies and axles (12):

$$m_c\ddot{y}_c + 2k_{cy}[2(y_c + h_{cc}\varphi_c) - (y_{b1} + y_{b2}) + h_{cb}(\varphi_{b1} + \varphi_{b2})] + 2\rho_{cy}[2(\dot{y}_c + h_{cc}\dot{\varphi}_c) - (\dot{y}_{b1} + \dot{y}_{b2}) + h_{cb}(\dot{\varphi}_{b1} + \dot{\varphi}_{b2})] = 0$$

$$
\begin{aligned}
&(I_{cz}/2)\ddot{\psi}_c + 2(\rho_{cy}l^2 + \rho_{cx}d_c^2)\dot{\psi}_c + 2(k_{cy}l^2 + k_{cx}d_c^2)\psi_c - \rho_{cy}l(\dot{y}_{b1} - \dot{y}_{b2}) + \rho_{cy}l(\dot{\varphi}_{b1} - \dot{\varphi}_{b2})h_{cb} - \rho_{cx}d_c^2(\dot{\psi}_{b1} + \dot{\psi}_{b2}) - \\
&- k_{cy}l(y_{b1} - y_{b2}) + k_{cy}l(\varphi_{b1} - \varphi_{b2})h_{cb} - k_{cx}d_c^2(\psi_{b1} + \psi_{b2}) = 0
\end{aligned}
$$

$$
\begin{aligned}
&(I_{cx}/2)\ddot{\varphi}_c + 2(\rho_{cy}h^2{}_{cc} + \rho_{cz}d_c^2)\dot{\varphi}_c + 2(k_{cy}h_{cc}^2 + k_{cz}d_c^2)\varphi_c + 2\rho_{cy}h_{cc}\dot{y}_c - \rho_{cy}h_{cc}(\dot{y}_{b1} + \dot{y}_{b2}) + (\rho_{cy}h_{cc}h_{cb} - \rho_{cz}d_c^2)\cdot \\
&\cdot(\dot{\varphi}_{b1} + \dot{\varphi}_{b2}) + 2k_{cy}h_{cc}y_c - k_{cy}h_{cc}(y_{b1} + y_{b2}) + (k_{cy}h_{cc}h_{cb} - k_{cz}d_c^2)(\varphi_{b1} + \varphi_{b2}) = 0
\end{aligned}
$$

$$
\begin{aligned}
&m_b\ddot{y}_{bj} + 2\rho_{cy}\dot{y}_{bj} + 2(k_{cy} + 2k_{oy})y_{bj} - 2\rho_{cy}\dot{y}_c - 2\rho_{cy}h_{cc}\dot{\varphi}_c - 2(-1)^{j+1}\rho_{cy}l\dot{\psi}_c - 2(\rho_{cy}h_{cb})\dot{\varphi}_{bj} - 2k_{cy}y_c - 2k_{cy}h_{cc}\varphi_c - \\
&- 2(-1)^{j+1}k_{cy}l\psi_c - 2(k_{cy}h_{cb} + 2k_{oy}h_{ob})\varphi_{bj} - 2k_{oy}(y_{2j-1} + y_{2j}) = 0
\end{aligned}
$$

$$(I_{bz}/2)\ddot{\psi}_{bj} + \rho_{cx}d_c^2\dot{\psi}_{bj} + (k_{cx}d_c^2 + 2k_{oy}a^2 + 2k_{ox}d_o^2)\psi_{bj} - \rho_{cx}d_c^2\dot{\psi}_c - k_{cx}d_c^2\psi_c - k_{oy}a(y_{2j-1} - y_{2j}) - k_{ox}d_o^2(\psi_{2j-1} + \psi_{2j}) = 0$$

$$
\begin{aligned}
&(I_{bx}/2)\ddot{\varphi}_{bj} + (\rho_{cy}h_{cb}^2 + \rho_{cz}d_c^2 + 2\rho_{oz}d_o^2)\dot{\varphi}_{bj} + (k_{cy}h_{cb}^2 + k_{cz}d_c^2 + 2k_{oy}h_{ob}^2 + 2k_{oz}d_o^2)\varphi_{bj} + \rho_{cy}h_{cb}\dot{y}_c + (\rho_{cy}h_{cb}h_{cc} - \rho_{cz}d_c^2)\dot{\varphi}_c + \\
&+ (-1)^{j+1}l\rho_{cy}h_{cb}\dot{\psi}_c - \rho_{cy}h_{cb}\dot{y}_{bj} + k_{cy}h_{cb}y_c + (k_{cy}h_{cb}h_{cc} - k_{cz}d_c^2)\varphi_c + (-1)^{j+1}lk_{cy}h_{cb}\psi_c - (k_{cy}h_{cb} + 2k_{0y}h_{ob})y_{bj} + \\
&+ k_{oy}h_{ob}(y_{2j-1} + y_{2j}) = 0
\end{aligned}
$$

$$m_o\ddot{y}_i + 2\frac{\chi Q}{v}\dot{y}_i + 2\left(k_{oy} + \frac{c_g(1-\chi_s)}{2}\right)y_i - 2\chi Q\psi_i - 2k_{oy}y_{bj} + 2(-1)^{j+1}k_{oy}a\psi_{bj} + 2k_{oy}h_{ob}\varphi_{bj} = c_g(1-\chi_s)\eta_i$$

$$I_{oz}\ddot{\psi}_i + \left(2\chi Q\frac{e^2}{v}\right)\dot{\psi}_i + 2\left(k_{ox}d_o^2 + \chi_s Q r_0\right)\psi_i - \left(2\chi_s\frac{Qr_0}{v}\right)\dot{y}_i + 2\chi Q\frac{e\gamma}{r_o}y_i - 2k_{ox}d_o^2\psi_{bj} = 2\chi Q\frac{e\gamma}{r_0}\eta_i$$

4. The response of the system

The general form of the movement equations for the system with more degrees of freedom (12) is:

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F(t)\} \tag{13}$$

where, $[M]$ – mass matrix, $[C]$ – damping matrix, $[K]$ – stiffness matrix, $\{\ddot{q}\}$ - acceleration vector, $\{\dot{q}\}$ - speed vector, $\{q\}$ - displacements vector, $\{F(t)\}$ - force vector.

Thus, for the system with 17 degrees of freedom, the displacements vector is: $\{q\} = [y_c \quad \psi_c \quad \varphi_c \quad y_{b1} \quad \psi_{b1} \quad \varphi_{b1} \quad y_{b2} \quad \psi_{b2} \quad \varphi_{b2} \quad y_1 \quad \psi_1 \quad y_2 \quad \psi_2 \quad y_3 \quad \psi_3 \quad y_4 \quad \psi_4]$.

The mass matrix is a square matrix of order 17, with mass on diagonal and moments of inertia of the concentrated mass composing the mechanical model associated to the coach previously presented. The $[C]$ and $[K]$ matrices are square matrices of order 17 made up of the damping coefficients and the stiffnesses of the mechanical system. Because it is not possible to establish analytical expressions in relation to the system's response or the critical speed, both the study of movement stability and the determination of the hunting oscillations amplitudes are made using a numerical integration method of the movement equations, the Runge – Kutta method of 4th order, for which the MATLAB program package has specific procedures.

Table 1

Construction data of the passenger coach

Body case mass	m_c= 30760 kg
Bogie mass	m_b= 2300 kg
Wheelset mass	m_o= 1410 kg
Body case moments of inertia	I_{cx}=53596 kgm^2 I_{cz}=1661732 kgm^2
Bogie moments of inertia	I_{bx}=2240 kgm^2 I_{bz}= 2965 kgm^2
Axles moments of inertia	I_{oy}=980 kgm^2 I_{oz}=100 kgm^2
Central suspension stiffness	k_{cx}=133 kN/m k_{cy}=133 kN/m
	k_{cz}=473 kN/m
Axle suspension stiffness	k_{ox}= 256 kN/m k_{oy}= 885 kN/m
	k_{oz}= 904 kN/m
Central suspension damping	ρ_{cx}= 0 kN/m/s ρ_{cy}= 25 kN/m/s
	ρ_{cz}= 18 kN/m/s
Damping of the axle suspension	ρ_{oz}=3,67 kN/m/s
Wheel tread radius	r_0=0,460 m
The track's gauge	$2e$=1,435 m
The bogie's wheelbase	$2a$ =2,560 m
The distance between bogies	$2l$ = 17,2 m

The distance between the central suspension's springs	$2d_c$= 2 m
The distance between the axle's suspension springs	$2d_o$= 2 m
The distance case center – central suspension	h_{cc}=1,24m
The distance axles suspension - bogie center	h_{ob}=0,01 m
The distance central suspension - bogie center	h_{cb}= 0,06 m
Load on wheel	Q=51250 N
The creepage coefficient	χ=190
The spin creepage coefficient	χ_s=0,83
The effective wheel conicity	γ=0,14
The maximum testing speed	v_{max}= 50 m/s

The data presented above helped in accomplishing a working numerical simulation using the MATLAB program. In the simulation it was considered that the coach is launched on a tangent track and runs with a constant speed. In the movement equations' general expressions the coach was considered as an oscillating system activated by the tracks' irregularities. The elements' response was thus established – concentrated masses that make up the coach's mechanical model, translated in the generalized displacements' diagrams in relation to time at the maximum speed at which the coach is checked in the test polygon – 180 km/h, presented in fig. 5-12. The diagram study indicates that the tracks' perturbations effect is not felt at the coach case level, as opposed to the bogie and axles where it persists during the coach's circulation. The coach's main suspension acts correspondingly and meets the comfort demands inside the coach.

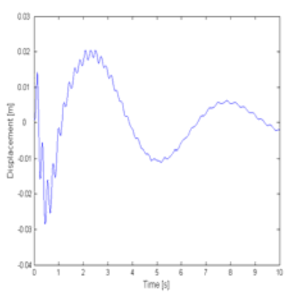

Fig.5 Case lateral displacement

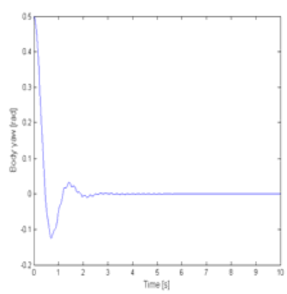

Fig.6 Coach's case yaw

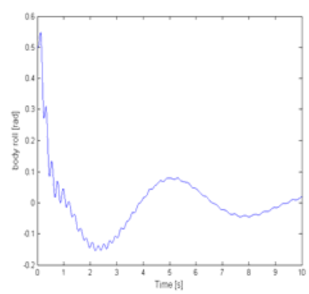

Fig.7 Coach's case roll

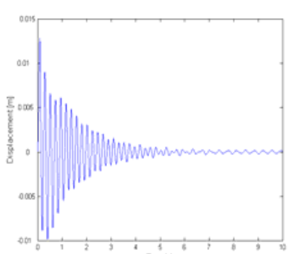

Fig. 8 Bogie's lateral displacement

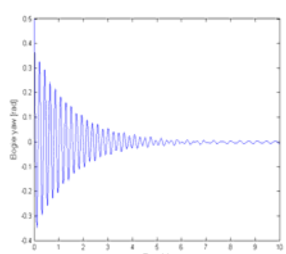

Fig.9 Bogie's yaw

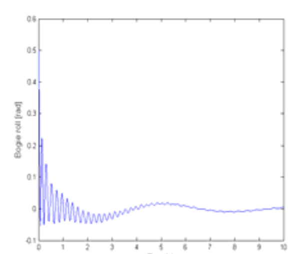

Fig. 10 Bogie's roll

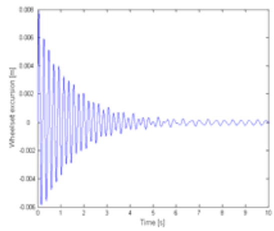

Fig.11 Wheelset's lateral displacement

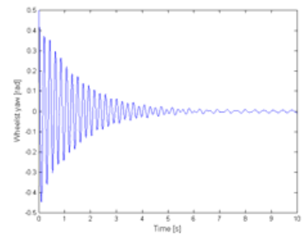

Fig.12 Wheelset's yaw

5. The vehicle's critical speed

The critical speed is the speed where the vehicle becomes unstable due to the fact that, on the wheel – rail contact, the creepage becomes pure slip [1]. The vehicle's maximum circulation speed must be lower than the critical speed. According to [6], vehicles with great speeds are operated mainly on straight tracks therefore the stability of the vehicle should be studied on tangent tracks.

The equations system describing the vehicle's movement is considered as a continuous dynamic system in time. The internal stability of this type of system solely depends on the distribution of the eigenvalues of the characteristic matrix in the complex plan.

The coach's critical speed is determined using the construction characteristics of the coach model seen above and the movement equations given by (12). We proceed then in calculating the eigenvalues of the characteristic matrix of the order I system resulted through the variable change:

$$\{y\} = \left\{ \frac{\{q\}}{\{\dot{q}\}} \right\} \tag{13}$$

that has the form:

$$\{\dot{y}\} = [E]\{y\} + \{F^*(t)\} \tag{14}$$

with

$$[E] = \begin{bmatrix} [0] & [I] \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \tag{15}$$

The dynamic system is asymptotically stable if and only if all the eigenvalues of the matrix E have a negative real part.

Determination of the eigenvalues of the matrix E was accomplished in MATLAB using the "eig" routine and increasingly varying the coach's circulation speed. As long as the real part of all the eigenvalues obtained is negative – the coach's movement is stable. If detecting a speed value for which at least one determined eigenvalue has the real part positive the speed's variation step is refined up to the necessary precision.

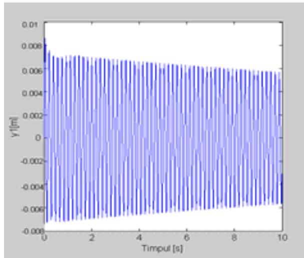

Fig.13 Axle lateral displacement , *v*<230 km/h

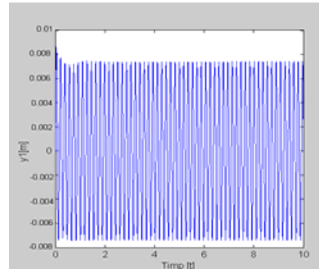

Fig.14Axle lateral displacement , *v*=230 km/h

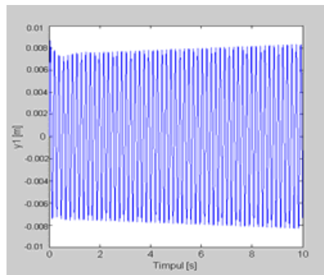

Fig.15 Axle lateral displacement , *v*>230 km/h

In what regards the coach used in the simulation, the critical speed was determined at a value of 63,7 m/s (~ 230 km/h). In a loaded state, the coach's critical speed will increase as a result of the stabilizing centering effect.

The coach's response is presented in the fig. 13–15 – the lateral displacement of the first axle – at inferior, equal and superior speeds to the critical value.

6. The construction characteristics' influence on the vehicle's stability

The mathematical model determined in the previous chapter can be used for improving the design of railway vehicles. Thus, applying the eigenvalues method, the influence of several construction characteristics of the vehicle over the critical speed can be studied.

The papers [4], [5], [9] contain studies of vehicle's stability with respect to the construction parameters of the wheelsets and suspensions. Paper [9] explicitly assumes the critical speed of the vehicle as unique optimization criteria. Paper [6] features the conclusions of similar studies and formulates recommendations for the suspension construction.

Image 16 presents the passenger coach's critical speed variation for conicity values equivalent to 0.12 , 0.13 and 0.14.

It is noticeable that the critical speed decreases along with the tracks' equivalent coning growth. Among the most frequent causes of the equivalent coning growth we can mention wear due to the wheels' rolling pattern exploitation or the tracks' radius of curvature growth.

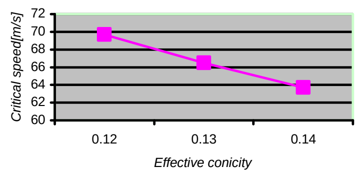

Fig.16 The influence of the effective conicity on the stability

The axles' suspension construction holds a particular importance over the vehicle stability on a horizontal plan. In general, a growth in the axles' suspension stiffness leads to a significant stability growth. Thus, if a longitudinal rigidity growth is accomplished, from 250 to 300 kN/m, the vehicle's critical speed can be augmented with up to 18 km/h.

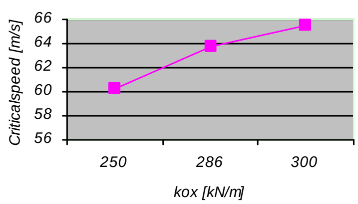

Fig.17 The influence of the longitudinal stiffness of the axle suspension

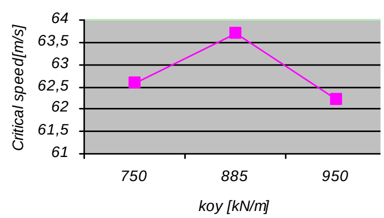

Fig.18 The influence of the transversal stiffness of the axle suspension

In the case of the studied vehicle it was noticed that a maximum critical speed of 64 m/s can be accomplished under the conditions of an axle suspension with a transversal stiffness around 885 kN/m, according to fig. 18.

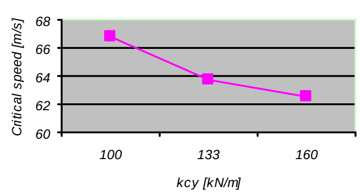

Fig.19 The influence of the transversal stiffness of the central suspension

Using more and more rigid suspensions also brings an intensification of the wear of the bogies subassemblies.

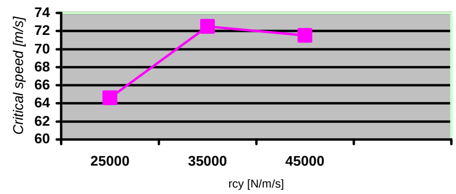

Fig.20 The influence of the transversal damping

Image 19 suggests that central suspension stiffness growth on a vertical direction reduces the coach's critical speed. Thus, a vertical stiffness growth of 60% in the central suspension produces a critical speed decrease of 15 km/h.

If the coach's central suspension transversal damping value grows from 20 kN/m/s to 30 kN/m/s, the critical speed increases with more than 20 km/h. If the damping has very high values the dampers become very rigid and have the tendency of behaving like bogie – case coupling elements, transmitting oscillations from the rolling apparatus to the coach case, reducing thus both dynamic performance and vehicle comfort.

7. Conclusions

The article presents a mathematical model with 17 degrees of freedom for a passenger coach reaching a maximum speed of 160 km/h.

The model considers the coach's lateral oscillations, respectively the lateral displacement, yawing and rolling motions of the concentrated mass building up the associated mechanical model: the coach case, the bogies and the mounted axles. The mathematical model proposed in this paper applies the equations featured in [1] for the bogies and wheelsets lateral movements and extend them to a passenger coach. A mathematical model to study the lateral dynamics of an entire vehicle solved with an original computer program is an original approach at national level.

The present study is conducted for the creep domain with linear friction coefficient characteristic. Previous studies proved that in the stability studies, the critical speed evaluated through linear methods it is higher than the non-linear critical speed, a fact that should be considered in the design activity. The non-linear systems offer a better accuracy in the evaluation of the dynamic behavior of the railway vehicle almost simulating the real phenomenon. The linear approach of the vehicle’s stability is useful in the design phase because it allows the investigation of the influence of the construction parameters on the critical speed. In that way it is possible to identify the optimal value domains of those parameters.

The equations system describing the vehicle's movement on a lateral direction was treated through numerical methods in order to determine its components' response to the coach's movement on an irregular track .

The critical speed of the coach used to exemplify the mathematical model was determined and applications of the study of the influence of construction parameters of the coach over its performances were presented.

It was shown that vehicle performance optimization is possible, allowing the increase of the critical speed with 20 km/h exclusively through an adequate suspension design. However, this undertaking must be the result of an optimization and adequacy process of the suspension's construction parameters in relation with the domain in which the railway vehicle is used and according to the specific operating conditions. Under this extent the presented mathematical model can represent an useful instrument in the calculation, design and optimization of the dynamic performances of railway vehicles.

The presented mathematical model offers developing opportunities considering the non-linearities of the wheel – rail contact and the situations when the vehicle runs in a curve.

R E F E R E N C E S

[1] *I. Sebesan,* Dinamica vehiculelor de cale ferata (Railway Vehicle Dynamics), Editura Tehnica, 1995 (in Romanian)

[2] *I. Sebesan, I. Copaci,* Teoria sistemelor elastice la vehiculele feroviare (Railway Vehicles Elastic Systems Theory), Editura Matrix Rom Bucuresti, 2008 (in Romanian)

[3] *I. Sebesan, D. Hanganu,–* Proiectarea suspensiilor pentru vehicule pe sine (Railway Vehicles Suspension Design), Editura Tehnica, 1993 (in Romanian)

[4] *V.K.Garg, R.V. Dukkipati,* Dynamics of railway vehicles systems, Academic Press Toronto, 1984

[5] *S.Y.Lee, Y.C. Cheng,* Hunting stability analysis of high-speed railway vehicle trucks on tangent tracks, Journal of sound and Vibration, 2005

[6] *A.H. Wickens,* Fundamentals of rail vehicles dynamics, Swets& Zeitlinger, 2002

[7] *J.J. Kalker,* Wheel-rail rolling contact theory, Wear,1991

[8] *Y.T.Fan, W.F. Wu,* Stability analysis of railway vehicles and its verification through field test data, Journal of Chinese Institute of Engineers, vol.29, 2006

[9] *Y. He, J. McPhee,* Optimization of the lateral stability of rail vehicles, Vehicle System Dynamics, 2002

[10] *S.D. wnicki, A.H. Wickens,* Validation of a Matlab railway vehicle simulation using a scale roller rig, Manchester Metropolitan University, 2004

[11] *M. Messouci,* Lateral stability of rail vehicles – a comparative study, ARPN Journal of Engineering and applied Sciences, vol.4, 2009

[12] *H. True,* On the theory of nonlinear dynamics and its applications in vehicle system dynamics, Vehicle System Dynamics, 1999

[13] *D.H. Wang, W.H. Liao,* Semi-active suspension systems for railway vehicles using magnetorheological dampers, Vehicle Systems Dynamics, 2009

[14] *K. Zboinski,, M. Dusza,* Extended study of railway vehicle lateral stability in a curved track, Vehicle System Dynamics, 2011