ACTIVE FLUTTER SUPPRESSION OF A 3D-WING: PRELIMINARY DESIGN AND ASSESSMENT

PART I - PROBLEM STATEMENT & CONTROLLER DESIGN

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The paper presents the problem and techniques of preliminary designing an active flutter suppressor, with reference to 3D-wings dynamically controlled through conventional trailing-edge flaps.

Two aspects are essential in such a project: building the wing dynamical model and designing the controller itself.

The present text makes use of standard representations in aeroelasticity and modern control theory; for the sake of clarity, explicit formulae are derived and numerical results are given for a test-case.

The validity of the active-flutter-suppression concept is well recognized.

The concept's performances will be discussed in Part II of this paper.

Keywords: aeroelasticity, flutter, active flutter suppression

1. Introduction

▪ The term *aeroelasticity* refers to a class of phenomena specific to aircraft constructions that result through an interaction process between the deformations of the elastic structure and the aerodynamic forces induced by the deformations themselves [1]-[2].

Flutter, the most significant of these phenomena, describes a self-excited oscillation which, by definition, exhibits an explosive behavior with detrimental or even catastrophic consequences.

For a given structure, a *flutter speed* can be determined that marks the *flutter onset* or, in mathematical terms, separates the stable from the unstable condition. The structure must be *flutter-free* in the entire aircraft's flight envelope.

▪ A structure that does not meet this criterion must be re-designed in order to completely eliminate any possibility of flutter occurrence or at last to raise the flutter speed a certain amount above the aircraft's limit speed. In a traditional approach, this objective would be attained through stiffening the structure and/or mass adding or redistribution, both leading usually to some extra weight.

▪ The *active flutter suppression* appears as a "modern" alternative to the "classical" one described above. It means controlling through some *active devices*

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- typically through activated controls - the *natural instabilities* of the structure, so as to make this flutter-free well *beyond* its "nominal" flutter boundary.

In must be mentioned, in context, that this concept can be seen as part of a rather new technology in aviation, the so-called *control-configured-vehicle* (*CCV*) [3]-[4] - a term introduced in the last decades of the 20th century - definitely determined by the remarkable progress in avionics that initiated in those years.

The validity of the concept has been proven both theoretically and experimentally ([5]-[6], etc.). With all that, it must be said that [14] "…*currently there is no vehicle in production that uses active flutter suppression*" and "…*much remains to be done before one can consider the routine incorporation of such systems in production aircraft*". Despite this, the concept is still valuable. One of the main reasons is that, within the present-day tendencies in transport aircraft design, the wings become *more flexible* and thus *more sensitive* to aeroelastic effects. For these, apart from possible flutter occurrence, a "much more realistic" problem can be considered, namely that of controlling the "conventional" aeroelastic structural vibrations which affect both the structure *strength* and *durability*. Consequently, the active structural control is basically investigated and designed both for *flutter prevention* and *structural load alleviation*.

▪ The active-flutter-suppression concept appears, over the last decades, as an attractive research field both for aeroelasticians and automatists. Numerous research programs have been conducted worldwide with the aim of demonstrating the feasibility of the concept or designing efficient ad-hoc controllers ([7]-[13], etc.). An indigenous research program referring to a typical 2D fluttering system [15] - which otherwise established a national priority - adds to all these.

▪ The present paper approaches the problem of designing a flutter suppressor for a conventional 3D-wing. Two are the main objectives of this work: firstly, to establish a relatively concise and reliable methodology for the preliminary design (Part I) and, secondly, to assess the system performance in the critical and in the subcritical domain as well (Part II).

The proposed techniques are based on the authors' past experience in flutter calculation as well as on that gained in the above mentioned program.

2. The mathematical model of the aeroelastic wing

▪ **In the present study**, the configuration to be analyzed is that of a typical *straight* cantilever wing of relatively *large* aspect ratio with a usual *trailing-edge flap* (aileron) serving as control device; further, for convenience, subsonic *incompressible* regime will be presumed. The system geometry and parameters - all in standard notations - are completely defined by Figs. 1 and 2.

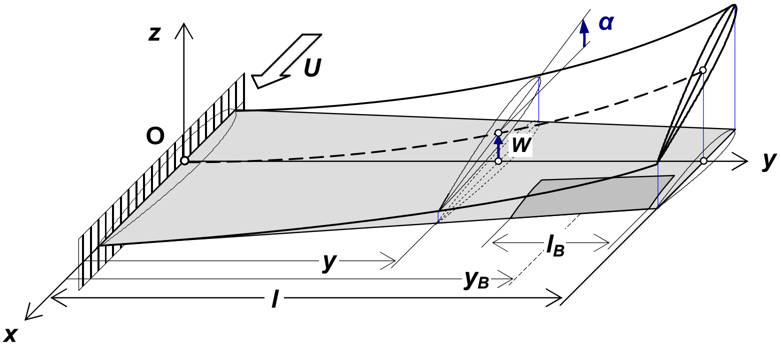

Fig. 1. Wing with an active trailing-edge flap: geometry and deformations

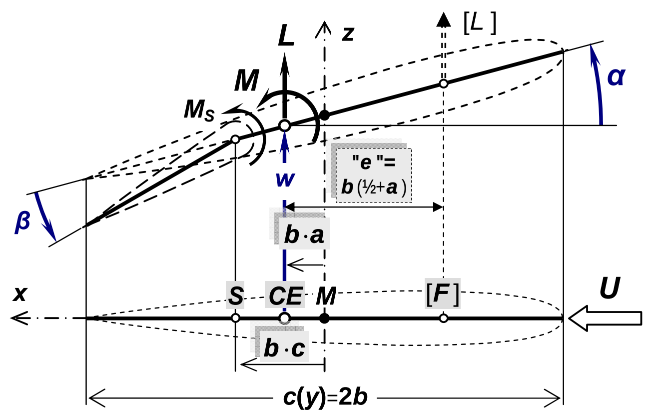

Fig. 2. Detailed wing section with flap: displacements and aerodynamic forces

▪ **For the proposed** wing, the engineering *beam theory* and the method of *assumed modes* [1]-[2] will be used for setting-up the equations of motion.

According to these, the deformed state of the wing is completely described by the *bending displacement* $w(y,t)$ and the *torsional rotation* $\alpha(y,t)$ - both measured in or relative to the local *elastic center*.

Further, these two last functions will be considered to be of the form

$$w(y,t) = \sum_{i=1}^{NW} Fw_i(y) \cdot w_i(t) \quad ; \quad \alpha(y,t) = \sum_{j=1}^{NA} F\alpha_j(y) \cdot \alpha_j(t) \tag{1}$$

where $Fw_i(y)$, $F\alpha_j(y)$ are *known space functions* satisfying the wing boundary conditions and $w_i(t)$, $\alpha_j(t)$ are the corresponding *generalized displacements* (*NW* and *NA* define the *order* of the approximation).

Note. For dynamical problems, the method of assumed modes usually takes for the approximating functions in (1) the free (*uncoupled*) bending and torsion *vibration modes* respectively, which leads to significant simplifications…

With the representation above one builds the cumulative vector of *generalized displacement* as

$$\mathbf{q} = \left\{ q_{k,k=1,2,\cdots(NW+NA)} \right\} = \begin{Bmatrix} \left\{ w_{i,i=1,2,\cdots NW} \right\} \\ \left\{ \alpha_{j,j=1,2,\cdots NA} \right\} \end{Bmatrix} \tag{2}$$

▪ **The system dynamics** can now be set-up through the well-known *Lagrange* equations

$$\frac{d}{dt}\left(\frac{\partial E_c}{\partial \dot{q}_k}\right) + \frac{\partial E_p}{\partial q_k} = Q_k \quad \cdots \qquad k = 1, 2, \cdots (NW + NA) \tag{3}$$

in which the *kinetic* and *potential* energies E_c and E_p relate to the inertial and elastic *conservative* forces, while all other "exterior" forces - aerodynamic, command, perturbations - enter in the system through the *generalized forces* Q_k (a *structural damping* can be included in the system equations if necessary).

▪ **The detailed derivation** of these equations is rather straightforward (it can be found in any textbook on aeroelasticity) and will not be shown here; some comments will be made instead in view of the actual objective of this work:

– The equations (3) lead to the following 2nd order differential system

$$\underbrace{\begin{bmatrix} \mathbf{Mww} & \mathbf{Mw\alpha} \\ \mathbf{M\alpha w} & \mathbf{M\alpha\alpha} \end{bmatrix}}_{\mathbf{M}} \cdot \begin{Bmatrix} \{\ddot{w}_i\} \\ \{\ddot{\alpha}_j\} \end{Bmatrix} + \underbrace{\begin{bmatrix} \mathbf{Kww} & \mathbf{Kw\alpha} \\ \mathbf{K\alpha w} & \mathbf{K\alpha\alpha} \end{bmatrix}}_{\mathbf{K}} \cdot \begin{Bmatrix} \{w_i\} \\ \{\alpha_j\} \end{Bmatrix} = \{Q\} \tag{4}$$

in which **M** and **K** are the *mass* and *stiffness* matrices.

Their expressions depend on system *mass* and *stiffness* distributions like

$$\begin{aligned} &\mathbf{Mww} : m_{ij} = \int_0^l m(y) \cdot Fw_i(y)\, Fw_j(y) \cdot dy \text{ ; and so on...} \\ &\mathbf{Kww} : k_{ij} = \int_0^l EIx(y) \cdot Fw_i''(y)\, Fw_j''(y) \cdot dy \text{ ; and so on...} \end{aligned} \tag{5}$$

– Alternatively, the equations can be re-written through the use of the *Laplace* transform (symbolized "$\mathscr{L}$") in a form amenable to subsequent setting-up of a control problem; further, for convenience, these can be put in *nondimensional* form. For clarity we show this process explicitly for the generic term in **M**:

$$m_{ij} = \int_0^l m(y)\,Fw_i(y)\,Fw_j(y)dy \;\xrightarrow{\eta=\frac{y}{l}}\; m_R l \int_0^1 \frac{m(\eta)}{m_R} Fw_i(\eta)\,Fw_j(\eta)d\eta \;\overset{\text{not!}}{=}\; m_R l \cdot Fww_{ij}$$

$$\Rightarrow m_{ij}\cdot \ddot{w}_j(t) \xrightarrow{L} m_{ij}\cdot s^2 w_j(s) = m_{ij}\cdot \omega_R^2 \left(\frac{s}{\omega_R}\right)^2 w_j(s) \overset{\text{not!}}{=} m_{ij}\cdot \omega_R^2 \bar{s}^{-2} w_j(\bar{s}) =$$
$$= [\pi\rho b_R^3 \omega_R^2 \cdot l]\frac{m_R}{\pi\rho b_R^3 \omega_R^2}\cdot Fww_{ij}\cdot \bar{s}^2 \frac{w_j(s)}{b_R} \overset{\text{not!}}{=} [\cdots]\,\mu_R \cdot Fww_{ij}\cdot \bar{s}^2 \frac{w_j(s)}{b_R} \tag{6}$$

$$\bar{s} = \frac{s}{\omega_R} \overset{\text{def!}}{=} \frac{\sigma + j\omega}{\omega_R}\;;\; \mu_R \overset{\text{def!}}{=} \frac{m_R}{\pi\rho b_R^3 \omega_R^2}\;;\; Fww_{ij} \overset{\text{not!}}{=} \int_0^1 \frac{m(\eta)}{m_R} Fw_i(\eta)\,Fw_j(\eta)d\eta$$

In this writing, m_R[kg/m] is a reference *mass* (the value of the mass distribution $m(y)$ at some reference station y_R) and b_R is a reference *semichord*; ω_R is a reference *frequency* while $\bar{s}$ denotes a nondimensional *Laplace variable*

$$s[1/\text{sec}] \to \bar{s} = \frac{s}{\omega_R} = \frac{\sigma + j\omega}{\omega_R} \overset{\text{not!}}{=} \bar{\sigma} + j\bar{\omega} \tag{7}$$

In the same manner, a *dimensional factor* […] (with ρ the air density) has been separated; further, in the final expression a nondimensional *mass parameter* μ_R has been entered, together with a nondimensional *influence coefficient* Fww_{ij}.

▪ **The uniform wing.** In the following text, the "uniform" or "constant" wing will be considered as a test case (Fig. 3). As well known, such a wing is completely described by 4 mechanical parameters: *mass* (*m*), *static unbalance* ($S\alpha$), *mass moment of inertia* about the elastic center ($I\alpha$), and the ratio of the bending and torsional *rigidities* (EIx/GId); the corresponding nondimensional definitions are (for convenience, the 4$^{\text{th}}$ parameter has been introduced in accord with [16] as was also the *reference frequency*):

$$\frac{m}{\pi\rho b^2} \overset{\text{not!}}{=} \mu \;;\; \frac{S\alpha}{\pi\rho b^3} \overset{\text{not!}}{=} \mu\cdot x_\alpha \;;\; \frac{I\alpha}{\pi\rho b^4} \overset{\text{not!}}{=} \mu\cdot r_\alpha^2 \;;\; p \overset{\text{def!}}{=} \frac{EIx}{GId}\cdot\frac{b^2}{l^2}$$
$$\omega_R \overset{\text{def!}}{=} \frac{1}{l}\sqrt{\frac{GId}{I\alpha}} \tag{8}$$

(Note that ω_R is a number proportional with the fundamental torsional frequency)

▪ **The 1×1 approximation.** All equations in (4) are treated the same way. For clarity let's show the system for the basic case *NW*=*NA*=1 in matrix form (the dimensional factors $[\pi\rho b^3\omega_R^2 \cdot l]$ and $[\pi\rho b^4\omega_R^2 \cdot l]$ appearing in the first and second equation respectively have been discarded…):

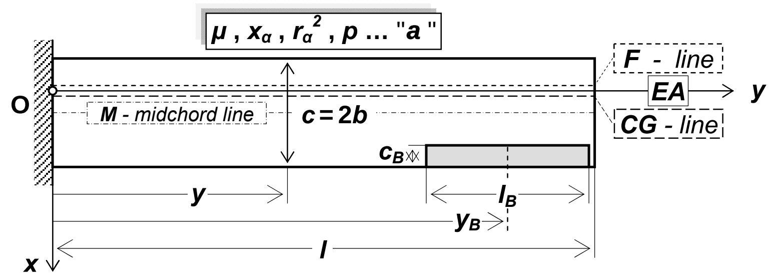

Fig. 3. The uniform wing: *EA*-elastic axis; *F*-aerodynamic center ("focar"); *CG*-center of gravity

$$\left(\underbrace{\begin{vmatrix} \mu \cdot Fww_{11} & -\mu x_\alpha \cdot Fw\alpha_{11} \\ -\mu x_\alpha \cdot Fw\alpha_{11} & \mu r_\alpha^2 \cdot F\alpha\alpha_{11} \end{vmatrix}}_{\mathbf{M}} \cdot \bar{s}^2 + \underbrace{\begin{vmatrix} \mu r_\alpha^2 p \cdot Gww_{11} & 0 \\ 0 & \mu r_\alpha^2 \cdot G\alpha\alpha_{11} \end{vmatrix}}_{\mathbf{K}}\right) \cdot \begin{Bmatrix} \dfrac{w_1(\bar{s})}{b} \\ \alpha_1(\bar{s}) \end{Bmatrix} = \begin{Bmatrix} Qw_1 \\ Q\alpha_1 \end{Bmatrix} \quad (9)$$

In this last, Fww_{11},… and Gww_{11},… denote *universal* constants depending only on the chosen *space functions* in (1):

$$\begin{cases} Fww_{11} = \int_0^1 Fw_1^{\,2}(\eta) \cdot d\eta \\ Fw\alpha_{11} = \int_0^1 Fw_1(\eta) F\alpha_1(\eta) \cdot d\eta \\ F\alpha\alpha_{11} = \int_0^1 F\alpha_1^2(\eta) \cdot d\eta \end{cases} ; \quad \begin{cases} Gww_{11} = \int_0^1 \left[Fw_1''(\eta) \right]^2 \cdot d\eta \\ G\alpha\alpha_{11} = \int_0^1 \left[F\alpha_1'(\eta) \right]^2 \cdot d\eta \end{cases} \quad (10)$$

Note. In the "basic" 1×1 case, the space functions are usually chosen as the fundamental *uncoupled* bending and torsion vibration modes of the wing. Further, the equations above describe the wing *coupled* bending/torsion oscillation…

▪ **The aerodynamics must** now be entered in the equations of motion (4) through the generalized forces $\{Q\}$.

Within the beam theory adopted in this text, the generalized forces associated with the generalized coordinates are determined from the running *lift* and *moment in CE* (*L* and *M*) through known definitions.

We exemplify for the $w_i(t)$:

$$Qw_{i,i=1,2,...NW} = \frac{\int_0^l L(y,t) \cdot \left[Fw_i(y) \cdot \delta w_i(t) \right] \cdot dy}{\delta w_i(t)} = \int_0^l L(y,t) \cdot Fw_i(y) \cdot dy = \cdots (11)$$

(A similar expression can be formulated for $Q\alpha_{j,j=1,2,...NA} \ldots$)

The actual derivations depend on the aerodynamic model to be used.

For control problems, such as flutter suppression is, the aerodynamic loads should be entered - in principle - through a model valid for *arbitrary motions*. Unfortunately [1] no simple mathematical expressions exist for this regime; therefore, for exploratory investigations one can rely on more simple models.

Two of these will be mentioned in this text:

– In the ***stationary regime*** the lift and moment depend *only* on and are *in phase* with the local *angle of attack* (*strip theory* and $C_L^\alpha = 2\pi \ldots$) - see Fig. 2:

$$\begin{cases} L^{ST}(y,t)[\text{ N/m }] = \rho/2\, U^2 c(y)\, C_L^\alpha \cdot \alpha(y,t) = \cdots = 2\pi\rho b U^2 \cdot \alpha(y,t) \\ M^{ST}(y,t)[\text{Nm/m}] = L^{ST} \cdot e_{F\to CE} = \cdots = 2\pi\rho b^2 (1/2 + a) U^2 \cdot \alpha(y,t) \end{cases} \quad (12)$$

Applying the definitions like (11) and the transformations indicated above, lead to the following compact expressions (for the "uniform" wing!):

$$\begin{Bmatrix} Qw_i \\ Q\alpha_j \end{Bmatrix}^{ST} = \begin{matrix} [\pi\rho b^3\omega_R^2 \cdot l] \times \\ [\pi\rho b^4\omega_R^2 \cdot l] \times \end{matrix} \begin{bmatrix} 0 & \dfrac{C_L^\alpha}{\pi} \cdot \bar{U}^2 \cdot Fw\alpha_{ij} \\ 0 & \dfrac{C_L^\alpha \cdot (\frac{1}{2}+a)}{\pi} \cdot \bar{U}^2 \cdot F\alpha\alpha_{ij} \end{bmatrix} \cdot \begin{Bmatrix} \left\{ \dfrac{w_i(\bar{s})}{b} \right\} \\ \left\{ \alpha_j(\bar{s}) \right\} \end{Bmatrix} \quad (13)$$

In these formulae "$F\ldots$" denote nondimensional *influence factors* like those in (6) or (10), while $\bar{U}$ is the nondimensional *speed*

$$U[\text{m/sec}] \to \bar{U} = \frac{U}{b\omega_R} \quad (14)$$

– The ***quasistationary model*** offers the simplest way to account for nonstationary effects; for this text we propose the formulation given by *Fung* [17] which results from the standard linear profile theory with the "geometric" incidence replaced by its "effective" instantaneous value:

$$\begin{cases} L^{QS-F} = 2\pi\rho U b \cdot \left[-\dot{w} + U\alpha + b\left(\frac{1}{2} - a\right)\dot{\alpha} \right] \\ M^{QS-F} = \pi\rho b^2 \cdot \left[-\frac{1}{2} U b \dot{\alpha} \right] + 2\pi\rho U b^2 \left(\frac{1}{2} + a\right) \cdot \left[-\dot{w} + U\alpha + b\left(\frac{1}{2} - a\right)\dot{\alpha} \right] \end{cases} \quad (15)$$

(Note that the aerodynamic forces depend on the *displacements* and their *rates as well*...)

The corresponding generalized forces will then be of the following form:

$$\begin{Bmatrix} Qw_i \\ Q\alpha_j \end{Bmatrix}^{QS-F} = \begin{matrix} [\pi\rho b^3\omega_R^2 \cdot l] \times \\ [\pi\rho b^4\omega_R^2 \cdot l] \times \end{matrix} \left(\Big[\; Q \; \Big] + \Big[\; Q' \; \Big] \cdot \bar{s} \right) \cdot \begin{Bmatrix} \left\{ \dfrac{w_i(\bar{s})}{b} \right\} \\ \left\{ \alpha_j(\bar{s}) \right\} \end{Bmatrix} \quad (16)$$

▪ **The "control" terms** can be derived in an analogous manner. In a first approximation, the flap can be entered through the parameters defined in its *mean section* y_B; with standard notations, the associated generalized forces will be (for the uniform wing! - where, as in Fig. 3, l_B is the flap span)

$$\begin{Bmatrix} Qw_{i\_\beta} \\ Q\alpha_{j\_\beta} \end{Bmatrix}^{ST} = \begin{matrix} [\pi\rho b^3\omega_R^2 \cdot l]\times \\ \cdots\cdots \\ [\pi\rho b^4\omega_R^2 \cdot l]\times \end{matrix} \begin{bmatrix} \left[\dfrac{l_B}{l}\right]\cdot\dfrac{C_L^\beta}{\pi}\cdot\bar{U}^2\cdot Fw_i(\eta_B) \\ ------------------ \\ \left[\dfrac{l_B}{l}\right]\cdot\dfrac{2C_M^\beta}{\pi}\cdot\bar{U}^2\cdot F\alpha_j(\eta_B) \end{bmatrix}\cdot\beta(\bar{s}) \tag{17}$$

In these last, C_L^β and C_M^β (the lift and moment slopes) are given by the well known formulae (in which "$T_{...}$" denote the so-called *Theodorsen functions* depending on the position of the flap hinge "c" - see Fig. 2):

$$\frac{C_L^\beta = 2T_{10}\ ;\ C_M^\beta = \tfrac{1}{2}\left[-(T_4+T_{10})+2(\tfrac{1}{2}+a)T_{10}\right]\ ;\ C_{Ms}^\beta = \cdots}{T_4 = -\cos^{-1}c + c\sqrt{1-c^2}\ ;\ T_{10} = \cos^{-1}c + \sqrt{1-c^2}} \tag{18}$$

▪ **A structural damping** can be introduced for the sake of generality. Here we will enter this as fraction of the *critical damping* in the vibration equation like (4) or (9) through a *diagonal matrix* $[C]$ defined as follows:

$$\left(\mathbf{M}\cdot\bar{s}^2+\mathbf{C}\cdot\bar{s}+\mathbf{K}\right)\begin{Bmatrix}\left\{\dfrac{w_i(\bar{s})}{b}\right\} \\ \left\{\alpha_j(\bar{s})\right\}\end{Bmatrix} = \cdots\ ;\ c_{ll} = \zeta_l\cdot 2\sqrt{m_{ll}k_{ll}} \tag{19}$$

The values for ζ_l can be set-up through identification with some standard recommendations from textbooks like [1] or [2] (see next chapter…).

▪ **The complete system** builds up from the previously derived entries; the most general nondimensional form, in compact notation, is

$$\mathbf{q} = \begin{Bmatrix}\left\{\dfrac{w_i(\bar{s})}{b}\right\} \\ \left\{\alpha_j(\bar{s})\right\}\end{Bmatrix} \Rightarrow \left(\mathbf{M}\cdot\bar{s}^2+\mathbf{C}\cdot\bar{s}+\mathbf{K}\right)\cdot\mathbf{q} = \Big(\underbrace{\mathbf{Q_q}+\mathbf{Q_{sq}}\cdot\bar{s}}_{\mathbf{Q}^M}\Big)\cdot\mathbf{q}+\mathbf{Q}^\beta\cdot\beta(\bar{s}) \tag{20}$$

in which, for convenience, the aerodynamic forces *associated with the motion* $\mathbf{Q}^M$ have been introduced according to the *quasistationary* model mentioned above.

Both $\mathbf{Q}^M$ and $\mathbf{Q}^\beta$ depend on the speed $\bar{U}$ as indicated.

▪ **State-space representation.** The system of equations (20) is the starting point for *flutter* or *flutter suppression* analysis. Alternatively, this can be cast into a state-space form. We choose as "natural" states the generalized displacements $\mathbf{q}$ and their successive "derivatives", that is

$$\mathbf{X} \overset{\text{def!}}{=} \begin{Bmatrix}\mathbf{q}_{NWA} \\ \bar{s}\cdot\mathbf{q}_{NWA}\end{Bmatrix}\ ;\ NWA = NW + NA \tag{21}$$

From (20) we extract the term having the highest power of $\bar{s}$ to

$$\bar{s}^2\cdot\mathbf{q} = -\mathbf{M}^{-1}\left[\left(\mathbf{K}-\mathbf{Q_q}(\bar{U})\right)+\left(\mathbf{C}-\mathbf{Q_{sq}}(\bar{U})\right)\cdot\bar{s}\right]+\mathbf{M}^{-1}\mathbf{Q}^\beta(\bar{U})\cdot\beta(\bar{s}) \tag{22}$$

With all these, we build the *canonic* system as

$$\bar{s}\cdot\mathbf{X}=\left[\!\left[\begin{array}{c|c}\mathbf{0}_{\dots} & \mathbf{I}_{\dots}\\ \hline -\mathbf{M}^{-1}\left(\mathbf{K}-\mathbf{Q_q}(\bar{U})\right) & -\mathbf{M}^{-1}\left(\mathbf{C}-\mathbf{Q_{sq}}(\bar{U})\right)\end{array}\right]\!\right]\cdot\mathbf{X}+\left[\begin{array}{c}\mathbf{0}_{\dots}\\ \hline \mathbf{M}^{-1}\mathbf{Q}^{\beta}(\bar{U})\end{array}\right]\cdot\beta(\bar{s}) \quad (23)$$

or, compact with generic notations

$$\bar{s}\cdot\mathbf{X}=\mathbf{A}(\bar{U})\cdot\mathbf{X}+\mathbf{B}(\bar{U})\cdot\beta(\bar{s}) \quad (24)$$

Note. The dimension of this system depends on the order of approximation $NWA = NW + NA$; further, "$\mathbf{0}_{\dots}$" and "$\mathbf{I}_{\dots}$" above stand for *zero* and *unity* matrices of appropriate orders…

3. Flutter - Numerical investigations

▪ **The conventional flutter** *problem* is set up by simply discarding the inhomogeneous part in (24), that is through building the system's "free dynamics"

$$\bar{s}\cdot\mathbf{X}=\mathbf{A}(\bar{U})\cdot\mathbf{X} \quad (25)$$

▪ **Relative to the order** of approximation in (1) one can prove [18]-[19] that, for wings of relatively large aspect ratios, only the first and second vibration modes have a significant contribution to flutter, while, for some cases, sufficiently accurate results are obtained even with the fundamental modes. In view of this reality, the primary model "1×1" can be used for preliminary studies.

▪ **For the numerical** investigations to follow, a uniform wing (Fig. 3) will be considered as in [16]. All system parameters are listed in Table 1; the explicit approximation functions $Fw_i(y)$ and $F\alpha_j(y)$ are given at the end of this text.

Note. The values for the damping factors ζ_w, ζ_α have been derived [18] through identification from the corresponding standard "complex modulus" coefficients $g_w, g_\alpha \subset (0.02 \div 0.05)$ from [1]-[2]; the resulting formulae are

$$\zeta_w = g_w\frac{1}{2\bar{s}_F}\sqrt{\frac{k_{11}}{m_{11}}} = \; ; \; \zeta_\alpha = g_\alpha\frac{1}{2\bar{s}_F}\sqrt{\frac{k_{22}}{m_{22}}} \quad (26)$$

in which k_{11}, etc. are the mass and stiffness terms in the vibration equation (9) and, consistent with the actual flutter frequency, a value $\bar{s}_F \approx 1$ has been used.

▪ **The standard root-locus** method applied on the flutter system (25) is fully illustrative for the dynamic characteristics of that system - Figure 4. As well known, the flutter onset is associated with the point where one branch of the root-locus enters the right half-plane… From figure (a) one can "read" this point to

$$\underline{\bar{U}_F \approx 5.5 \;;\; \bar{s}_F \approx 1.2} \quad (27)$$

Table 1

System parameters

Wing overall			
Wing chord	c (= $2b$)	2 m	Reference value
Wing aspect ratio (semi-span!)	$\lambda^* = l / c$	~ 5	Reference value
Reference frequency	$\omega_R = 1/l\sqrt{GId / I\alpha}$	33 rad/s	Reference value
Nondimensional mass and elastic properties			
Mass	μ	40	Test case
Static unbalance	x_α	0.1	
Mass moment of inertia about *EC*	r_α^2	0.25	
Stiffness (modified) ratio	$p = EIx / GId \times b^2 / l^2$	0.04	
Elastic center (*EC*) location (Fig. 2)	a	– 0.4	
Flap geometry (Figures 2 & 3)			
Relative span	l_B / l	0.3	Adopted in design
→ Mean chord position	y_B / l	~ 0.85	…
Relative chord	c_B / c	0.2	Adopted in design
→ Hinge position from midchord	"c"	0.6	*Theodorsen* notation
Aerodynamic coefficients			
C_L^α	Formula (12)	2π	Teoretic 2D…
C_M^α	$C_M^\alpha = \frac{1}{2}\left(\frac{1}{2} + a\right) \cdot C_L^\alpha$	0.3142	Formula (12)
C_L^β	Formula (18)	3.4546	…
C_M^β	Formula (18)	– 0.4673	…
Structural damping coefficients (fractions of critical damping)			
Bending	ζ_w	0.01	Derived!...
Torsion	ζ_α	0.05	Derived!...

4. The flutter suppression problem

▪ **The objective of this section** is to design a controller that, through the use of an active flap, eliminates flutter. In mathematical terms this problem reads (see also Fig. 1 for illustration and Fig. 4 for illumination):

Let's denote U_F the system nominal flutter speed. Given a design speed $U_{des} = k \cdot U_F$ ($k > 1$), a feed-back command law $\beta(t) = f(w, \alpha, \dot{w}, \dot{\alpha}, \cdots)$ should be determined that stabilizes the system in the entire interval $U \in [0, U_{des}]$.

Notes. In this formulation one can readily recognize a standard theme of the *Control theory*. In the case of flutter suppression, the problem raises two distinct questions: the demonstration of the *feasibility* of the concept and the determination of the *controller* itself.

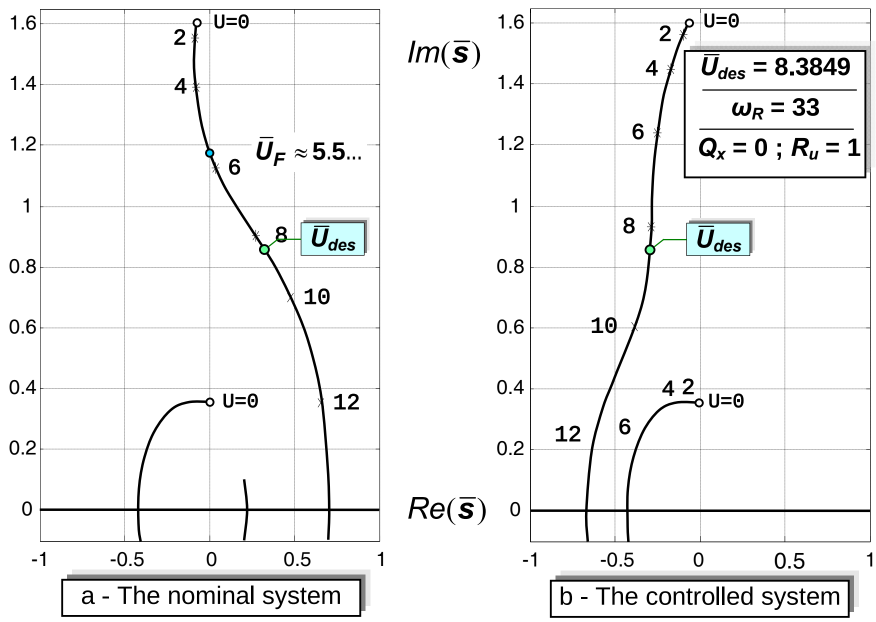

Fig. 4. System "1×1" - Numerical simulation

In a historical perspective on the subject, one should mention the attempts made by some authors [5] to assess the efficiency of simple *proportional* laws in which the (generic) displacements or accelerations are fed-back to the flap, like

$$\beta(t) = k_w \cdot w(t)\ ;\ \beta(t) = k_\alpha \cdot \alpha(t)\ ;\ \beta(t) = k_{\ddot{w}} \cdot \ddot{w}(t)\ ;\ \beta(t) = k_{\ddot{\alpha}} \cdot \ddot{\alpha}(t) \quad (28)$$

(These laws work as simple "mechanical connections" from wing to flap!)

▪ **The modern control theory,** due to its power and versatility, has become a standard tool to treat active structures; in particular, the **LQR regulator** is an efficient algorithm for solving the problem of stabilizing a dynamic system:

– Let's consider the linear system (generic notations!)

$$\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}\ ;\ \mathbf{y} = \mathbf{C} \cdot \mathbf{x} \quad (29)$$

and introduce a *full-state* feedback connection, that is

$$\mathbf{u}(t) = -\mathbf{K} \cdot \mathbf{x}(t) \Rightarrow \dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B} \cdot \mathbf{K}) \cdot \mathbf{x} + \mathbf{B} \cdot \mathbf{u}\ ;\ \mathbf{y} = \mathbf{C} \cdot \mathbf{x} \quad (30)$$

– The *optimal regulator* results through minimization of a *quadratic index*

$$I[\mathbf{u}(\mathbf{t})] = \frac{1}{2} \int_0^\infty \left[\mathbf{x}^T(t) \cdot \mathbf{Q} \cdot \mathbf{x}(t) + \mathbf{u}^T(t) \cdot \mathbf{R} \cdot \mathbf{u}(t) \right] dt \to \min \quad (31)$$

– With **Q** and **R** prescribed, the solution to this is the *Riccati* regulator

$$\frac{\mathbf{u}_{\text{opt}} \overset{\text{not!}}{=} -\mathbf{K}_{\text{opt}} \cdot \mathbf{x} \to \mathbf{K}_{\text{opt}} \overset{\text{def!}}{=} \mathbf{R}^{-1}\mathbf{B}^T\mathbf{P}}{\mathbf{PA} + \mathbf{A}^T\mathbf{P} - \mathbf{PBR}^{-1}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = 0} \quad (32)$$

– As well known [20] the optimal regulator also stabilizes the system in the sense of *Lyapunov*…

▪ **The application of this technique** to the subject in the present article is rather straightforward. For conformity with the LQR algorithm, the system equations like (24) - which are formulated in *Laplace* variable - must first be re-written in "*physical*" time. This is always possible due to the known "duality" property (with zero initial conditions presumed...)

$$\text{Let } \mathcal{L}\left[f(t)\right]=F(s);\ \text{then } \frac{d^k f}{dt^k}\Leftrightarrow s^k\cdot F(s);\ \text{i.e. } \dot{f}\Leftrightarrow s\cdot F(s);\ \ddot{f}\Leftrightarrow s^2\cdot F(s);\ \cdots \quad (33)$$

– Since in (21) to (24) the *nondimensional* variable $\bar{s}$ has been used, a transformation of the states is introduced as follows

$$\{X\}\overset{\text{def!}}{=}\begin{Bmatrix}\{q\}\\ \bar{s}\cdot\{q\}\end{Bmatrix}\overset{\text{def!}}{=}\begin{Bmatrix}\{q\}\\ \frac{1}{\omega_R}\cdot\{s\cdot q\}\end{Bmatrix}\Leftrightarrow\begin{Bmatrix}\{q\}\\ \frac{1}{\omega_R}\cdot\{\dot{q}\}\end{Bmatrix}\overset{!}{\equiv}\begin{bmatrix}[I_{2\times 2}] & [0_{2\times 2}]\\ [0_{2\times 2}] & \frac{1}{\omega_R}\cdot[I_{2\times 2}]\end{bmatrix}\cdot\begin{Bmatrix}\{q\}\\ \{\dot{q}\}\end{Bmatrix}=$$
$$\cdots\overset{\text{not!}}{=}\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}\cdot\begin{Bmatrix}\{q\}\\ \{\dot{q}\}\end{Bmatrix}\overset{!}{\equiv}\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}\cdot\{x\} \quad (34)$$

in which $\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}$, depending on the *reference frequency* ω_R, is clearly nonsingular.

With this transformation, the left-hand side in (24) reads

$$\bar{s}\cdot\mathbf{X}\overset{\text{def!}}{=}\frac{s}{\omega_R}\mathbf{X}\overset{!}{\equiv}\frac{1}{\omega_R}\left(s\cdot\mathbf{X}\right)\Leftrightarrow\frac{1}{\omega_R}\cdot\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}\cdot\{\dot{x}\}\approx\frac{1}{\omega_R}\cdot\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}\cdot\dot{\mathbf{x}}\ \cdots \quad (35)$$

By introducing (34) and (35) into (24) one gets

$$\frac{1}{\omega_R}\cdot\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}\cdot\dot{\mathbf{x}}=\mathbf{A}\left(\bar{U}\right)\cdot\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}\cdot\mathbf{x}+\mathbf{B}\left(\bar{U}\right)\cdot\beta \quad (36)$$

and, finally

$$\dot{\mathbf{x}}=\underbrace{\omega_R\cdot\left[\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}{}^{-1}\cdot\mathbf{A}\left(\bar{U}\right)\cdot\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}\right]}_{\tilde{\mathbf{A}}}\cdot\mathbf{x}+\underbrace{\omega_R\cdot\left[\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}{}^{-1}\cdot\mathbf{B}\left(\bar{U}\right)\right]}_{\tilde{\mathbf{B}}}\cdot\beta \quad (37)$$

– The *Riccati* regulator $\tilde{\mathbf{K}}_{\text{opt}}$ can be set-up as in (32) with the matrices $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ above (and properly selected $\tilde{\mathbf{Q}}$ and $\tilde{\mathbf{R}}$ …); the controlled system reads

$$\tilde{\mathbf{K}}_{\text{opt}}\overset{\text{def!}}{=}\tilde{\mathbf{R}}^{-1}\tilde{\mathbf{B}}^T\tilde{\mathbf{P}}\quad\Rightarrow\quad\dot{\mathbf{x}}=\left[\tilde{\mathbf{A}}-\tilde{\mathbf{B}}\cdot\tilde{\mathbf{K}}_{\text{opt}}\right]\cdot\mathbf{x} \quad (38)$$

– To assure a consistency of the analysis with the basic system (24), the controlled system (38) can be "back"-transformed to the $\bar{s}$ variable; thus, through simple operations one gets

$$\bar{s}\cdot\mathbf{X}=\left[\mathbf{A}-\mathbf{B}\cdot\mathbf{K}_{\text{reg}}\right]\cdot\mathbf{X}\ ,\ \text{with } \mathbf{K}_{\text{reg}}=\tilde{\mathbf{K}}_{\text{opt}}\cdot\mathbf{T}_{\boldsymbol{\omega}_\mathbf{R}}^{-1} \quad (39)$$

in which a "regulator matrix" $\mathbf{K}_{\text{reg}}$ appears, that again depends on ω_R.

– With this last, the characteristics of the controlled system - through the root-locus method - can be directly compared with those of the nominal one.

▪ **Reference case study.** The LQR technique will be applied to the previously proposed test case. The solution obviously depends on the weighting matrices **Q** and **R** in the quadratic index; in practice these are selected on the basis of a "technical-economical" analysis" [20] through which the performances of the system can be set to a desired level.

In this text a special case will be considered, namely the so-called *minimum energy* regulator that results by relieving any conditions on the states, that is by putting $\mathbf{Q}_{(\mathbf{x})} = \mathbf{0}$; in these conditions, the value of $\mathbf{R}_{(\mathbf{u})} = \mathbf{0}$ in the quadratic index becomes *irrelevant* and thus can be set simply to "unity": $\mathbf{R} = \mathbf{I}$.

(As known, this particular type of regulator has the remarkable effect of *reflecting* the unstable roots of the system about the imaginary axis while *keeping* the stable ones unaffected).

For the present analysis a (rather high!) *design speed* has been selected - for evaluation purposes - to

$$\bar{U}_{des} \ \ (\approx 1.5 \times \bar{U}_F) \ = 8.3849 \tag{40}$$

All the values of interest are summed up in Table 2; the root locus of the controlled system is given again in Fig. 4 along with that of the nominal one. It is clear that the controlled system remains stable *up to* (and even *beyond*!) $\bar{U}_{des}\ldots$

Table 2

System "1×1" (quasistationary aero) - The flutter suppression controller

$\bar{U}_{des} = 8.3849$ | $\boldsymbol{\zeta}_w = 0.01$, $\boldsymbol{\zeta}_\alpha = \mathbf{0.05}$ | $\boldsymbol{\omega}_R = 33$ | $Q_X = 0$, $R_u = I$

$$u = \boldsymbol{\beta} \ ; \ X = \left[\frac{w_1}{b} \;\vdots\; \boldsymbol{\alpha}_1 \;\vdots\; \bar{s}\frac{w_1}{b} \;\vdots\; \bar{s}\boldsymbol{\alpha}_1 \right]^T \ ; \ u_{opt} = K_{reg} \cdot X$$

$\bar{s}_{des} = \{$ -1.2620 ¦ -0.1262 ¦ 0.3160 + 0.8692i ¦ 0.3160 - 0.8692j $\}$

$K_{reg} =$

$X \rightarrow$	$\frac{w_1}{b}$	$\boldsymbol{\alpha}_1$	$\bar{s}\frac{w_1}{b}$	$\bar{s}\boldsymbol{\alpha}_1$
$\boldsymbol{\beta}$	0.0728	-0.9296	0.3547	-1.2832

$\bar{s}_{reg} = \{$ - 1.2620 ¦ -0.1262 ¦ - 0.3160 + 0.8692i ¦ - 0.3160 - 0.8692j $\}$

*

Note. The approximation functions $Fw_i(\eta)$ and $F\alpha_j(\eta)$, $\eta = y/l$ used in this text are (their derivation is also addressed in Part II of this paper):

$$Fw_i = \left[\left(\frac{\sin \pi N_i - \sinh \pi N_i}{\cosh \pi N_i + \cos \pi N_i} \right) \left(\sinh \pi N_i \eta - \sin \pi N_i \eta \right) + \cosh \pi N_i \eta - \cos \pi N_i \eta \right]$$

N_1 = 0.596864…	N_2 = 1.494175…	N_3 = 2.500246…	N_4 = 3.499989…	N_5 = 4.500000…	…

$$F\alpha_j = \sin(2j-1)\frac{\pi}{2}\eta \ \ (j = 1,2,3,\cdots)$$

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