U.P.B. Sci. Bull., Series C, Vol. 85, Iss. 2, 2023 ISSN 2286-3540

AN ALGORITHM FOR SOLVING GRAPH COLORING PROBLEMS BASED ON AN IMPROVED ANT COLONY OPTIMIZATION

Supei ZHOU[1]

The Graph Coloring Problem (GCP) is a significant research area in graph theory, and new breakthroughs are constantly being made, while the ant colony optimization shows its outstanding ability to solve path planning problems. In this research, the ant colony optimization method is made full use of to improve it using one kind of greedy algorithm, and the effectiveness of this proposed solution algorithm for the coloring problem in a realistic directed and undirected network environment is validated. It is demonstrated that the proposed GAC-GC graph coloring solution method has better efficiency and accuracy than the classical algorithm for coloring problems. In future research, further improvements can be made from various perspectives, such as application scenarios, integrated application of algorithmic data pre-processing, and hybrid algorithm improvement.

Keywords: graph coloring problem; ant colony optimization; greedy algorithm; social networks; link prediction

1. Introduction

GCP [1], one significant research area in graph system, can be investigated in several directions such as coloring for vertexes, edges and entire graphs. It has been applied to practical problems such as channel allocation in mobile wireless networks [2], natural time expansion [3] and traffic scheduling planning [4]. Problems related to the permutation and combination of vertex coloring in graph theory can be solved using tree-based maximum independent set algorithms [5], properly ordered coloring algorithms [6], intelligent algorithms [7], etc. Ant Colony Optimization (ACO) [8] is an intelligent bionics algorithm which can deal with graph coloring problems. However, the convergence rate of the traditional ACO is slow when it faces the coloring problem with large data size as the result of its time complexity of $O(Nn^3)$. Meanwhile, the traditional ACO is prone to be trapped in local optimal solutions, but it can be improved from directions such as the fusion of multiple algorithms, and it owns the ability to be applied to solving the coloring problem of the graph which is focused on vertexes in question.

1.1 Related work on improving the ACO

First and foremost, some researchers tried to integrate ACO into more application backgrounds. In the Travelling Salesman Problem (TSP) ACO based

[1] School of Science of Zhejiang Sci-Tech University, Hangzhou, Zhejiang, 310018, China; E-mail: 2430188678@qq.com

on optimizing parameters studied by Wang et al. [9], the remaining parameters were determined by adaptive optimization with hybrid symbiotic search, which well solved the fixed parameter problem in the ACO after assigning some parameters of the ACO. In the ACO algorithm for semi-supervised association classification studied by Awan et al. [10], they construct association classification rules which concluded labeled and pseudo-labeled examples. Also, they integrated three aspects of SSL's transduction self-training mechanism, association classification principles and ACO rules, well complemented the advantages of data mining direction algorithms and traditional ACO algorithms.

On top of that, some researchers realized the hybrid application of ACO and other algorithms. In the hybrid genetic ACO based on the cloud model, which was embedded, researched by Wang et al. [11], half of the solutions were optimally populated using Genetic Algorithm (GA) and the other half were populated using uniform sampling. In this way, the initialization of the ACO solution was achieved. Besides, the embedded cloud model was used to achieve adaptive parameter selection based on the ambiguity of the current solution archiving. In the hybrid algorithm combined with Simulated Annealing (SA), Tabu Search (TS) and ACO studied by Addou et al. [12], they successfully achieved the advantages and performance fusion of multiple algorithms. The advantages of SA's high explorability of new solutions and TS's high potential for optimization enhancement in narrow search spaces were incorporated into ACO in a coupled way with multiple approximation algorithms, cleverly solving the problem of classical k-Minimum Spanning Tree (k-MST) based on the graph with the new optimal value of the base.

Moreover, others improved ACO with advanced mathematical and computer science methods. Liu et al. [13] proposed one new type of ACO, which was structured to solve 3D multi-task planning problems with finite time constraints. They made fruitful improvements to the pheromone mechanism and probability selection mechanism by initializing all the pheromones of the paths to zero and introducing the manual preemption coefficient matrix into the transfer probability formula. Additionally, they introduced the variable dimensional vector coefficients which were defined about their probability to achieve transfer and the adaptive factor with regard to time, so as to optimize results of the algorithm and accelerate the convergence rate of this algorithm. And Zhao et al. [14] developed a discrete multi-objective optimization ACO which was based on the method called decomposition, using an adaptive pheromone update strategy based on reinforcement learning, a multi-objective planning ACO with adaptive scheme construction and selection techniques. It enhanced the ability to search the high-dimensional target space during the solution construction stage while seeking for the ACO solution.

1.2 Related work on ACO for solving graph coloring problems

In the Physarum-based ACO for solving graph coloring problems studied by Lv et al. [15], the update of the pheromone matrix was achieved with the help of the physarum mathematical model with one type of positive feedback mechanism, which brought greater efficiency and stability of the algorithm. In a new clustering method based on vertex coloring to accelerate the ACO studied by Li et al. [16], the vertices in the graph were colored with the idea of clustering to obtain a divided urban cluster, and then each class was solved using the ant colony optimization algorithm to arrive at a local optimal solution. Finally, the individual localities were concatenated to arrive at a global optimal solution, which made good use of the idea of vertex coloring and improved the algorithm's operational efficiency by a factor of 15-750. In the improved graph coloring optimization algorithm studied by Dowsland et al. [17], algorithm improvements were implemented in a combination of visibility functions, multiset options in the stage of construction and a forbidden search improvement stage to achieve algorithm balancing and arithmetic implementation at medium-scale.

1.3 Introduction and organization of this study

Methods to solve GCP are mainly classified as coloring for vertexes, edges and entire graphs, while both edge coloring and total coloring of graphs may be equivalently translated into vertex coloring of graphs, therefore, this paper is limited to achieve the coloring problem of the graph for points. Point coloring, one way to make vertices of a graph get their colors, requires each vertex being colored in the form that any vertex which has the adjacent relationship has a disparate color. If it is possible to color every vertex of a graph according to the rule above, the graph is colored with respect to those nodes. And since the algorithm complexity of GCP has reached the level of NP-hard, one form of improved greedy ACO with an approximation ratio of $1 + ln\,(\gamma/\epsilon)$ is chosen to deal with GCP more skillfully for this study.

This study designs an approximate greedy algorithm and improves the ACO based on this greedy algorithm. The improved algorithm studied in this research is applied to obtain the final answer of the graph vertex coloring problem which owns the content of social network link prediction for the opinion propagation problem. To verify the superiority of the improved algorithm studied in this research, 30 recent hot comment user nodes of Facebook and Twitter are selected and constructed as undirected graphs and directed graphs respectively used as real network data set. In addition, the algorithm experimental results, which are obtained by the method proposed in this paper and other similar methods, demonstrate the superiority of this research according to high solution quality and low time complexity when facing a real social network environment. Meanwhile, they outperform the traditional ACO for both directed and undirected graphs with large vertex counts based on the numerical solution results of several comparative levels of arithmetic cases.

The algorithm proposed in this paper has high novelty, which can be described from three aspects. First and foremost, in this study, ACO, GA and some frontier methods are fused and improved, so that a new hybrid improved ant colony algorithm is obtained. Furthermore, in order to solve the vertex coloring problem of a larger scale graph, this research proposes a hybrid innovation technology based on decomposition and heuristic methods and carries out graph coloring in the advantageous network environment to fully reflect the advantages of the innovation method. Besides, the research background of this paper is novel. This research combines the link prediction methods in social networks and verifies the algorithm performance in the real data set of large-scale social networks, realizing the new improved fusion of a variety of traditional graph theory algorithms and link prediction algorithms.

This paper not only illustrates and verifies the improved algorithm and its superiority, but also discusses the frontier research directions for future graph coloring problem solution algorithms. This paper indicates the direction of improvement for further in-depth research on graph coloring problem solution algorithms to obtain higher efficiency and better precision which belong to the algorithm solving. The concrete contents of this paper are as follows: the first part mentions the background of the research and related work; the second part constructs the algorithm model; the third part presents outcomes of numerical example based on real social network data set; the fourth part concludes the paper with an outlook.

2 Algorithm Design

$G = (V, E)$ in this study is set up as a graph which is directed, using nodes V and directed edges E which all exist in a collective form to indicate a social network. This study first introduces one kind of greedy algorithm as the method in Algorithm 1 based on $h(\cdot)$ as a submodular monotonic function in Algorithm 1, and an approximate greedy algorithm whose ratio is calculated as $(1 - 1/e - \epsilon)$ has shown in Algorithm 1.

Algorithm 1: Greedy Algorithm

Input: Graph $G = (V, E), S = \{s_k\}$
Output: Node subset A
1: $A \leftarrow \emptyset$
2: **while** $h(\cdot) < \gamma - \epsilon$ **do**
3: $u = argmax\ h(v) - h(\cdot)$
4: $A \leftarrow A \cup \{u\}$
5: Node subset A

ACO is one of many intelligent algorithms, which is a bionics algorithm formed by imitating ants to look for food in nature [18], and its algorithmic characteristics of concurrency, robustness and positive feedback are initially mostly used in TSP such as path planning based on graph theory [19]. The traditional ACO is based on visibility related to the reciprocal value of the Euclidean distance from one point to another, and the probability of moving to the next node is decided by a roulette wheel method. Through continuous iteration, many pheromones will be left on the paths where those ants pass through, and the pheromones originally left by the ants will evaporate with the passage of time. In this way, the pheromone concentration of each global position will be continuously updated, guiding the ant colony to seek for the convergence path solution which is optimal [20]. During this study, a weighted graph $G = (V, E)$ is set up. V is defined as the set consisting of all points in Fig. G. E is defined as the set consisting of all edges in Fig. G, and all elements in E have corresponding weights. The calculation formula of traditional ACO in Fig. G is described as follows. Setting i and j as any two points in the point set V, $\tau_{ij}(t)$ denoting the pheromone concentration between point i and point j after time t, and d_{ij} as the distance which is named about Euclidean between point i and point j. The heuristic function $\eta_{ij}(t)$ corresponds to the visibility from location i to location j, and its expression is shown in equation (1).

$$\eta_{ij}(t) = \frac{A}{d_{ij}} \tag{1}$$

Next, making $allowed_k \subseteq \mathrm{V}$ be the set of nodes not yet visited, s be a point in the point set $allowed_k$, α be the pheromone factor, and β be the heuristic function factor. Defining the probability of ant k moving from point i to point j as P_{ij}^k by roulette, the non-zero calculation is shown in equation (2), and the update rule of pheromone is designed as shown in equation (3).

$$P_{ij}^k(\cdot) = \frac{\tau_{ij}^{\alpha}(\cdot) * \eta_{ij}^{\beta}(\cdot) * A}{\sum_{s \in allowed_k} \tau_{is}^{\alpha}(\cdot) * \eta_{is}^{\beta}(\cdot)}, i, j \in allowed_k \tag{2}$$

$$\tau_{ij}(t+1) = \tau_{ij}(t) * A * (1-\rho) + \Delta\tau_{ij}(t), 0 < \rho < 1 \tag{3}$$

Where, ρ denotes the information play factor, $\Delta\tau_{ij}(t)$ is the sum of the pheromone increments of m ants, setting $\Delta\tau_{ij}^k(t)$ as the information left by the kth ant pathway point i and point j, then the specific formula of $\Delta\tau_{ij}(t)$ is shown in equation (4).

$$\Delta\tau_{ij}(t) = A * \sum_{k=1}^{m} \Delta\tau_{ij}^k(t) \tag{4}$$

And the pheromone increments of the kth ant can be expressed as $\Delta\tau_{ij}^k =$

$\frac{Q}{L_k}$ after the path that between i and j. Q is the pheromone enhancement factor to adjust the pheromone increment on the path and L_k is the length of the object found by the kth ant. This study optimizes the classical ACO so that the ants prefer paths with a lower number of pheromones, allowing the ants to explore more of the unexplored parts of the graph, i.e., the probability whether ant k will chose the path of point *i* to point *j* as next route has been shown in equation (5).

$$P_{ij}^k(\cdot) = \frac{\left(\frac{1}{\tau_{ij}}\right)^{\alpha}}{\sum_{L\in N_i^k}\left(\frac{1}{\tau_{ij}}\right)^{\alpha}}, i, j \in allowed_k \tag{5}$$

In this research, the coloring of the graph G means that each of its vertices is painted with a color so that the color painted by any adjacent vertex is different. If the vertices of G will be colored with the help of k colors, i.e., G is colorable by k point; if G is colorable by k point but not by $k-1$ point, which can be said that G is k chromatic graph, and k is the chromatic number which belongs to G, showed as $x(G)$ which is defined as the minimum value of k which makes G coloring. Based on this, this study proposes the Greedy Ant Colony Graph Coloring (GAC-GC) technique for coloring distribution among network graph vertices, which is essentially a linear decomposition of the resulting graph for coloring purposes [21]. If the desire to make the recommended solution algorithm applicable to big graphs in size can be achieved, a hybrid technique based on decomposition and heuristics is proposed, the same as the contents in Algorithm 2.

Algorithm 2: Specific steps of the proposed GAC-GC algorithm

Input: Graph $G = (V, E), S, A, P, \tau, k$
Output: upper and lower bounds $B(+)/B(-)$
1: Set $B(\cdot) \leftarrow \emptyset$
2: The ACO improved by the greedy algorithm calculates the upper and lower bounds $B(+)/B(-)$
3: **while** $G(\cdot) = \sum x_{ik} * x_{jk} = 0, x_{ik}, x_{jk} \in \{0,1\}$ **do**
4: **if** there is no solution, make the lower bound $B(-) = B(-) + 1$
5: Calculate the upper bound $B(+)$
6: **end if** $B(-)! = B(+)$
7: **end**

3 Experimental Results

3.1 Data description and experimental setup

To demonstrate the powerful performance of the GAC-GC graph coloring solution algorithm proposed in this study, numerical simulations will be performed using real network [22] data set in order to observe how well the model fits realistic situations. Referring to the literature [23], the real network data set was used in the experiments (see Table 1). In this research, the user data set of Facebook (undirected) and Twitter (directed) were crawled as the base data for the experimental simulations (crawled from July 23, 2022 - August 21, 2022) using Python software with 30 recent hot comment user nodes as the initial nodes. This study generated a simple social network with the method of supposing every user as one node, using edges between nodes to represent the relationships between users, and selecting 30 highly influential users and their friend lists as the initial nodes of the network, respectively [24]. The algorithm structure in this research and the related algorithm used for comparison were implemented on Tensorflow 1.5.1. The experimental process was carried out in groups, dividing the data into 10 equal groups and test them in a crossing method, i.e., the data set was split into 10 parts equally, with one group of data selected each time as the set which is eager to be used to test and the other groups as the set for training, and the final average was taken. All data were saved in CSV format in a MySQL database for data processing. For the network data set, 10% of each user rating data was selected in a random way as the set to be tested using the Rapidminer data mining tool, and other 90% of data which belonged to users was considered as the set which will be trained. The experiments were all conducted on one server (Intel Xeon processor (34GHZ) and 32GB RAM) with a Linux operating system. The performance and efficiency comparisons of the models and algorithms conducted in this study were based on the same database, data pre-processing, servers and computing equipment, thus ensuring the scientificity of the experiments. As the graph coloring solution algorithm in the experiments may give different results in each operation, the results evaluated of the algorithm were set as the average of 1000 iterations with a standard deviation of 1.758. The conversion of directed graphs to weighted undirected graphs was carried out in the compiling environment of Matlab 7.0.

Table 1

Description of social network data set

Social network name	Type	Number of nodes	Boundary number of nodes	Average	Average path of nodes	Clustering coefficient
Facebook	Undirected	31940	759610	34.18	4.41	0.450
Twitter	Directed	41285	755592	38.12	3.24	0.305

And the GAC-GC solution algorithm structured in this research will be got together with those algorithms below for contrast: ① the link-based Greedy Link Algorithm (GLA) [21], ② the node-based Greedy Node Algorithm (GNA) [25], ③ the Classic Greedy Algorithm [26] (CGA), ④ Classic Ant Colony Algorithm [27] (CACA), ⑤ Simulated Annealing Ant Colony Algorithm [28] (SAACA), ⑥ Bee Colony Algorithm [29] (BCA), etc. In conjunction with the literature [9], two accuracy functions which will be calculated such as Mean Absolute Error (MAE) and Root Mean Square Error (RMSE), will be made full use of to show the power of the GAC-GC, which are calculated as shown in equations (6) and (7) respectively. N represents the total number of samples, f_i represents the model predicted value of the ith sample, and y_i represents the true value of the ith sample.

$$MAE = \frac{1}{N}\sum_{i=1}^{N}|f_i - y_i| \tag{6}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(f_i - y_i)^2} \tag{7}$$

Whereas the opinion propagation problem in social networks is usually considered as a binary classification task, in the confusion matrix for evaluating binary classification tasks with two categories [30]. True Positive (TP), which shows the number which can achieve the effect of predicting links correctly; True Negative (TN) shows the number of the right links which have not been predicated; False Positive (FP) indicates the number of the predicted links which have been predicated in an incorrect way; and False Negative (FN) shows the number of the false links which have not been predicated. Based on this, the following metrics can be obtained, such as True Positive Rate/Recall Rate/Sensitivity. The calculation of True Positive Rate (TPR), False Positive Rate (FPR), True Negative Rate (TNR) and Precision can be referred to Ruchansky et al. [30]. It is also evaluated based on two kinds of metrics, Area Under the Receiver Operating Characteristics Curve (AUROC) [31] and Average Precision (AP) [31]. The ROC curve is defined as the curve between the true positive rate on the Y-axis (sensitivity) and the false positive rate (1-specificity) on the X-axis. The summary statistics which belong to the single point between the area below the ROC curve of $[0,1]$ will be calculated with the rule of trapezoid, summing all trapezoids whose positions are below the curve. Link prediction methods should have an AUROC value which should be higher than 0.5; the bigger the AUROC value, the more excellent the behavior of the link prediction method. The precision of average is a summary value with some single points calculated based on different recall thresholds, and it is the precision value in average that links to the interval recall values, which is shown as equation (8), where p denotes the precision rate under different recall r thresholds and R is the set of different thresholds.

$$AP = \sum_{k=1}^{R} p(k)\Delta r(k) \tag{8}$$

3.2 Experimental outcomes

Table 2 reports the precision outcomes after being compared between the

GAC-GC algorithm and other classical solution computing methods in Facebook and Twitter networks. It shows that the proposed GAC-GC algorithm has more stable and scientific operation performance than the classical graph coloring solution algorithms and can better solve the graph coloring problems in real environment. Table 3 reports the results of the area under the curve of the proposed GAC-GC algorithm and other classical solution algorithms in Facebook and Twitter networks. This study finds that the proposed GAC-GC algorithm has better experimental results in real network data set.

Table 2

Experimental results of model comparison

Solution algorithm name	Facebook		Twitter	
	RMSE	MAE	RMSE	MAE
GLA	0.8351	0.7635	0.8341	0.8151
GNA	0.7546	0.7437	0.7384	0.7320
CGA	0.7462	0.6583	0.7291	0.6271
CAC	0.6964	0.6673	0.6372	0.5768
SAAC	0.6860	0.6564	0.5201	0.5291
BCA	0.6785	0.5733	0.5392	0.5202
GAC-GC	**0.6039**	**0.4295**	**0.4102**	**0.3251**

Note: The values displayed in bold indicate good performance of the corresponding algorithm.

Table 3

Area under curve for each data set in different methods

Data set name	GLA	GNA	CGA	CAC	SAAC	BCA	GAC-GC
Facebook	0.3251	0.3591	0.5361	0.5724	0.6019	0.6389	**0.7283**
Twitter	0.3262	0.4252	0.5392	0.5901	0.6128	0.5692	**0.6928**
Sina Weibo	0.2104	0.4210	0.5418	0.6382	0.6281	0.4280	**0.5182**

Note: The values displayed in bold indicate good performance of the corresponding algorithm.

Finally, this study analyses the time complexity of the proposed GAC-GC graph coloring algorithm. For this purpose, the average operation time of the algorithms was compared (Fig. 1 has expressed the outcomes).

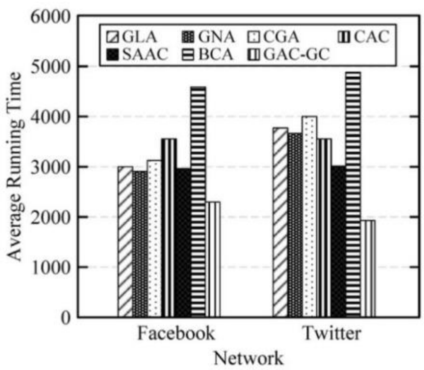

Fig. 1 Time complexity of the solution algorithm for each data set in different graph colorings

Obviously, the structured GAC-GC graph coloring algorithm is more computationally efficient compared to other methods due to the fact that this study combines the advantages of GA and ACO for graph coloring in a network environment with high computational efficiency.

4. Summary and Outlook

In this research, ACO for solving graph coloring problems based on the greedy algorithm optimization is introduced, and the GAC-GC algorithm based on the improvement of the classical ACO is proposed. The experimental outcomes, which were obtained with real data sets reveal that the improved ACO, can better solve the coloring problem of the graph vertex set by the greedy algorithm. The GAC-GC algorithm owns many obvious advantages, such as improving the behavior of the algorithm solution and getting less time complexity belonging to this algorithm. On top of that, it shows better superiority than the traditional ACO when facing large scale directed and undirected graphs whose number of vertices is quite big.

This study achieves innovations from several aspects. Firstly, this study provides an improvement of the original ACO algorithm, namely the addition of an approximate greedy method. Meanwhile, this study proposes a hybrid technique based on decomposition and heuristics, whose essence is linear decomposition and coloring of graphs, solving the coloring problem of large-scale graphs. Secondly, the large-scale graph applied in this study has the characteristics of real-time and high realism, unlike the type of graphs often applied by the ACO in practical life problems. This study constructs an undirected graph and a directed graph with 30 recent hot comment user nodes of Facebook and Twitter respectively and completes the social network link prediction for the opinion propagation problem. In addition, the strengths and weaknesses of algorithmic in this study were compared in various and effective ways. This study compares the algorithmic solutions of the GAC-GC algorithm in this study with those of the GLA algorithm, GNA algorithm, CGA algorithm, CAC algorithm, SAAC algorithm, and BCA algorithm. This study uses the algorithmic efficiency measures of RMSE and MAE, which are precision functions, and AUROC values, which are algorithmic measures of testing curve area values and time complexity. Meanwhile, the arithmetic example of Sina Weibo is also added for the comparison of the area values of AUROC curves, which fully reflects the superiority of this study.

Although this study has presented the above findings of great significance, it has certain limitations, some of which may point the way for further research in the future. Firstly, in terms of future algorithm improvements for solving graph coloring problems, three aspects can be explored: broadening the application context, integrated application of algorithmic data pre-processing, and hybrid algorithm improvement applications. In addition, the application context of the graph coloring problem can be broadened first, then algorithmic pre-processing of the data of the arithmetic example will be performed, and then a new hybrid algorithm will be structured to obtain the solution of GCP in the way of combining multiple intelligent algorithms. Finally, advanced deep learning techniques such as auto-encoder or neural networks can be considered to be introduced into the ACO to further improve the efficiency and precision which belong to the algorithm in the research for solving GCP.

REFERENCES

[1] Lima P T, Leeuwen E J, Wegen M. Algorithms for the Rainbow Vertex Coloring Problem on Graph Classes. Theoretical Computer Science, 2021, 887: 122-142.
[2] Rosiane de F, Bruno D, Nelson M, et al. On Distance Graph Coloring Problems. International Transactions in Operational Reseaech, 2021, 28(3): 1213-1241.
[3] George B. M, Hendrik M, Viktor Z. Sliding Window Temporal Graph Coloring. 2021, 120: 97-115.
[4] Hajdu L, Toth A, Kresz M. Graph Coloring Based Heuristic for Crew Rostering. Acta Cybernetica, 2020, 24(4): 643-661.
[5] Sharma P C, Chaudhari N S. A Tree Based Novel Approach for Graph Coloring Problem Using Maximal Independent Set. Wireless Personal Communications, 2020, 110(3): 1143-1155.
[6] Fujita S, Kitaev S, Sato S, et al. On Properly Ordered Coloring of Vertices in a Vertex-Weighted Graph. Order-A Journal on the Theory of Ordered Sets and Its Applications, 2021, 38(3): 515-525.
[7] Ardelean S M, Udrescu M. Graph Coloring Using the Reduced Quantum Genetic Algorithm. Peerj, 2022, 8: e836-e836.
[8] Natarajan Y, Kannan S, Dhiman G. Task Scheduling in Cloud Using ACO. Recent Advances in Computer Science and Communications, 2022, 15(3): 348-353.
[9] Wang Y, Han Z. Ant Colony Optimization for Traveling Salesman Problem Based on Parameters Optimization. Applied Soft Computing Journal, 2021, 107.
[10] Awan H H, Shahzad W. Semi-Supervised Associative Classification Using Ant Colony Optimization Algorithm. Peerj, 2021, 7, e676-e676.
[11] Wang P, Bai J Y, Meng J. A Hybrid Genetic Ant Colony Optimization Algorithm with an Embedded Cloud Model for Continuous Optimization. Journal of Information Processing Systems, 2022, 16(5): 1169-1182.
[12] Addou E H, Serghini A, Mermri E B. Hybrid Approaches Based on Simulated Annealing, Tabu Search and Ant Colony Optimization for Solving The K-Minimum Spanning Tree Problem. Revstat-Statistical Journal, 2021, 12(2): 708-712.
[13] LiuW H, Zheng X. Three-Dimensional Multi-Mission Planning of UAV Using Improved Ant Colony Optimization Algorithm Based on the Finite-Time Constraints. International Journal of Computational Intelligence Systems, 2021, 14(1): 79-87.
[14] Zhao H T, Zhang C S, Zheng X Y, et al. A Decomposition-Based Many-Objective Ant Colony Optimization Algorithm with Adaptive Solution Construction and Selection Approaches. Swarm and Evolutionary Computation, 2022, 68, 100977.
[15] Lv L Y, Gao C, Chen J J, et al. Physarum-based ant colony optimization for graph coloring problem. 10th International Conference on Swarm Intelligence, ICSI 2019, 2019: 210-219.
[16] Li X, Liu J B. A Novel Approach to Speed Up Ant Colony Algorithm via Applying Vertex

Coloring. International Journal of Parallel, Emergent and Distributed Systems, 2018, 33(6): 608-617.
[17] Dowsland K A, Thompson J M. An Improved Ant Colony Optimisation Heuristic for Graph Colouring. Discrete Applied Mathematics, 2008, 156(3): 313-324.
[18] Kumar C T, Karpurapu A, Singh Y P. Reduction of PAPR for FBMC-OQAM System Using Ant Colony Optimisation Technique. Soft Computing, 2022, 26(9): 4295-4302.
[19] Fei T, Wu X X, Zhang L Y, et al. Research on Improved Ant Colony Optimization for Traveling Salesman Problem. Mathematical Biosciences and Engineering: MBE, 2022, 19(8): 8152-8186.
[20] Kumazawa T, Takimoto M, Kambayashi Y. Exploration Strategies for Balancing Efficiency and Comprehensibility in Model Checking with Ant Colony Optimization. Journal of Information and Telecommunication, 2022, 6(3): 341-359.
[21] Tong G, Wu W, Du D Z. On Misinformation Containment in Online Social Networks. Advances in neural information processing systems 31: annual conference on neural information processing systems 2018, Montréal, Canada, 2018: 339-349.
[22] Page L, Brin S, Motwani R, et al. The Pagerank Citation Ranking: Bringing Order to The Web. Technical Report, Stanford Digital Library Technologies Project, 1998.
[23] Nguyen H T, Cano A, Tam V, et al. Blocking Self-Avoiding Walks Stops Cyber-Epidemics: A Scalable GPU-Based Approach. IEEE Transactions on Knowledge and Data Engineering, 2020, 32(7): 1263-1275.
[24] Nguyen N P, Yan G, Thai M T. Analysis of misinformation containment in online social networks. Computer Networks, 2013, 57(10): 2133-2146.
[25] Zhang H, Alim M A, Li X, et al. Misinformation in Online Social Networks: Detect Them All with A Limited Budget. Acm Transactions on Information Systems, 2016, 34(3): 1-24.
[26] Naghizadeh A, Abavisani M, Metaxas D N. Greedy AutoAugment. Pattern Recognition Letters, 2020, 138: 624-630.
[27] Liu C Y, Mu F R, Zhang W L. Cloud Computing Demand Elasticity Algorithm Based on Ant Colony Algorithm. Recent Advances in Electrical & Electronic Engineering, 2021, 14(1): 37-43.
[28] Gai Z M, Liu R T, Jiang Q X. A New Model of Projection Pursuit Grade Evaluation Model Based on Simulated Annealing Ant Colony Optimization Algorithm. International Journal of Cognitive Informatics and Natural Intelligence, 2018, 12(4): 69-80.
[29] Han X M, Wang Y D, Cai C, et al. An Efficient Universal Bee Colony Optimization Algorithm. Tehnicki Vjesnik-Technical Gazette, 2020, 27(1): 320-332.
[30] Ruchansky N, Seo S, Liu Y. CSI: A Hybrid Deep Model for Fake News Detection. 2017 ACM Conf. Inf. Knowl. Manage., Singapore, 2017: 797-806.
[31] Zhang H, Nguyen D T, Zhang H, et al. Least Cost Influence Maximization Across Multiple Social Networks. IEEE/ ACM Transactions on Networking, 2016, 24(2): 929-939.