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FIXED POINTS OF A CLASS OF CONTRACTIVE-TYPE MULTIFUNCTIONS ON FUZZY METRIC SPACES

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We prove a fixed point theorem for a contractive type multifunction on a nonempty and closed subset of a complete fuzzy metric spaces. An example is given.

Keywords: Contraction, fixed point, Hausdorff fuzzy metric, multifunction.
MSC2000: 47H10, 54H25.

1. Introduction

In 1922, Banach proved the principle contraction result ([9]). As we know, there have been published many works about fixed points theory for different kinds of contractions on some spaces such quasi-metric spaces [11, 23], cone metric spaces [4, 15, 39], convex metric spaces [33], partially ordered metric spaces [3, 5, 8, 10, 13, 16, 38, 42, 44], G-metric spaces [6, 7, 14, 43, 46, 47], b-metric spaces [48], partial metric spaces [2, 25, 41], quasi-partial metric spaces [45], Menger spaces [32] and fuzzy metric spaces [19, 20]. Also, studies either on approximate fixed point or on qualitative aspects of numerical procedures for approximating fixed points are available in literature; please, see [22, 29, 30, 33].

The concept of fuzzy sets introduced by Zadeh in 1965 ([56]). In 1975, Kramosil and Michalek introduced the notion of fuzzy metric spaces ([27]) and George and Veeramani modified the concept in 1994 ([18]). Some researchers have been provided different fixed point results in fuzzy metric spaces (see for example, [1, 12, 17, 24, 26, 31, 35, 49, 50, 51, 52, 54, 55]).

In this paper, we prove a fixed point theorem for a class of contractive type multifunctions, defined on a nonempty and closed subset of a complete fuzzy metric space. An example is given to support the usability of our main result.

2. Preliminaries

We shall use the notion of Hadzic-type t-norms in the sense of [21] in this paper.

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Let X be a non-empty set, $*$ a Hadzic-type t-norm and M a fuzzy set on $X^2 \times [0, \infty)$ such that

1) $M(x, y, 0) = 0$,
2) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
3) $M(x, y, t) = M(y, x, t)$,
4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$, for all $x, y, z \in X$ and $s, t > 0$,
5) $M(x, y, \cdot)\colon [0, \infty) \to [0, 1]$ is continuous and $\lim_{t\to\infty} M(x, y, t) = 1$, for all $x, y \in X$.

Then, $(X, M, *)$ is called a *fuzzy metric space.* ([34]).

Let $(X, M, *)$ be a fuzzy metric space. Put

$$B(x, r, t) = \{y \in X \,:\, M(x, y, t) > 1 - r\},$$

for all $x \in X$, $t > 0$ and $0 < r < 1$.

Denote the generated topology by the sets $B(x, r, t)$ by τ_M.

It has been proved that in an fuzzy metric space every compact set is closed and bounded ([34]).

A sequence $\{x_n\}$ in $(X, M, *)$ is called a *Cauchy sequence* whenever for each $\varepsilon > 0$ and $t > 0$, there exists a natural number n_0 such that $M(x_n, x_m, t) > 1 - \varepsilon$ for all $n, m \ge n_0$. Also, the triple $(X, M, *)$ is called a *complete fuzzy metric space* whenever every Cauchy sequence is convergent with respect τ_M.

A selfmap f on a fuzzy metric space $(X, M, *)$ is called a *fuzzy contraction* whenever there exists $k \in (0, 1)$ such that

$$\frac{1}{M(f(x), f(y), t)} - 1 \le k \left(\frac{1}{M(x, y, t)} - 1 \right),$$

for all $x, y \in X$ and $t > 0$; please, see [36].

Let B be a nonempty subset of a fuzzy metric space $(X, M, *)$.

According to [53], for $x \in X$ and $t > 0$, define

$$M(x, B, t) = \sup_{b\in B} M(x, b, t).$$

For a fuzzy metric space $(X, M, *)$, denote by $\mathcal{CB}(X)$ and $\mathcal{H}(X)$ the set of nonempty closed bounded subsets and the set of nonempty compact subsets of (X, τ_M).

Let B be a nonempty subset of a fuzzy metric space $(X, M, *)$, $x \in X$ and $t > 0$. In this case, H_M stands the Hausdorff fuzzy metric space on $\mathcal{H} \times \mathcal{H} \times (0, \infty)$ which defined by

$$H_M(A, B, t) = \min \left\{ \inf_{a\in A} M(a, B, t), \inf_{b\in B} M(b, A, t) \right\}$$

for all $A, B \in \mathcal{H}$ and $t > 0$ ([40]).

3. **The result**

Now, we are ready to state and prove our main result. For this purpose, we use the technique of of Reem, Reich and Zaslavski [37]. Here, we suppose that there is a metric d on X and

$$M(x,y,t)=\frac{t}{t+d(x,y)},\quad \forall\, t\geq 0.$$

Theorem 3.1. *Suppose that K be a non-empty closed subset of a complete fuzzy metric space $(X,M,*)$, $c\in[0,1)$ and $T\colon K\to\mathcal{H}(X)$ satisfies*

$$\frac{1}{H_M(Tx,Ty,t)}-1\leq c\left(\frac{1}{M(x,y,t)}-1\right),$$

for all $x,y\in X$. Let K_0 be a bounded subset of K and $\{x_n\}_{n\geq 1}$ a sequence in K_0 such that $\cup_{i=1}^{n}T^ix_n\subseteq K$ for all $n\geq 1$. Then T has a fixed point.

Proof. First of all, note that

$$\frac{1}{H_M(T^{i+1}x,T^{i+1}y,t)}-1\quad\leq\quad c\left(\frac{1}{H_M(T^ix,T^iy,t)}-1\right)\tag{3.1}$$

for all $i\geq 1$ and $x,y\in K$, with T^ix, $T^iy\subseteq K$.

Since K_0 is bounded, there exist $\theta\in K$ and $c_0>0$ such that $\frac{1}{M(\theta,z,t)}-1\leq\frac{c_0}{t}$ for all $z\in K_0$.

Now, we continue the proof in several steps.

$\star$Step 1. For each $\varepsilon>0$ there exists $n_0\geq 1$ such that

$$\frac{1}{H_M(T^ix_n,T^{i+1}x_n,t)}-1\leq\frac{\varepsilon}{t}\tag{3.2}$$

for all $n>n_0$ and $n_0\leq i<n$.

Proof of Step 1. If (3.2) does not hold, then for each $m\geq 1$ there exist n_m and i_m such that $m\leq i_m<n_m$ and

$$\frac{1}{H_M(T^{i_m}x_{n_m},T^{i_m+1}x_{n_m},t)}-1>\frac{\varepsilon}{t}\tag{3.3}$$

for some $\varepsilon>0$.

Choose a natural number m such that $m>\frac{2c_0+t(\frac{1}{H_M(\{\theta\},T\theta,t)}-1)}{(1-c)\varepsilon}$.

Since $c<1$, by using (3.1) and (3.2) we conclude that

$$\frac{1}{H_M(T^ix_{n_m},T^{i+1}x_{n_m},t)}-1>\frac{\varepsilon}{t},$$

for all $i=1,2,\ldots,i_m$.

Since

$$\frac{1}{H_M(T^{i+2}x_{n_m},T^{i+1}x_{n_m},t)}-1\leq c\left(\frac{1}{H_M(T^{i+1}x_{n_m},T^ix_{n_m},t)}-1\right),$$

for all $i=1,2,\ldots,i_m-1$, by using (3.3), we get

$$\left(\frac{1}{H_M(T^{i+2}x_{n_m},T^{i+1}x_{n_m},t)}-1\right)-\left(\frac{1}{H_M(T^{i+1}x_{n_m},T^ix_{n_m},t)}-1\right)$$
$$\leq(c-1)\left(\frac{1}{H_M(T^{i+1}x_{n_m},T^ix_{n_m},t)}-1\right)<\frac{(c-1)\varepsilon}{t}.$$

On the other hand,

$$\frac{\varepsilon}{t} < \frac{1}{H_M(T^2x_{n_m}, Tx_{n_m}, t)} - 1 \leq c\left(\frac{1}{H_M(\{x_{n_m}\}, Tx_{n_m}, t)} - 1\right) \leq \frac{1}{H_M(\{x_{n_m}\}, Tx_{n_m}, t)} - 1.$$

Hence,

$$\left(\frac{1}{H_M(T^2x_{n_m}, Tx_{n_m}, t)} - 1\right) - \left(\frac{1}{H_M(\{x_{n_m}\}, Tx_{n_m}, t)} - 1\right) \leq (c-1)\left(\frac{1}{H_M(\{x_{n_m}\}, Tx_{n_m}, t)} - 1\right) < \frac{(c-1)\varepsilon}{t}.$$

Thus,

$$\begin{aligned} -\left(\frac{1}{H_M(\{x_{n_m}\}, Tx_{n_m}, t)} - 1\right) &\leq \left(\frac{1}{H_M(T^{i_m+1}x_{n_m}, T^{i_m}x_{n_m}, t)} - 1\right) - \left(\frac{1}{H_M(\{x_{n_m}\}, Tx_{n_m}, t)} - 1\right) \\ &\leq \sum_{i=1}^{i_m-1}\left[\left(\frac{1}{H_M(T^{i+2}x_{n_m}, T^{i+1}x_{n_m}, t)} - 1\right) - \left(\frac{1}{H_M(T^{i+1}x_{n_m}, T^{i}x_{n_m}, t)} - 1\right)\right] \\ &\quad + \left[\left(\frac{1}{H_M(T^2x_{n_m}, Tx_{n_m}, t)} - 1\right) - \left(\frac{1}{H_M(\{x_{n_m}\}, Tx_{n_m}, t)} - 1\right)\right] \\ &\quad + \frac{(1-c)\varepsilon i_m}{t} \leq \frac{m(c-1)\varepsilon}{t}. \end{aligned} \tag{3.4}$$

Since H_M is a fuzzy metric on $\mathcal{H}(X)$, from (3.1) and (3.4), we obtain

$$\begin{aligned} \frac{m(1-c)\varepsilon}{t} &\leq \frac{1}{H_M(\{x_{n_m}\}, Tx_{n_m}, t)} - 1 \\ &\leq \left(\frac{1}{H_M(\{x_{n_m}\}, \{\theta\}, t)} - 1\right) + \left(\frac{1}{H_M(\{\theta\}, T\theta, t)} - 1\right) \\ &\quad + \left(\frac{1}{H_M(T\theta, Tx_{n_m}, t)} - 1\right) \\ &\leq \left(\frac{1}{M(x_{n_m}, \theta, t)} - 1\right) + \left(\frac{1}{H_M(\{\theta\}, T\theta, t)} - 1\right) \\ &\leq c\left(\frac{1}{M(\theta, x_{n_m}, t)} - 1\right) \leq \frac{2c_0}{t} + \left(\frac{1}{H_M(\{\theta\}, T\theta, t)} - 1\right), \end{aligned}$$

which is a contradiction. This implies that (3.2) holds.

$\star$STEP 2. For each $\delta > 0$ there exists $n_0 \geq 1$ such that

$$\frac{1}{H_M(T^ix_n, T^jx_n, t)} - 1 \leq \frac{\delta}{t}, \tag{3.5}$$

for all $n > n_0$ and $n_0 \leq i, j < n$.

PROOF OF STEP 2. Let us consider $\varepsilon < \frac{1}{4}\delta(1-c)$. Choose $n_0 \geq 1$ such that (3.2) holds, for all $n > n_0$ and $n_0 \leq i < n$.

We claim that (3.5) holds for all $n_0 \le i, j < n$.

If $\frac{1}{H_M(T^i x_n, T^j x_n, t)} - 1 > \frac{\delta}{t}$ for some i and j, then

$$\begin{aligned} \frac{1}{H_M(T^i x_n, T^j x_n, t)} - 1 \quad &\le \quad \left(\frac{1}{H_M(T^i x_n, T^{i+1} x_n, t)} - 1\right) \\ &\quad + \left(\frac{1}{H_M(T^{i+1} x_n, T^{j+1} x_n, t)} - 1\right) \\ &\quad + \left(\frac{1}{H_M(T^{j+1} x_n, T^j x_n, t)} - 1\right) \\ &\le \quad \frac{2\varepsilon}{t} + \left(\frac{1}{H_M(T^{i+1} x_n, T^{j+1} x_n, t)} - 1\right) \\ &\le \quad \frac{2\varepsilon}{t} + c\left(\frac{1}{H_M(T^i x_n, T^j x_n, t)} - 1\right). \end{aligned}$$

This implies that

$$\frac{\delta}{t} < \frac{1}{H_M(T^i x_n, T^j x_n, t)} - 1 \le \frac{2\varepsilon}{t(1-c)},$$

which is a contradiction.

$\star$STEP 3. For each $\varepsilon > 0$ there exists a natural number n_0 such that

$$\frac{1}{H_M(T^{n_0} x_{n_1}, T^{n_0} x_{n_2}, t)} - 1 \quad \le \quad \frac{\varepsilon}{t}, \tag{3.6}$$

for all $n_1, n_2 > n_0$.

PROOF OF STEP 3. Choose a natural number n_0 such that $n_0 > \frac{4c_0}{\varepsilon(1-c)}$. Now, suppose that $n_1, n_2 > n_0$.

We claim that

$$\frac{1}{H_M(T^{n_0} x_{n_1}, T^{n_0} x_{n_2}, t)} - 1 \le \frac{\varepsilon}{t}. \tag{3.7}$$

If (3.7) does not hold, then $\frac{1}{H_M(T^i x_{n_1}, T^i x_{n_2}, t)} - 1 > \frac{\varepsilon}{t}$ for all $i = 1, 2, \ldots, n_0$, because $c < 1$.

Note that,

$$\frac{\varepsilon}{t} < \frac{1}{H_M(T x_{n_1}, T x_{n_2}, t)} - 1 \le c\left(\frac{1}{M(x_{n_1}, x_{n_2}, t)} - 1\right) \le \frac{1}{M(x_{n_1}, x_{n_2}, t)} - 1.$$

Hence, $-(\frac{1}{M(x_{n_1}, x_{n_2}, t)} - 1) < -\varepsilon$.

Since

$$\frac{1}{H_M(T^{i+1} x_{n_1}, T^{i+1} x_{n_2}, t)} - 1 \le c\left(\frac{1}{H_M(T^i x_{n_1}, T^i x_{n_2}, t)} - 1\right)$$

for all $i = 1, 2, \ldots, n_0 - 1$, we get

$$\begin{gathered} \left(\frac{1}{H_M(T^{i+1} x_{n_1}, T^{i+1} x_{n_2}, t)} - 1\right) - \left(\frac{1}{H_M(T^i x_{n_1}, T^i x_{n_2}, t)} - 1\right) \\ \le (c-1)\left(\frac{1}{H_M(T^i x_{n_1}, T^i x_{n_2}, t)} - 1\right) < -(1-c)\frac{\varepsilon}{t}, \end{gathered}$$

for all $i = 1, 2, \ldots, n_0 - 1$.

This implies that

$$-\left(\frac{1}{M(x_{n_1},x_{n_2},t)}-1\right)\leq\left(\frac{1}{H_M(T^{n_0}x_{n_1},T^{n_0}x_{n_2},t)}-1\right)-\left(\frac{1}{M(x_{n_1},x_{n_2},t)}-1\right)$$
$$\leq\sum_{i=1}^{n_0-1}\left[\left(\frac{1}{H_M(T^{i+1}x_{n_1},T^{i+1}x_{n_2},t)}-1\right)-\left(\frac{1}{H_M(T^{i}x_{n_1},T^{i}x_{n_2},t)}-1\right)\right]$$
$$+\left[\left(\frac{1}{H_M(Tx_{n_1},Tx_{n_2},t)}-1\right)-\left(\frac{1}{M(x_{n_1},x_{n_2},t)}-1\right)\right]\leq -n_0(1-c)\frac{\varepsilon}{t}.$$

Thus,

$$n_0(1-c)\frac{\varepsilon}{t}\leq\frac{1}{M(x_{n_1},x_{n_2},t)}-1\leq\left(\frac{1}{M(x_{n_1},\theta,t)}-1\right)+\left(\frac{1}{M(\theta,x_{n_2},t)}-1\right)\leq\frac{2c_0}{t}$$

and so $n_0\leq\frac{2c_0}{\varepsilon(1-c)}$.

This contradiction shows that (3.7) holds.

$\star$STEP 4. For each $\varepsilon>0$ there exists a natural number $m(\varepsilon)\geq 1$ such that

$$\frac{1}{H_M(T^{i}x_{n_1},T^{i}x_{n_2},t)}-1\leq\varepsilon,$$

for all $n_1,n_2>m(\varepsilon)$ and natural numbers $i\in[m(\varepsilon),n_1)$ and $j\in[m(\varepsilon),n_2)$.

Proof of Step 4. Let $\varepsilon>0$ be given.

By using (3.6), choose $m_1\geq 1$ such that

$$\frac{1}{H_M(T^{m_1}x_{n_1},T^{m_1}x_{n_2},t)}-1\leq\frac{\varepsilon}{4t}$$

for all $n_1,n_2>m_1$.

By using (3.5), choose $m_2\geq 1$ such that

$$\frac{1}{H_M(T^{i}x_{n},T^{j}x_{n},t)}-1\leq\frac{\varepsilon}{4t}$$

for all $n>m_2$ and $m_2\leq i,j<n$.

Put $m(\varepsilon):=m_1+m_2$. Suppose that $n_1,n_2>m(\varepsilon)$ and i and j are natural numbers such that $i\in[m(\varepsilon),n_1)$ and $j\in[m(\varepsilon),n_2)$. Then,

$$\frac{1}{H_M(T^{m(\varepsilon)}x_{n_1},T^{m(\varepsilon)}x_{n_2},t)}-1\leq\frac{1}{H_M(T^{m_1}x_{n_1},T^{m_1}x_{n_2},t)}-1\leq\frac{\varepsilon}{4t}.$$

Also,

$$\frac{1}{H_M(T^{m(\varepsilon)}x_{n_1},T^{i}x_{n_1},t)}-1\leq\frac{\varepsilon}{4t},\quad\frac{1}{H_M(T^{m(\varepsilon)}x_{n_2},T^{j}x_{n_2},t)}-1\leq\frac{\varepsilon}{4t}.$$

Thus,

$$\frac{1}{H_M(T^{i}x_{n_1},T^{j}x_{n_2},t)}-1\leq\left(\frac{1}{H_M(T^{m(\varepsilon)}x_{n_1},T^{i}x_{n_1},t)}-1\right)$$
$$+\left(\frac{1}{H_M(T^{m(\varepsilon)}x_{n_1},T^{m(\varepsilon)}x_{n_2},t)}-1\right)+\left(\frac{1}{H_M(T^{m(\varepsilon)}x_{n_2},T^{j}x_{n_2},t)}-1\right)<\frac{\varepsilon}{t},$$

and this completes the proof of the step.

Now, we complete the proof of the theorem.

Consider the sequences $\{T^{n-2}x_n\}_{n\geq 3}$ and $\{T^{n-1}x_n\}_{n\geq 2}$. For each $\varepsilon > 0$, take $N = m(\varepsilon) + 2$. Let $m, n \geq N$, $i = m - 2$, $j = n - 2$, $n_1 = m$ and $n_2 = n$. Then, it is easy to see that $i \in [m(\varepsilon), n_1)$ and $j \in [m(\varepsilon), n_2)$. By using STEP 4, we get $\frac{1}{H_M(T^{m-2}x_m, T^{n-2}x_n, t)} - 1 < \frac{\varepsilon}{t}$. Thus, $\{T^{n-2}x_n\}_{n\geq 3}$ is a Cauchy sequence.

By using a similar argument, one can conclude that $\{T^{n-1}x_n\}_{n\geq 2}$ is a Cauchy sequence and

$$\lim_{n\to\infty} \frac{1}{H_M(T^{n-2}x_n, T^{n-1}x_n, t)} - 1 = 0.$$

Note that, the sequences $\{T^{n-2}x_n\}_{n\geq 3}$ and $\{T^{n-1}x_n\}_{n\geq 2}$ lie in K and $(\mathcal{H}(K), H_M, *)$ is a complete fuzzy metric space. Hence, there exists $A \in \mathcal{H}(K)$ such that

$$\lim_{n\to\infty} \frac{1}{H_M(T^{n-2}x_n, A, t)} - 1 = \lim_{n\to\infty} \frac{1}{H_M(T^{n-1}x_n, A, t)} - 1 = 0.$$

Since

$$\begin{aligned}
&H_M(T^{n-1}x_n, T(A), t)\\
&= \min\left\{ \inf_{a\in T^{n-2}x_n, a_1\in Ta} \sup_{b\in A, b_1\in Tb} M(a_1, b_1, t), \inf_{d_1\in Td, d\in A} \sup_{c_1\in Tc, c\in T^{n-2}x_n} M(c_1, d_1, t) \right\}\\
&= \min\left\{ \inf_{a\in T^{n-2}x_n} \sup_{b\in A} \inf_{a_1\in Ta} \sup_{b_1\in Tb} M(a_1, b_1, t), \inf_{d\in A} \sup_{c\in T^{n-2}x_n} \inf_{d_1\in Td} \sup_{c_1\in Tc} M(c_1, d_1, t) \right\}\\
&\geq \min\left\{ \inf_{a\in T^{n-2}x_n} \sup_{b\in A} H_M(Ta, Tb, t), \inf_{d\in A} \sup_{c\in T^{n-2}x_n} H_M(Tc, Td, t) \right\}\\
&\geq c\min\left\{ \inf_{a\in T^{n-2}x_n} \sup_{b\in A} M(a, b, t), \inf_{d\in A} \sup_{c\in T^{n-2}x_n} M(c, d, t) \right\}\\
&= cH_M(T^{n-2}x_n, A, t) \to 1,
\end{aligned}$$

we get $A = T(A)$. Thus, $T|_A \colon A \to \mathcal{H}(A)$ is a contractive multifunction.

Since $(A, M|_{A\times A}, *)$ is a complete fuzzy metric space, by using main result of [28], $T|_A$ has a fixed point in A and so there exists $x_0 \in A \subseteq K$ such that $x_0 \in Tx_0$. $\square$

Example 3.1. Let $X = [0, \infty)$, $M(x, y, t) = \frac{t}{t+|x-y|}$, $m \geq 2014$ a fixed natural number, $K = \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}}, \ldots\right\} \cup \{0, 1\}$ and $K_0 = \left\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{4}}, \ldots\right\}$.

Now, define the multifunction

$$T\colon K \to \mathcal{H}(X), \quad Tx = \left\{\frac{x}{\sqrt{2}}, \frac{x}{\sqrt{3}}, \frac{x}{\sqrt{4}}, \ldots, \frac{x}{\sqrt{m}}\right\}, \quad \text{for all } x \in K.$$

Note that, the values of T are compact and

$$\frac{1}{H_M(Tx, Ty, t)} - 1 \leq \frac{1}{2}\left(\frac{1}{M(x, y, t)} - 1\right), \quad \text{for all } x, y \in K.$$

Since $Tx \subseteq K$ for all $x \in K_0$, it is easy to see that $\cup_{i=1}^{n} T^i x_n \subseteq K$ for all $n \geq 1$, where $\{x_n\}_{n\geq 1}$ is a sequence in K_0. Thus, T satisfies the conditions of Theorem 3.1 and so has a fixed point.

4. **Conclusions**

Using the technique of of Reem, Reich and Zaslavski [37], in this article we proved a fixed point theorem for a class of contractive type multifunctions, defined on a nonempty and closed subset of a complete fuzzy metric space. An example is given to support the usability of our main result.

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