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WAVE PROPAGATION IN MEDIA OBEYING A THERMOVISCOANELASTIC MODEL

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The aim of this work is to investigate mechanical phenomenological and state coefficients taking in account irreversible processes in isotropic viscoanelastic media of order one. In the contest of Kluitenberg-Ciancio theory we consider the case of linear transverse acoustic wave of high frequency propagates in such a medium showing that the aforementioned coefficients assume a particular form as function of complex wave number. By mean of considerations on linear phenomenological acoustic theory, we determine a connection between complex wave number and shear complex modulus, and so we are able to express phenomenological and state coefficients as function of complex shear modulus. The experimental knowledge of such a modulus, as function of the frequency, allow us to experimental evaluation of the aforementioned coefficients. This approach has been applied to polymeric materials as PolyIsobutilene.

1. Introduction

In a previous paper [1], considering Kluitenberg-Ciancio (K.C.) theory on isotropic viscoanelastic medium of order one, we have introduced a method to calculate some mechanical phenomenological and state coefficients [2], as function of frequency dependent quantities experimentally measurable, when it is perturbed by an harmonic shear deformation. This has been possible by deriving a connection between aforementioned coefficients and complex shear modulus experimentally measurable.

Since mechanical response of materials depends on perturbation which it is subject, it can be useful to investigate the form which assume the aforementioned coefficients when the medium is subject to a different perturbation as transverse acoustic wave. To this hope let a displacement $\underline{u}$ be of the form:

$$u_3 = Ae^{i(kx_1 - \omega t)} \qquad u_1 = u_2 = 0 \tag{1}$$

where $i^2=-1$, K is the complex number, ω is the real angular frequency and A is a constant which may be complex. The relations (1) represent a plane wave which

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propagates in the direction of the x_1-axis, while u has the direction of the x_3-axis. Since in a viscoanelastic medium attenuation occur, it is useful to introduce a complex wave number

$$K = K_1 + iK_2 \tag{2}$$

where K_1 and K_2 are real and are connected to phase velocity V_s of the wave

$$V_S = \frac{\omega}{K_1} \tag{3}$$

and to attenuation respectively.

The equation (1) become

$$u_3 = Ae^{-K_2 x} e^{i(K_1 x - \omega t)} \tag{4}$$

We will remember that in K.C. theory for such a medium the following equation is derived

$$\frac{d\tilde{\tau}_{\alpha\beta}}{dt} + R_0^{(\tau)}\tilde{\tau}_{\alpha\beta} = R_0^{(\varepsilon)}\tilde{\varepsilon}_{\alpha\beta} + R_1^{(\varepsilon)}\frac{d\tilde{\varepsilon}_{\alpha\beta}}{dt} + R_2^{(\varepsilon)}\frac{d^2\tilde{\varepsilon}_{\alpha\beta}}{dt^2} \tag{5}$$

$$\begin{aligned} &R_0^{(\tau)} = a^{(1,1)}\eta_s^{(1,1)} = 1/\sigma \qquad && R_1^{(\varepsilon)} = a^{(0,0)} + a^{(1,1)}\eta_s^{(1,1)}\eta_s^{(0,0)} \\ &R_0^{(\varepsilon)} = a^{(0,0)}\left(a^{(1,1)} - a^{(0,0)}\right)\eta_s^{(1,1)} \qquad && R_2^{(\varepsilon)} = \eta_s^{(0,0)} \end{aligned} \tag{6}$$

in which $a^{(0,0)}, a^{(1,1)}$ are state coefficients while $\eta_s^{(1,1)}, \eta_s^{(0,0)}$ are phenomenological coefficients, and σ is relaxation time. It can be shown that by considerations on stress and strain the only component which are different from zero are

$$\tilde{\tau}_{13} = \tilde{\tau}_{31} = \tau_{13} = \tau_{31} = i\rho_0 \frac{\omega^2}{K} Ae^{(kx_1 - \omega t)}$$

$$\tilde{\varepsilon}_{13} = \tilde{\varepsilon}_{31} = \varepsilon_{13} = \varepsilon_{31} = \frac{1}{2} iKAe^{(kx_1 - \omega t)}$$

and the equation (5) furnish the following expressions for complex wave number [3] [4]:

$$K_1 = \omega\sqrt{\rho_0}\left\{B(\omega)\left(\sqrt{1+D(\omega)}+1\right)\right\}^{1/2} \tag{7}$$

$$K_2 = \omega\sqrt{\rho_0}\left\{B(\omega)\left(\sqrt{1+D(\omega)}-1\right)\right\}^{1/2} \tag{8}$$

$$|K_1| \geq |K_2|$$

where

$$B(\omega) = \frac{R_0^{(\tau)}R_0^{(\varepsilon)} + \omega^2\left(R_1^{(\varepsilon)} - R_0^{(\tau)}R_2^{(\varepsilon)}\right)}{\left(R_0^{(\varepsilon)} - \omega^2 R_2^{(\varepsilon)}\right)^2 + \left(\omega R_1^{(\varepsilon)}\right)^2} \tag{9}$$

$$D(\omega) = \frac{\omega^2\left(R_0^{(\varepsilon)} - R_1^{(\varepsilon)}R_0^{(\tau)} - \omega^2 R_2^{(\varepsilon)}\right)^2}{\left\{R_0^{(\tau)}R_0^{(\varepsilon)} + \omega^2\left(R_1^{(\varepsilon)} - R_0^{(\tau)}R_2^{(\varepsilon)}\right)\right\}^2} \tag{10}$$

$$B(\omega) > 0 \qquad D(\omega) > 0$$

2. Complete system for the coefficients in the k.c. Rheological equation.

Since K_1 and K_2 are, as we will see, connected to quantities experimental measurable the equations (7) and (8) make an algebraic system of two equations with three unknown functions $R_0^{(\varepsilon)}, R_1^{(\varepsilon)}, R_2^{(\varepsilon)}$ while $R_0^{(\tau)} = 1/\sigma$ is experimentally measurable (σ =relaxation time). To obtain the fourth equation to complete the system we observe that for high frequency the medium has a meanly elastic behaviour and so the sound velocity is such that

$$G_1 \cong \rho_0 V_S^2 = \rho_0 \frac{\omega^2}{K_1^2} \tag{11}$$

where G_1 is the real part of complex shear modulus $G=G_1+iG_2$. Remembering the physical meaning of the ratio $\dfrac{R_0^{(\varepsilon)}}{R_0^{(\tau)}}$ and equation (3) we can write:

$$R_0^{(\varepsilon)} \cong R_0^{(\tau)} G_1 \cong \frac{\rho_0 \omega^2}{K_1^2} R_0^{(\tau)} \tag{12}$$

This is the fourth equation which complete the system make by equations (7), (8), (12). The solution of such a system is:

$$\begin{aligned}
&R_0^{(\tau)} = 1/\sigma \\
&R_0^{(\varepsilon)} = \frac{\rho_0 \omega^2}{K_1^2 \sigma} \\
&R_1^{(\varepsilon)} = \frac{1}{\omega}\left(\frac{4\rho_0 \omega^2 K_1 K_2}{\sigma\left(K_1^2 + K_2^2\right)^2} + \frac{2\rho_0 \omega^3 \left(K_1^2 - K_2^2\right)}{\left(K_1^2 + K_2^2\right)^2} \right) \\
&R_2^{(\varepsilon)} = \frac{1}{\omega^2}\left(\frac{\rho_0 \omega^2}{K_1^2 \sigma} - \frac{2\rho_0 \omega^2 \left(K_1^2 - K_2^2\right)}{\sigma\left(K_1^2 + K_2^2\right)^2} + \frac{4\rho_0 \omega^3 K_1 K_2}{\left(K_1^2 + K_2^2\right)^2} \right)
\end{aligned} \tag{13}$$

Considering complex shear modulus $G=G_1+iG_2$ it is possible to obtain the relations between complex wave number and G

$$K=K(G)$$

3. Phenomenological approach.

If a shear wave propagates in the direction of the x_1-axis it can be represented as

$$A \approx e^{i(\omega t - h_1 x)} e^{-\alpha x} \tag{14}$$

If α is attenuation coefficient and V_C complex shear velocity it follows [5]:

$$\Gamma = \frac{\omega}{V_S} - i\alpha \quad \Rightarrow \quad V_C = \left(\frac{\Gamma}{\omega}\right)^{-1} = \left(\frac{1}{V_S} + \frac{\alpha}{i\omega}\right)^{-1} \tag{15}$$

We introduce complex shear impedance Z_S [5] as:

$$Z_S = \rho_0 V_C = \rho_0 \left(\frac{1}{V_S} + \frac{\alpha}{i\omega}\right)^{-1} = R_S + iX_S \tag{16}$$

where R_S and X_S are real and imaginary part of Z_S; consider the following complex relation [6][7][8]

$$V_C^2 = \frac{G}{\rho_0} = \frac{G_1 + iG_2}{\rho_0} \tag{17}$$

combined with (16) it follows:

$$\frac{R_S^2 - X_S^2}{\rho_0} = G_1 \quad ; \qquad \frac{2R_S X_S}{\rho_0} = G_2 \tag{18}$$

and solving such a system with respect to R_S^2, X_S^2

$$R_S^2 = \frac{\rho_0 G_1}{2}\left(\sqrt{1+\left(\frac{G_2}{G_1}\right)^2} + 1\right) \tag{19}$$

$$X_S^2 = \frac{\rho_0 G_1}{2}\left(\sqrt{1+\left(\frac{G_2}{G_1}\right)^2} - 1\right) \tag{20}$$

Moreover from (18), after some manipulation, by equating real and imaginary part, it follows

$$\alpha = \frac{\rho_0 \omega X_S}{R_S^2 + X_S^2} \tag{21}$$

$$V_S = \frac{R_S^2 + X_S^2}{\rho_0 R_S} \tag{22}$$

Substituting (19) and (20) into (21) and (22) we obtain

$$\alpha = \frac{\omega\sqrt{\frac{\rho_0 G_1}{2}\left(\sqrt{1+\left(\frac{G_2}{G_1}\right)^2} - 1\right)}}{G_1\sqrt{1+\left(\frac{G_2}{G_1}\right)^2}} \tag{23}$$

$$V_S = \frac{G_1\sqrt{1+\left(\frac{G_2}{G_1}\right)^2}}{\sqrt{\frac{\rho_0 G_1}{2}\left(\sqrt{1+\left(\frac{G_2}{G_1}\right)^2}+1\right)}} \tag{24}$$

4. Phenomenological coefficients

Now we are able to obtain the relations K=K(G) between complex wave number and G combining equation (3) , (14) and (4)

$$K_1 = \frac{\omega}{V_S} = \frac{\omega\sqrt{\frac{\rho_0 G_1}{2}\left(\sqrt{1+\left(\frac{G_2}{G_1}\right)^2}+1\right)}}{G_1\sqrt{1+\left(\frac{G_2}{G_1}\right)^2}};$$

$$K_2 = \alpha = \frac{\omega\sqrt{\frac{\rho_0 G_1}{2}\left(\sqrt{1+\left(\frac{G_2}{G_1}\right)^2}-1\right)}}{G_1\sqrt{1+\left(\frac{G_2}{G_1}\right)^2}} \tag{25}$$

Taking in account equations (25) from equations (13) it follows:

$$\begin{aligned}
R_0^{(\tau)} &= 1/\sigma \\
R_0^{(\varepsilon)} &= \frac{2}{\sigma}\left(\frac{G_1^2+G_2^2}{\sqrt{G_1^2+G_2^2}+G_1}\right) \\
R_1^{(\varepsilon)} &= 2\left(\frac{G_2}{\omega\sigma}+G_1\right) \\
R_2^{(\varepsilon)} &= \frac{1}{\omega^2}\left\{\frac{2\left(G_2^2-G_1\sqrt{G_1^2+G_2^2}\right)}{\sigma\left(\sqrt{G_1^2+G_2^2}+G_1\right)}+2\omega G_2\right\}
\end{aligned} \tag{26}$$

And finally, from equations (6), it follows:

$$a^{(0,0)} = 2G_1 - \frac{2}{\omega^2\sigma^2}\left\{\frac{G_2^2 - G_1\sqrt{G_1^2+G_2^2}}{\sqrt{G_1^2+G_2^2}+G_1}\right\}$$

$$a^{(1,1)} = \frac{\sigma^2\left[G_1 - \frac{1}{\omega^2\sigma^2}\left\{\frac{G_2^2 - G_1\sqrt{G_1^2+G_2^2}}{\sqrt{G_1^2+G_2^2}+G_1}\right\}\right]^2}{\left(G_1\sqrt{G_1^2+G_2^2} - G_2^2\right)\left(1+\omega^2\sigma^2\right)}$$

$$\eta_s^{(0,0)} = \frac{1}{\omega^2}\left\{\frac{2\left(G_2^2 - G_1\sqrt{G_1^2+G_2^2}\right)}{\sigma\left(\sqrt{G_1^2+G_2^2}+G_1\right)} + 2\omega G_2\right\}$$

$$\eta_s^{(1,1)} = \frac{\left(G_1\sqrt{G_1^2+G_2^2} - G_2^2\right)\left(1+\omega^2\sigma^2\right)}{\sigma^3\left[G_1 - \frac{1}{\omega^2\sigma^2}\left\{\frac{G_2^2 - G_1\sqrt{G_1^2+G_2^2}}{\sqrt{G_1^2+G_2^2}+G_1}\right\}\right]}$$

These are the expressions of phenomenological and state coefficients as function of frequency dependent complex modulus for a isotropic viscoanelastic medium of order one when it is perturbed by a linear transverse acoustic wave. The following figures represents material coefficients G_1 and G_2 for PolyIsobutilene [7], phenomenological and state coefficients as function of frequency and other parameters related to this material.

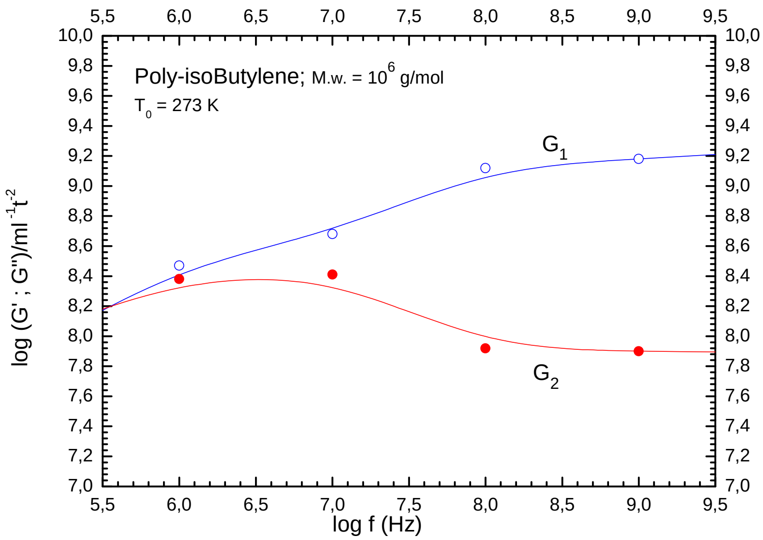

Fig. 1: G_1, G_2 for Poly-isoButylene; M.w. = 10^6 g/mol; T_0 = 273 K; $\sigma \cong 10^{-6}$ s

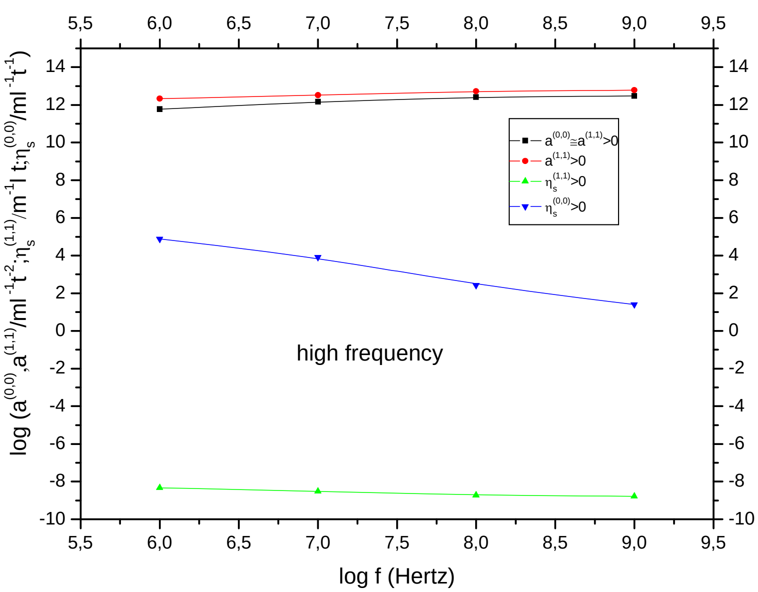

Fig. 2: Poly-isoButylene; M.w. = 10^6 g/mol; T_0 = 273 K

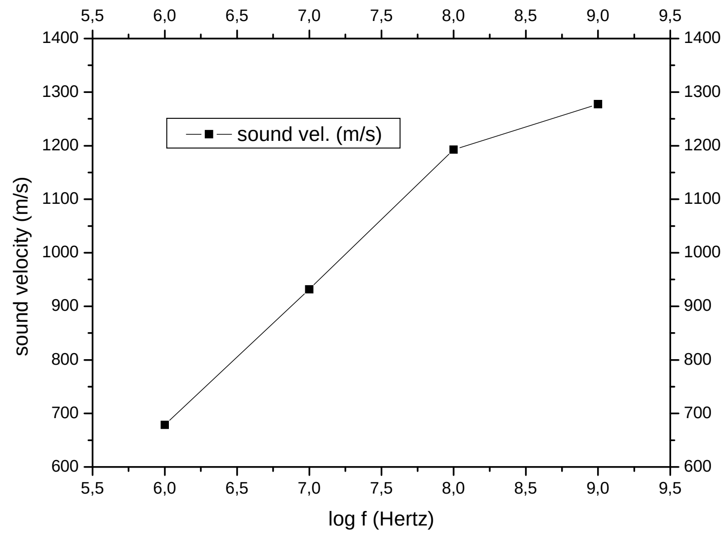

Fig. 3: Poly-isoButylene; M.w. = 10^6 g/mol; T_0 = 273 K

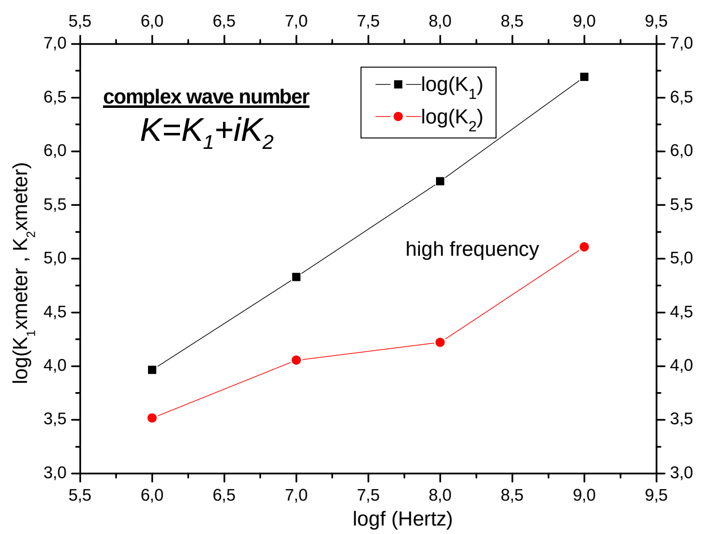

Fig. 4: Poly-isoButylene; M.w. = 10^6 g/mol; T_0 = 273 K

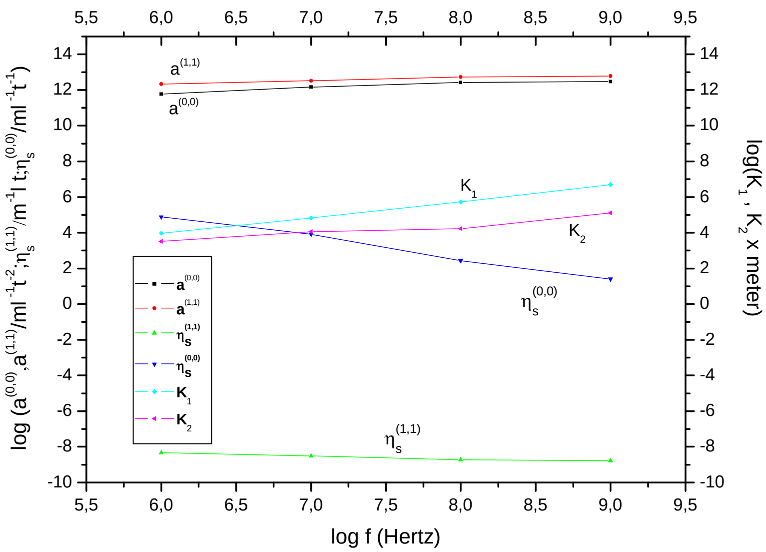

Fig. 5 **:** Poly-isoButylene; M.w. = 10^6 g/mol; T_0 = 273 K

The fig. 5 shows the phenomenological and state coefficients when the medium (PolyIsobutilene) is subject to a shear deformation at a constant temperature. The comparison with fig. 2 shows that the coefficients have the same trend.

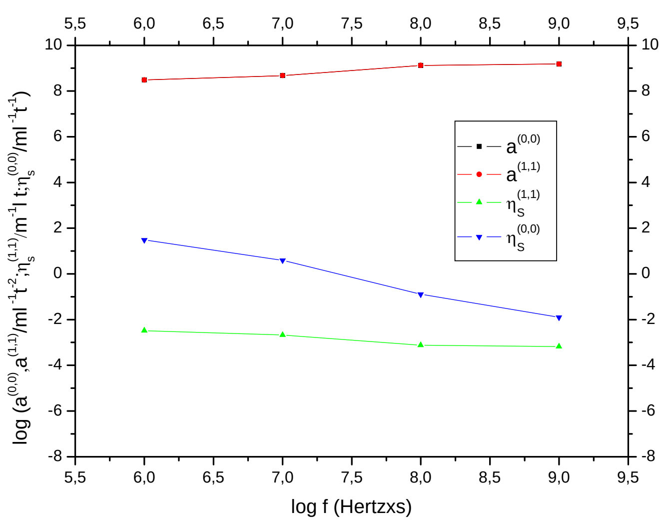

Fig. 5: Poly-isoButylene; M.w. = 10^6 g/mol; T_0 = 273 K

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