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INCREASING BENDING STRENGTH IN SPUR GEARS USING SHAPE OPTIMIZATION OF CUTTING TOOL PROFILE

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The geometrical shape of gear tooth fillet profile, usually cut out by the cutter tip, plays a significant role in the evaluation of the gear bending stresses. In order to improve the teeth bending strength, the research detailed hereby introduced a novel curve (quadratic rational Bezier curve) to describe the cutter tip. The gear tooth finite element model was founded by APDL in the ANSYS software. With the maximum bending stress (von Mises stress) as the objective function, sub-problem and first-order optimization methods in ANSYS were used to optimize the cutter tip. The study reveals that the relationship between the design variable and tooth root bending stress is nonlinear, and the gear cut by the optimized cutter exhibits higher bending strength rather than the gear cut by standard cutting tool.

Keywords: gear；tooth fillet；bending stress；cutter tip；optimization

1. Introduction

The gear tooth fillet profile, usually generated by the cutter tip trajectory, is the place of maximum stress concentration. If the bending stress is too high, larger module has to be used to decrease the bending stress, but this will increase the tooth size. The objective of the gear drive is to transmit high power with higher load-carrying and lower weight. But the gear load capacity may be limited by tooth contact conditions or by the bending strength. With the development of gear heat treatment technology, such as carburizing, high-frequency hardening and nitriding technology, the tooth contact strength has been significantly improved, so the bending fatigue failure may happen with the high-speed heavy gear.

Researchers in the gear field have proposed many solutions to tackle the problem of failure. Lewis suggested the idea of considering the tooth as a cantilever beam and some researchers still used this approach to analyze the bending stress [1]. L.Wilcox and W.Coleman used FEM to analyze the stress of gear[2]. A.L. Kapelevich and Y.V. Shekhtman proposed the idea of optimizing the fillet profile of symmetric and asmmetric teeth based on FEA and a random search method [3]. V. Senthil Kumar and D.V. Muni introduced the use of the asymmetric toothed gear to improve the fillet capacity in bending [4]. Hebbal and Math proposed the idea of using the stress redistribution techniques by introducing internal stress relieving

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features of different shapes to reduce the root filet stress in the spur gear [5]. Niels L. Pedersen used asymmetric gears and shape optimization to reduce bending stress in spur gears. He used super elliptical shape to describe the cutter tip and a custom tool must be designed depending on the number of teeth [6]. Senthilvelan and Gnanamoorthy have analyzed the effect of gear tooth filet radius on the performance of injection molded nylon 6 gears[7]. Shanmugasundaram Sankar and Muthusamy Nataraj discussed a novel method to prevent the tooth failure in the spur gear by introducing circular root filet instead of standard trochoidal root filet in the gear [8]. El-Sayed S. Aziz and Constantin Chassapis reported on the application of the Stress-Strength Interference (SSI) theory to evaluate the tooth-root strength with FEM-based verification [9]. Tesfahunegn and Rosa investigated the influence of the shape of profile modifications on transmission error, root stress and contact pressure through non linear finite element approach [10].

The novelty of the work in this paper is that quadratic rational Bezier curve is used to describe the cutter tip(or hob tip) so that the fillet profile of the tooth root can be optimized through direct optimization of the cutter tip. In order to accelerate the optimization speed, two-dimensional finite element model of the tooth has been established by APDL .Sub-problem and first-order optimization method in ANSYS have been used to increase the tooth strength. In addition, three-dimensional finite element model with meshing gears has been established to verify the optimization results.

2. Description of the cutter tip by rational quadratic bezier curve

2.1 Description of the rational quadratic Bezier curve

As shown in Fig.1, the equation of rational quadratic Bezier curve can be expressed as:

$$\boldsymbol{P}=\boldsymbol{P}(\boldsymbol{u})=\frac{(\mathbf{1}-\boldsymbol{u})^{2}\boldsymbol{w}_{\mathbf{0}}\boldsymbol{P}_{\mathbf{0}}+\mathbf{2}\boldsymbol{u}(\mathbf{1}-\boldsymbol{u})\boldsymbol{w}_{\mathbf{1}}\boldsymbol{P}_{\mathbf{1}}+\boldsymbol{u}^{2}\boldsymbol{w}_{\mathbf{2}}\boldsymbol{P}_{\mathbf{2}}}{(\mathbf{1}-\boldsymbol{u})^{2}\boldsymbol{w}_{\mathbf{0}}+\mathbf{2}\boldsymbol{u}(\mathbf{1}-\boldsymbol{u})\boldsymbol{w}_{\mathbf{1}}+\boldsymbol{u}^{2}\boldsymbol{w}_{\mathbf{2}}} \qquad (\mathbf{1})$$

where P_0,P_1,P_2 are coordinate points in the coordinate system i-j,k; w_1,w_2,w_3 are weight factors and control the shape of the curves which are tangent to the line p_0p_1 and p_1p_2. u is a parameter and $0 \leq u \leq 1$.In particular, the equation (1) can be expressed as the standard form with $w_0=w_2=1$ when $p_0p_1= p_1p_2$, leaving a weight factor w_1 as the design variable [11-14].

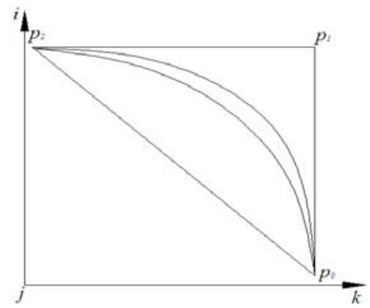


Fig. 1 Rational quadratic Bezier curve

2.2 Establishment of the cutter Tip parametric equations

As shown in Fig.2, The standard rack cutter tip curve is fillet, with radius R=0.38m (m is gear module).

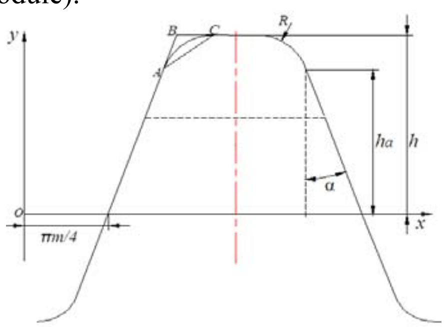


Fig. 2 Rack cutter tip fillet

*h*a is the Effective addendum, generating the involute of gear; h is the cutter tooth height and α is pressure angle. The coordinates of points A, B, C, are as follows: (x_0, y_0), (x_1, y_1), (x_2, y_2). So the equation (1) can be expressed as cartesian coordinate form:

$$\left.\begin{aligned} x(u) &= \frac{(1-u)^2 x_0 + 2u(1-u)w_1\, x_1 + u^2 x_2}{(1-u)^2 + 2u(1-u)w_1 + u^2} \\ y(u) &= \frac{(1-u)^2 y_0 + 2u(1-u)w_1\, y_1 + u^2 y_2}{(1-u)^2 + 2u(1-u)w_1 + u^2} \end{aligned}\right\} \tag{2}$$

Where (x_0, y_0) , (x_1, y_1), (x_2, y_2) can be solved by the cutter parameters, so the equation (2) contains two parameters : u $(0 \le u \le 1)$and w_1. w_1, as the only design variable, controls the shape of the curve. When the equation (2) determined, the parametric equations of the tooth fillet profile can be derived by engagement theory.

3. Gear tooth modeling and optimization

The standard cutter parameters are given in Table 1:

Table 1

Gear parameters

z	*α*	*m*[*mm*]	*ε*	*w*[*mm*]	*x*	*R*[*mm*]
20/20	20°	5	1.5298	10	0	0.38*m*

Standard cutter tip fillet radius is 0.38m, the addendum coefficient ha*=1 and bottom clearance coefficient c* = 0.25, and the remaining parameters are shown in table 1. Where z represents tooth number, α is pressure angle, m is Module, ε is contact ratio, w is tooth width, *x* is modification coefficient, R is hob tip fillet radius. Gear material constants: the elastic modulus E = 205 GPa, Poisson's ratio μ = 0.3. In order to accelerate the optimization speed, two-dimensional finite element model is used.

3.1 Gear tooth modeling.

APDL (ANSYS Parameter Design Language) is a kind of parametric design language, used for parameterized finite element analysis, analysis of the batch, the secondary development of a dedicated analysis system, as well as the optimal design. APDL language can create complex models to avoid the undesirable factors of transmission between different software models. In this paper, the APDL language is used to establish a finite element model of the tooth and optimize the design.

When the cutter tip fillet radius R =0.38*m* (After calculation, the quadratic Bezier curve represents Arc with R =0.38*m* when w_1=0.8192). A series of key points were established to generate standard gear tooth profile by B-Spline curve fitted. In order to accelerate the optimization speed, PLANE 82 is used to establish two-dimensional finite element model. Normal lord P = 254N/mm is applied to the highest point of single tooth contact. The line segments EF, FG, GH are full-constraints and free meshing is adopted .the results are shown in Fig. 3:

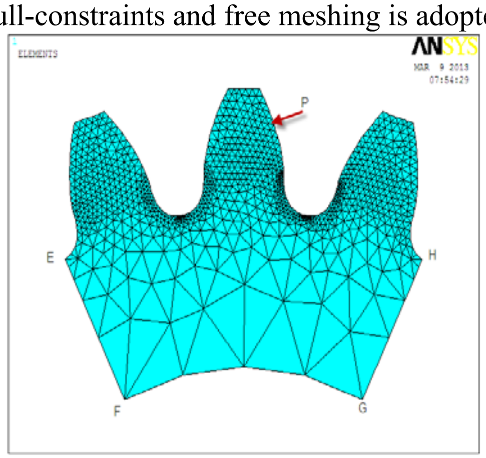


Fig. 3 Gear finite element model

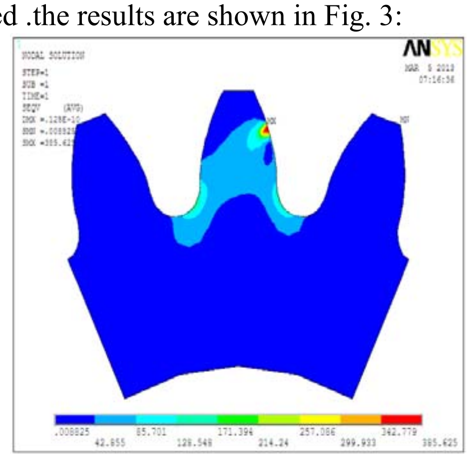


Fig. 4 von Mises stress(R=0.38m)

As shown in Fig. 4, the von Mises stress analysis results show that the maximum bending tensile stress of tooth root is 154.704 MPa when w_1=0.8192(R=0.38m).

3.2 Optimization

Sub-problem and first-order optimization methods in ANSYS have been used to decrease the tensile von Mises stress of the root in the spur gear.

3.2.1 Sub-problem and First-order optimization

In order to reduce the maximum bending tensile stress , the maximum bending tensile von Mises stress in tooth root is used as the objective function ,and the weight factor w_1 （$0<w_1\leq 1$）,whose initial value is w_1=0.5 , is used as the design variable .Firstly, a Sub-problem optimization method in ANSYS is used to optimize the shape of cutter tip to optimize the teeth root fillet. Fig. 5 describes the relationship between iteration times and von Mises stress. Fig. 6 shows the relationship between w_1 and von Mises stress is nonlinear. The total Iterations is 11 times and the minimum von Mises stress 138.95MPa occurred at the 5th iteration when w_1=0.23624.

The Sub-problem optimization can reach a minimum value but the value is not the best. So the first-order optimization method is used to optimize the result again. As shown in Fig.8, the relationship between the weight factor w_1 and tooth root bending stress is nonlinear. And the minimum bending Stress is 136.415MPa when w_1=0.16. So the first-order optimization can get a smaller value, compared to the Sub-problem optimization.

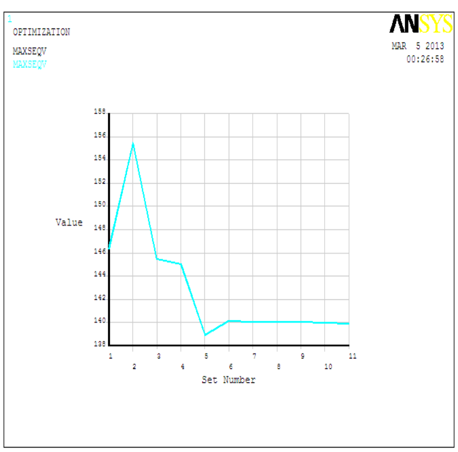


Fig. 5 Ralationship between Iteration times and von Mises stress (Sub-problem)

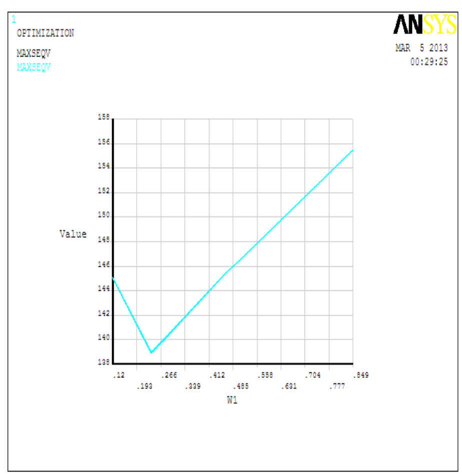


Fig. 6 Ralationship between w_1 and von Mises stress(Sub-problem)

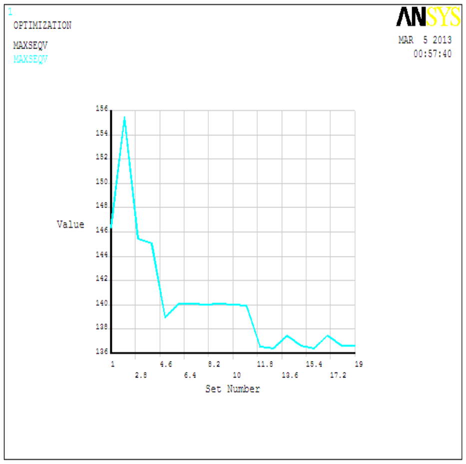


Fig. 7 Ralationship between Iteration times and von Mises stress (first-order)

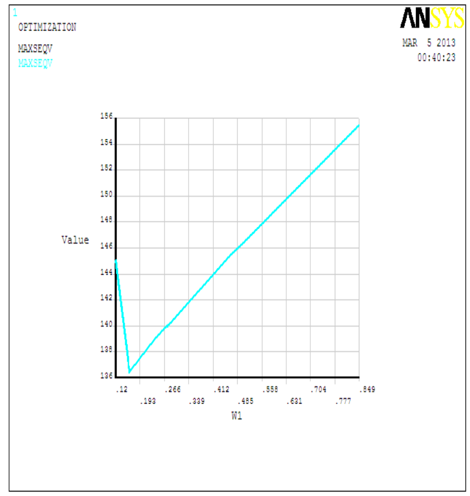


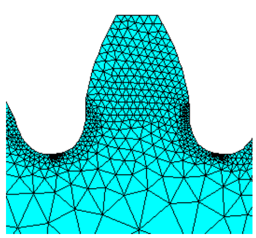
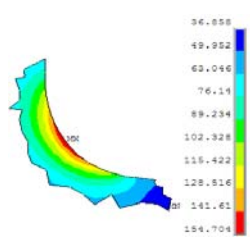
Fig. 8 Ralationship between w1 and von Mises stress(first-order)

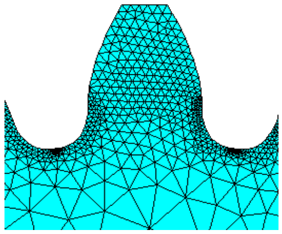
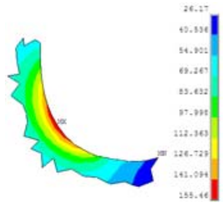
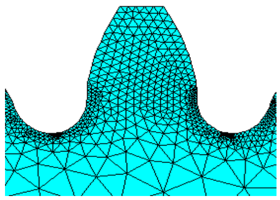
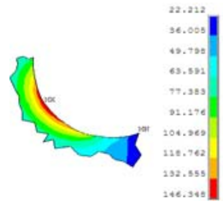
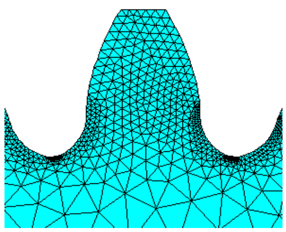
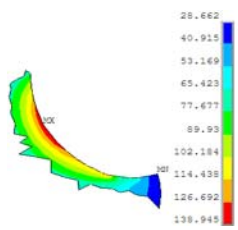
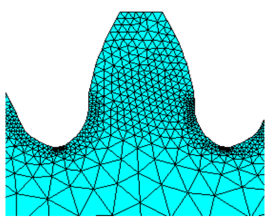
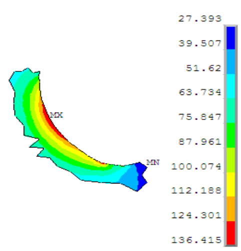
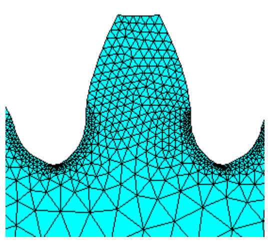
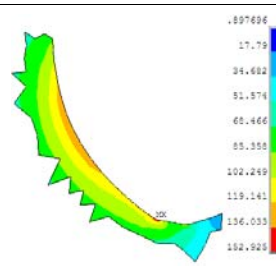
3.2.2 Optimization results and discussion

Table 2 presents the FEA and the bending tensile stress distribution with different parameters w_1. It indicates that the optimized fillet has the minimum stress 136.415 MPa when w_1=0.16, decreased 11.8% compared to the standard gear. It also shows the maximum stress is distributed along the tooth fillet when w_1=0.849、0.8192、0.5、0.2362、0.16. But the maximum stress is located at the intersection of the tooth root fillet and the tooth root circle when w_1=0.1, having significantly greater maximum stress that is sharply concentrated.

Table 2

FEA of different parameter w_1

w_1	Finite element model	Bending tensile stress in tooth root (von Mises stress)	Compared to standard gear
0.8192			standard gear (R=0.38*m*)

0.849			Increase 0.49%
0.5			decrease 5.4%
0.2362			decrease 10.2%
0.16			decrease 11.8%
0.1			decrease 1.15%

4. Finite element analyses of three-dimensional model verification

In order to verify the optimization results, three-dimensional finite element model with meshing gears has been established. Mesh200 is used to establish two-dimensional finite element model. Solid95 is used to drag two-dimensional finite element model to three-dimensional finite element model, shown in Fig.9 and Fig.11. Fig.10 and Fig.12 show the von Mises stress distribution between the meshing gears.

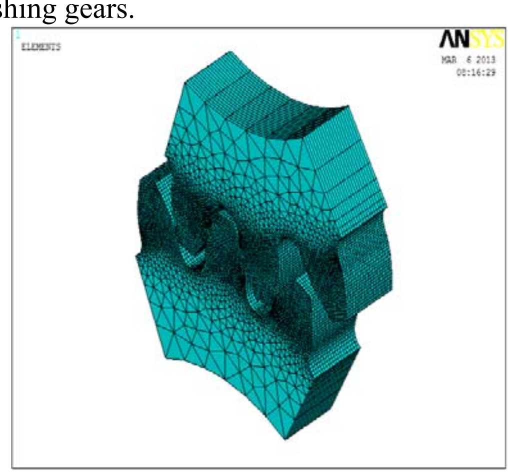


Fig. 9.Three-dimensional finite element model (w1=0.8192)

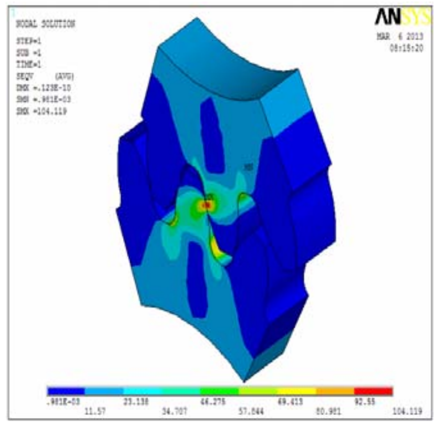


Fig. 10. von Mises stress (w1=0.8192)

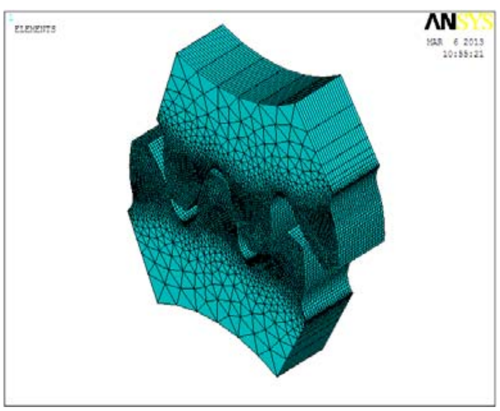


Fig. 11. Three-dimensional finite element model (w_1=0.16)

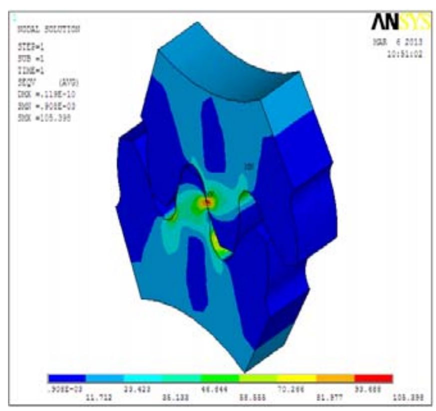


Fig. 12. von Mises stress (w_1=0.16)

As shown in Fig.10 and Fig.12, the contact stress is 104.119 MPa when w_1=0.8192(standard gear) . And the contact stress is 105.398 MPa when w_1=0.16, increased 1.2% compared to the standard gear. But compared to the contact stress increased, the bending stress decreased more, shown in table 3.

Table 3

Bending tensile stress distribution in tooth root of different parameter w_1

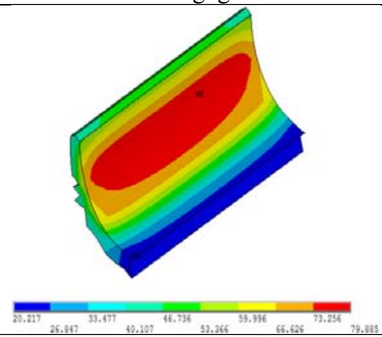
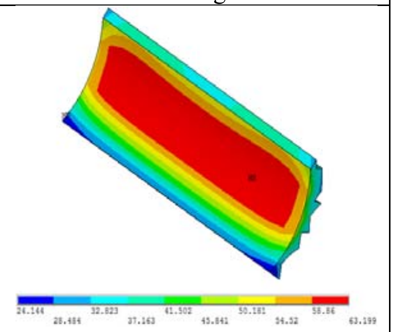
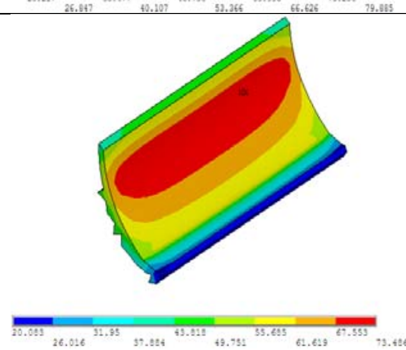
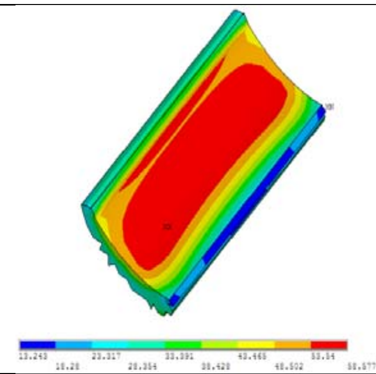
W1	Driving gear	Driven gear
0.8192		
0.16		

Table 3 presents the bending tensile stress distribution at the tooth root of the three-dimensional finite element model. The maximum bending tensile stress in tooth root of the driving gear is 79.885MPa when w_1=0.8192(standard gear) and the driven gear is 63.199MPa when w1=0.8192(standard gear). The driving gear is 73.486MPa when w_1=0.16, decreased 8% compared to the standard driving gear and the driven gear is 58.577MPa when w_1=0.16, decreased 7.3% compared to the standard driven gear. From the above analysis, the bending stress reduction percentage of three-dimensional finite element model is different from the two-dimensional finite element model. For the same three-dimensional finite element model, the percentage of bending stress reduction is different due to different force applied. But the general trend of optimization makes the bending tensile stress decreased, so this method is feasible.

5. Conclusions

Rational quadratic Bezier curve is used to describe the cutter tip by direct optimization of the cutter tip to optimize the tooth root fillet profile. With the use of the only use of only one design variable w_1, it is easy to optimize the solution by APDL. A significant improvement in tooth root bending strength can be got and smoothness of the tooth profile can be satisfied simultaneously. Results calculated also show that the bending stress is reduced by 11.8% when w_1=0.16, compared to the standard gear established by a two-dimensional finite element model. The amount of reduction is different when the finite element model and the force applied are different. But all the results show that optimizing the tooth root fillet profile can lead to the best value and the root fillet profile can be processed by the cutter, also seeking CNC milling or wire cutting.

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