KINEMATICS OF THE CASTER WHEEL OF A PLANAR MOBILE ROBOT

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Articolul analizează cinematica roţii de orientare a unui robot mobil planar constituit dintr-o platformă orizontală şi două roţi convenţionale. Vitezele şi acceleraţiile caracteristice ale sistemului mecanic neolonom sunt stabilite pe baza unor relaţii matriceale de conectivitate. Cunoscând mişcarea celor două roţi motoare, se obţin expresii şi grafice pentru unghiul relativ de rotaţie şi traiectoria centrului roţii de orientare.

Kinematics of the rolling caster wheel of a planar mobile robot, consisting of a horizontal platform and two conventional wheels, are analyzed in the paper. Based on several matrix relations of connectivity, the characteristic velocities and accelerations of the non-holonomic mechanical system are established. Knowing the motion of two driving wheels, expressions and graphs for the relative rotation angle and the trajectory of the caster wheel's centre are obtained.

Keywords: Connectivity relations; Kinematics; Mobile robot. Planar platform

List of symbols

$Ox_0y_0z_0$: inertial reference frame with origin in the ground surface

$a_{k,k-1}$: orthogonal transformation matrix

$\vec{u}_1, \vec{u}_2, \vec{u}_3$: three right-handed orthogonal unit vectors

θ_1, θ_2: rotation angles of two driving wheels

θ_3: rotation angle of the caster wheel;

ψ: rotation angle of the crank PO_3

l: distance between the wheel centres;

$a+b$: height of the triangular platform

r: radius of each driving wheel;

r_0: radius of the caster wheel

$\vec{r}_{21}^{\,A}, \vec{r}_{21}^{\,B}, \vec{r}_{32}^{\,C}$: relative position vectors of wheel centres

$\theta, x_{10}, y_{10}, H$: orientation angle and coordinates of the moving platform's centre

$\vec{\omega}_{k,k-1}$: relative angular velocity of T_k rigid body;

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$\vec{\omega}_{k0}$: absolute angular velocity of T_k;
$\tilde{\omega}_{k,k-1}$: skew-symmetric matrix associated to the angular velocity $\vec{\omega}_{k,k-1}$

1. Introduction

The wheeled mobile robots are pre-programmable multi-functional systems with autonomous control consisting of a mobile platform and some cylindrical wheels, which have a rolling with friction motion on a fixed or mobile surface. Such vehicles are used mainly for tasks of transport in automated industrial processes.

The mobile robot is capable of autonomous motion because it is equipped with motors that are driven by an embarked computer. The propriety of autonomy is understood as the ability to independently make intelligent decisions as the situation changes. These machines are used in inaccessible environments that are often cluttered with lots of unknown, moving and fixed obstacles, in agricultural workings or for special medical procedures. This is why the study of the mobile robots dynamics acquired more and more importance [1], [2], [3], [4], [5]. Recently, neural networks appeared as powerful tools for learning dynamic highly non-linear systems [6].

The non-integrable kinematical constraint conditions of rolling motion without slipping and side slipping between the wheel and the contact surface, demands the presence of the non-holonomic constraints, which represent the kinematics particularity of this kind of robot.

On the other hand, mobile robots are more complex to control than serial and parallel robots, because of non-holonomic constraints. But, at the instantaneous velocities level, mobile robots can be treated mathematically as a special type of parallel robot, having different connections to the ground in parallel.

An equivalent parallel robot, consisting of three legs, can model a differentially driven mobile robot with two moving actuators [7], [8]. Pathak et al. [9] analyse the dynamics modelling and the position control of a series of wheeled inverted pendulum (Segway, Quasimoro, JOE) by partial feedback linearization and from a controllability point of view. Using a recursive formulation, the kinematics model with a global singularity analysis is carefully discussed in [10]. Chakraborty and Ghosal presented in their works [11], [12] the kinematics and a set of differential equations for the kinematical modelling and simulation of a wheeled mobile robot. The Quasimoro prototype of the mobile wheeled-pendulum of Salerno and Angeles [13] is a special quasi-holonomic mechanical system, which comprise two driving wheels and an intermediate central body carrying the payload.

In the present paper we are going to analyze the kinematics of the caster wheel of a planar rolling robot, using a matrix method based on the kinematical conditions of connectivity.

2. Kinematics model of the robot

Let us consider a mobile robot with three conventional wheels that can roll without slipping on a horizontal surface (Fig. 1). This kind of differentially driven robots needs three non-collinear support points in order not to fall over. In practice, the robot can turn on the spot by giving opposite speeds to both actuated wheels.

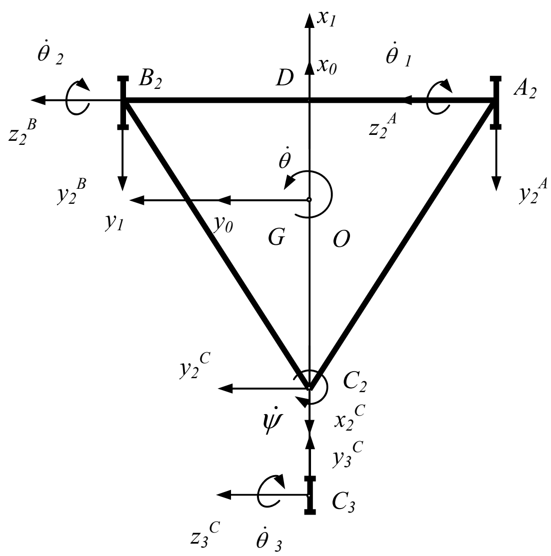

Fig. 2 Kinematical scheme of the mobile robot

The mobile robots are made up of a rigid frame with non-deformable wheels and sometimes they are moving on a fixed horizontal ground.

To simplify the graphical image of the kinematical scheme of the robot, in what follows we will represent the intermediate reference systems by only two axes, as it is used in many robotics papers [4], [8], [20]. The z_k axis is represented for each component element T_k. It is noted that the relative rotation with the angle θ_i must be always about the direction of the z_k axis.

The moving platform of the robot, linked to a central reference frame $Gx_1y_1z_1$, is an isosceles triangle with the dimension l for the base and $a+b$ for its height. Two cylindrical coaxial driving wheels of the same radius r are fixed to the frames $A_2x_2^Ay_2^Az_2^A$, $B_2x_2^By_2^Bz_2^B$ and connected to the chassis by means of revolute joints in the points $A_2 = O_1$ and $B_2 = O_2$. A crank $C_2C_3 = PO_3$ is jointed to the moving platform in the point $C_2 = P$ of the triangle. This rigid element can orientate permanently the motion of a passive rolling caster wheel of small radius r_0 . The caster wheel has no kinematical function; its only purpose is to keep the robot in balance.

In the forward velocity kinematics (FVK), we will consider that the input rotation angles θ_1, θ_2 of the driven wheels can determine completely the instantaneous position and orientation of the robot. Thus, since the platform has a planar motion, its position with respect to a fixed reference frame $Ox_0y_0z_0$ with origin O on the horizontal ground, is given by the coordinates x_{10}, y_{10}, H and by the angle of rotation θ, which form the following matrices:

$$\vec{r}_{10} = \begin{bmatrix} x_{10} \\ y_{10} \\ H \end{bmatrix} \quad , \quad a_{10} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{1}$$

Let us denote with θ_i $(i = 1, 2, 3)$ the rotation angles of the three wheels and with $\theta_4 = \psi$ the relative angle of rotation of the crank about the apex C_2 of the platform.

The non-holonomic constraints reduce the mobile robot's velocity degrees of freedom and hence the robot has only two actuated joints. In the study of the kinematics of mobile robots, we are interested in deriving a matrix equation relating the location of an arbitrary T_k body to the joint variables. When the change of coordinates is successively considered, the corresponding matrices are multiplied. We obtain the following orthogonal transformation matrices in the reference frames [14], [15]:

$$a_{21}^A = a_z^{\theta_1} a_1 \ , \ a_{21}^B = a_z^{\theta_2} a_1 \ , \ a_{21}^C = a_z^{\theta_4} a_2 \ , \ a_{32}^C = a_z^{\theta_3} a_1 \ , \tag{2}$$

Where

$$a_1 = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, a_2 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, a_z^{\theta_i} = \begin{bmatrix} \cos\theta_i & \sin\theta_i & 0 \\ -\sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \ (i = 1, 2, 3, 4) \cdot \tag{3}$$

If the distance $A_2B_2 = l$ between both actuated wheels is known, as well as the characteristic dimensions d, h of the crank PO_3, the following vectors give the relative invariable positions of the revolute joints A_2, B_2, C_2, C_3 :

$$\vec{r}_{21}^{\,A} = \begin{bmatrix} a \\ -l/2 \\ -h_0 \end{bmatrix}, \vec{r}_{21}^{\,B} = \begin{bmatrix} a \\ l/2 \\ -h_0 \end{bmatrix}, \vec{r}_{21}^{\,C} = \begin{bmatrix} -b \\ 0 \\ 0 \end{bmatrix}, \vec{r}_{32}^{\,C} = \begin{bmatrix} d \\ 0 \\ h \end{bmatrix}. \tag{4}$$

The absolute position of the caster wheel's centre C_3 in the fixed frame $Ox_0y_0z_0$ is given by the vector [16], [17], [18].

$$\vec{r}_{30}^{\,C} = \vec{r}_{10} + a_{10}^{T}\vec{r}_{21}^{\,C} + a_{10}^{T}a_{21}^{CT}\vec{r}_{32}^{\,C}. \tag{5}$$

Thus, the kinematics of the robot's elements is completely characterized by five relative angular velocities

$$\vec{\omega}_{10} = \dot{\theta}\vec{u}_3\,,\;\; \vec{\omega}_{21}^{A} = \dot{\theta}_1\vec{u}_3, \vec{\omega}_{21}^{B} = \dot{\theta}_2\vec{u}_3\,,\;\; \vec{\omega}_{21}^{C} = \dot{\theta}_4\vec{u}_3, \vec{\omega}_{32}^{C} = \dot{\theta}_3\vec{u}_3\,, \tag{6}$$

which are *associated* to the following skew-symmetric matrices

$$\tilde{\omega}_{10} = \dot{\theta}\tilde{u}_3\,, \tilde{\omega}_{21}^{A} = \dot{\theta}_1\tilde{u}_3,\;\; \tilde{\omega}_{21}^{B} = \dot{\theta}_2\tilde{u}_3\,, \tilde{\omega}_{21}^{C} = \dot{\theta}_4\tilde{u}_3,\;\; \tilde{\omega}_{32}^{C} = \dot{\theta}_3\tilde{u}_3$$

$$\vec{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \; \tilde{u}_3 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{7}$$

Since the analyzed system of three rolling wheels are characterized by non-holonomic constraints, *the matrix conditions of connectivity* (8) will establish five analytical relations between the characteristic velocities of a two-degrees-of-freedom mobile robot

$$\begin{aligned} &\vec{v}_{10} + \tilde{\omega}_{10}\vec{r}_{21}^{\,A} = [\, r\dot{\theta}_1 \quad 0 \quad 0\,]^{T},\;\; \vec{v}_{10} + \tilde{\omega}_{10}\vec{r}_{21}^{\,B} = [\, r\dot{\theta}_2 \quad 0 \quad 0\,]^{T} \\ &a_{21}^{C}(\vec{v}_{10} + \tilde{\omega}_{10}\vec{r}_{21}^{\,C}) + (a_{21}^{C}\tilde{\omega}_{10}a_{21}^{C\,T} + \tilde{\omega}_{21}^{C})\vec{r}_{32}^{\,C} = [-r_0\dot{\theta}_3 \;\; 0 \;\; 0]^{T}. \end{aligned} \tag{8}$$

These constraint conditions are satisfied if all wheels do not slip transversally and longitudinally, so that the distance over which the outer wheel surface rotates equals the distance traveled by the point on the rigid body to which the wheel axle is attached.

Indeed, we assume in *FVK* problem that the position and orientation of the mechanism at a given instant is completely determined by the input rotation angles of the two actuated wheels, namely:

$$\theta_1 = \theta_1^{*}[1-\cos(\frac{\pi}{3}t)],\;\; \theta_2 = \theta_2^{*}[1-\cos(\frac{\pi}{3}t)]. \tag{9}$$

Therefore, the relations (8) can provide first the Jacobian matrix and second the expressions of the characteristic velocities of the moving platform:

$$\omega_{10} = \dot{\theta} = \frac{r}{l}(\dot{\theta}_1 - \dot{\theta}_2),\ v_{10}^x = r\dot{\theta}_1 - \frac{1}{2}l\dot{\theta},\ v_{10}^y = -a\dot{\theta}$$
$$[\dot{x}_{10} \quad \dot{y}_{10} \quad 0]^T = a_{10}^T [v_{10}^x \quad v_{10}^y \quad 0]^T . \tag{10}$$

Concerning the kinematics of the crank PO_3 and the passive caster wheel jointed in the point $O_3 = C_3$, from the matrix conditions (7), a significant differential equation and a relation containing the angular velocities $\omega_{21}^C = \dot{\theta}_4 = \dot{\psi}, \omega_{32}^C = \dot{\theta}_3$ derive as follows

$$d\dot{\psi} + r\dot{\theta}_1 \sin\psi - [d + 0.5l \sin\psi + (a+b)\cos\psi]\dot{\theta} = 0$$
$$r_0\dot{\theta}_3 = [(a+b)\sin\psi - 0.5l\cos\psi]\dot{\theta} + r\dot{\theta}_1 \cos\psi . \tag{11}$$

In the forward position kinematics, the estimation of the relative angle of rotation ψ and the absolute pose of the moving platform must be performed by integration of above velocity equations (10), (`11).

Note that the absolute velocities $\vec{v}_{k0}^C$, $\vec{\omega}_{k0}^C$ of the joints C_2, C_3 of third leg of the robot, for example, can be calculated with some recursive matrix formulae [19], [21], [22]:

$$\vec{v}_{20}^C = a_{21}^C\{\vec{v}_{10} + \tilde{\omega}_{10}\vec{r}_{21}^C\}, \vec{v}_{30}^C = a_{32}^C\{\vec{v}_{20}^C + \tilde{\omega}_{20}^C\vec{r}_{32}^C\}, \tilde{\omega}_{20}^C = a_{21}^C\tilde{\omega}_{10}^C a_{21}^{CT} + \tilde{\omega}_{21}^C. \tag{12}$$

3. Kinematics simulation

As application, the motion of a robot, which has the following characteristics:

$$x_2^C = \frac{1}{2}d^2/(h+d),\ l = 0.5\,\text{m},\ a = l\sqrt{3}/6,\ b = 2a$$
$$d = 0.025\,\text{m},\ r = 0.1\,\text{m},\ r_0 = 0.025\,\text{m} \tag{13}$$
$$h_0 = 0.2\,\text{m},\ H = h_0 + r,\ h = H - r_0 .$$

is analyzed.

Two important manoeuvres can be implemented.

1°. *Rotation motion* about the vertical axis passing through the midpoint D of the actuated wheel axle A_2B_2, with

$$\theta_1^* = -\theta_2^* = \pi, \theta_{\max} = \pi, v_{10}^x = 0, v_{10}^y = -2a\frac{r}{l}\dot{\theta}_1, \dot{\theta} = 2\frac{r}{l}\dot{\theta}_1 . \tag{14}$$

Starting from the initial position, the crank and the caster wheel have first a transition motion. Angular velocities $\dot{\psi}, \dot{\theta}_3$ follow two functions:

$$\dot{\psi} = (1 + \frac{a+b}{d}\cos\psi)\dot{\theta},\ \ \dot{\theta}_3 = \frac{a+b}{r_0}\dot{\theta}\sin\psi \ . \tag{15}$$

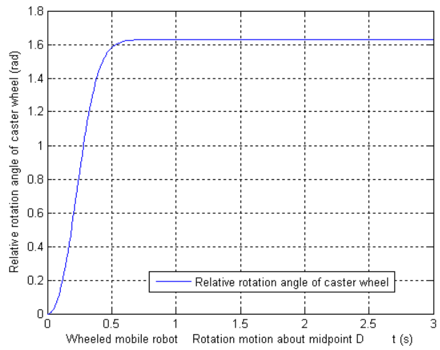

Fig. 2 Relative rotation angle of caster wheel

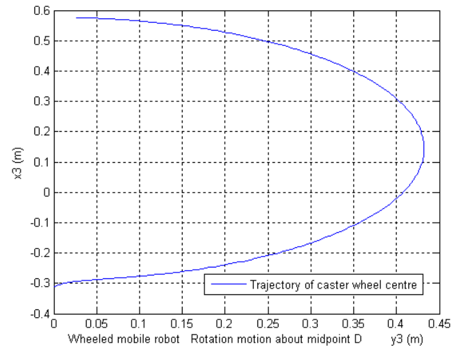

Fig. 3 Trajectory of caster wheel centre

We remark a quick increase of the relative angle ψ (Fig. 2) and a vanishing of the velocity $\dot{\psi}$ to a characteristic angle ψ_0, as follows

$$\cos\psi_0 = -\frac{d}{a+b} \ . \tag{16}$$

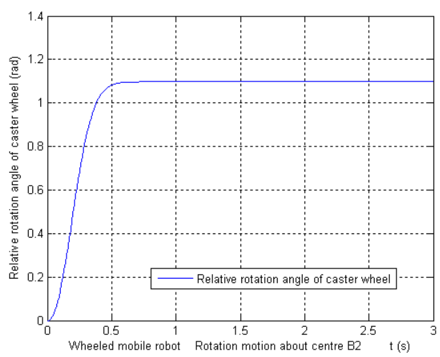

Fig. 4 Relative rotation angle of caster wheel

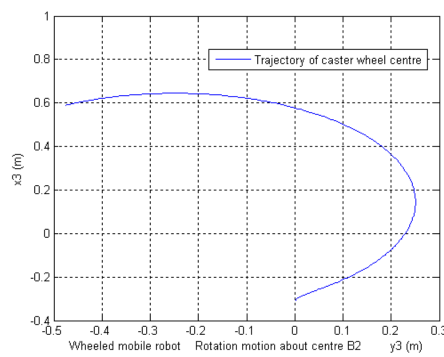

Fig. 5 Trajectory of caster wheel centre

In a stabilized rotation, the centre describes a circle of radius $R = (a+b)\sin\psi_0$ with an angular velocity

$$\dot{\theta}_3 = \frac{R}{r_0}\dot{\theta} \ . \tag{17}$$

Some irregularities in the graph of the plotted trajectory of the caster wheel's joint C_3 during the first 1.2 s correspond just to the period of transition to the stabilized rotation motion (Fig. 3).

2º. *Rotation motion* around the vertical axis passing through the centre B_2 of second driving wheel, with

$$\theta_1^* = 2\pi, \theta_2^* = 0, \theta_{\max} = \pi$$
$$v_{10}^x = \frac{r}{2}\dot{\theta}_1, v_{10}^y = -a\frac{r}{l}\dot{\theta}_1, \dot{\theta} = \frac{r}{l}\dot{\theta}_1. \quad (18)$$

We note a new radius $R = (a+b)\sin\psi_0 + 0.5l\cos\psi_0$ of the stabilized circle described by the centre C_3 and a smaller characteristic angle ψ_0 given by the trigonometric equation

$$l\sin\psi_0 - 2(a+b)\cos\psi_0 - 2d = 0. \quad (19)$$

The time-history of the relative rotation angle ψ (Fig. 4) and the trajectory described by the centre C_3 (Fig. 5) can be calculated through a computer program, using the MATLAB software.

4. Conclusions

1º. Kinematics for the motion of the caster wheel of a planar mobile robot is analyzed in the paper.

2º. Within the forward kinematics analysis some exact relations that can give in real-time the velocity of each element of the wheeled mobile robot have been established in the present paper.

3º. Numerical simulations using the MATLAB software program certify that one of the major advantages of the current matrix recursive formulation is a reduced number of additions or multiplications and consequently a smaller processing time of computation.

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