RESEARCH ON TENSION OPTIMIZATION OF UNDER-CONSTRAINED COLLABORATIVE TOWING SYSTEM WITH MULTI-HELICOPTERS IN INVERSE DYNAMICS

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In the under-constrained system that multiple unmanned helicopters cooperatively tow a load by cables, the tension of the cables cannot be obtained directly since the number of the equations is always smaller than the number of unknown quantities in inverse dynamics problem. In this paper, on the basis of established dynamic equation of the system, the problem of calculating the tension of cables in real time is converted into a nonlinear programming problem. Therefore, minimum solution and maximum solution of cable tensions that meet the requirements can be obtained. The optimal solution is between minimum solution and maximum solution, which can be expressed by linear interpolation. In the process of optimization, the optimal objective function is expressed by the p-norm approximately considering that the optimal solution may not be continuous in some cases. Finally, the feasibility of cable tension optimization method is illustrated simulatively by three unmanned helicopters cooperatively tow a load by cables.

Keywords: multi-helicopters, cable-traction, tension optimization, nonlinear programming

1. Introduction

Multiple helicopters are used to coordinate the traction of the same target object at different positions to realize the position and posture of the object in the air. It can be applied to rapid erection of bridges in wartime, the hoisting of large equipments in the case that cranes can not be used, as well as segmental aerial determination and connection of civil bridges. Obviously, it is a typical tightly coupled air multi-robot system, which can be classified as a multi-robot cable-traction system.

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Multi-robot cable-traction system is a system in which multiple robots cooperatively tow a load by cables, mainly including multiple robots, cables, loads (or actuators).With robots as driving parts and cable as the medium, the motion and force of the driving parts can be transmitted through the change of the position of robots and the change of cable length, so as to realize the motion of the load. The main difference between this kind of system and the traditional cable-traction parallel robots system is that: cable-traction parallel robots system mainly includes frame, cables, actuator, pulleys, compound joints and driver (such as a motor), etc., and it is still a robot in essence. In multi-robot cable-traction system, one end of the cable is connected to a movable robot, which is equivalent to that there is one movable robot at each branch chain of the cable-traction parallel robots system. It can be regarded as a large parallel lifting system composed of parallel robots systems.

The multi-robot cable-traction system can control the position and posture of large volume or overweight loads, which has a great prospect in practical application. Meanwhile, the manufacturing and maintenance costs of large traction equipment can be avoided since its structure can be recombined. As we know, in order to realize the overall positioning constraint on the lifted object, 7 single lifting mechanisms are required to form a parallel multi-robot collaborative towing system[1]. Although the research on the overall constraint system is relatively mature, it will make the suspension system of heavy load and large space too complex, thus adding too much difficulty to the implementation of its design. Therefore, in order to reduce the complexity of the design and manufacture of the system, it is a feasible solution to form a under-constrained parallel suspension system with fewer individual tandem lifting mechanisms.

Many scholars focus on the research of parallel robots with over-constraint cable traction and have achieved very effective results. Although the multi-robot cable-traction system is different from the cable-traction parallel robots system, there’re many similarities, which can be drawn on in terms of research method. Since the emergence of cable-traction parallel robots system in the 1980s, many researchers have been interested in it[2-5]. In the actual application of cable traction system, in order to realize the control of the load, the cables must always be in a state of tension, therefore, in the process of actual motion control, the real-time cable tension needs to be calculated. Verhoeven[6] converted the optimization of cable tension distribution into a nonlinear optimization in convex polyhedra for cable traction parallel robots system. On the basis of Verhoeven, Zheng Yaqing[7] considered the discontinuity of optimal solution in some cases, and came up with p-norm approximation for optimal solution,. Wang Xuanyao et al.[8] discussed the optimization of tension distribution from the perspective of energy optimization. Jonathan Fink et al.[9] discussed how to minimize the angle of potential energy and optimize the trajectory when three unmanned helicopters are lifting loads.

In the reference[10], the solutions of kinematics and dynamics have been classified and discussed in all cases. Among them, in inverse dynamics of the under-constrained system, if the additional constraints are not considered, no matter what relation is satisfied between m and n, there are infinite solutions in most cases, and the tension of the cable cannot be directly calculated. Therefore, the optimization needs to be taken into consideration. Next, the cable tension will be optimized from the perspective of cable tension, and it will be applicable no matter which kind of mechanism the system belongs to. It will provide foundation for the subsequent programming and control research.

2. Dynamic Modeling of Helicopter Towing System

The schematic diagram of m helicopters pulling *n* degree of freedom load through cables is shown in Fig.1, where each helicopter is connected to the lifted load by a cable. Since the helicopter itself is not studied, the helicopter can be regarded as a particle for research. The connection point P_m between the cable and the helicopter is the location of the helicopter, and the traction of the load can be realized through the change of the position of the helicopters and the change of cable length. Establishing the global coordinate system O-XYZ and the local coordinate O'-X'Y'Z' at the load centroid, the positioning of the load centroid O' in the global coordinate system O-XYZ is

$$\boldsymbol{r} = [x \ \ y \ \ z]^{\mathrm{T}} \tag{1}$$

Suppose the mass of the load be $\boldsymbol{M}$, then the gravity in the global coordinate system can be expressed with the unit spinor of zero pitch

$$\boldsymbol{G} = \text{-}\boldsymbol{Mg}[\boldsymbol{i} \ \ \boldsymbol{r}\times\boldsymbol{i}]^{\mathrm{T}}, \quad \boldsymbol{i} = [0 \ \ 0 \ \ 1]^{\mathrm{T}} \tag{2}$$

The velocity and angular velocity of the load in the global coordinate system are υ and ω respectively, then

$$\boldsymbol{v} = \dot{\boldsymbol{r}} = [\dot{x} \ \ \dot{y} \ \ \dot{z}]^{\mathrm{T}} \tag{3}$$

$$\boldsymbol{\omega} = \begin{bmatrix} \dot{\gamma} \\ \dot{\beta} \\ \dot{\alpha} \end{bmatrix} \tag{4}$$

The inertia matrix of the load in the local coordinate system is $\boldsymbol{I}$, then the inertia matrix $\boldsymbol{I}'$ in the global coordinate system is

$$\boldsymbol{I}' = \boldsymbol{R}\boldsymbol{I}\boldsymbol{R}^{\mathrm{T}} \tag{5}$$

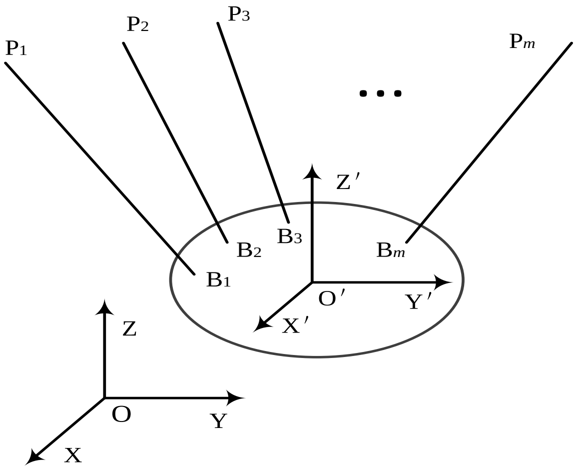


Fig. 1 Schematic diagram of the system

Where $\boldsymbol{R}$ represents the rotation transformation matrix of the local coordinate system in relative to the global coordinate system

$$\boldsymbol{R}=\begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix} \tag{6}$$

In the above equation, γ、β、α represents the rotation angle of the X-axis, Y-axis and Z-axis of the local coordinate system in relative to the global coordinate system; *c* represents cosine and *s* represents sine. Suppose the tension of each cable be T_1, $T_2 ..., T_m$, and for the load, with Newton euler equations it can be obtained:

$$\begin{bmatrix} \boldsymbol{MI}_3 & 0 \\ 0 & \boldsymbol{I}' \end{bmatrix}\begin{bmatrix} \dot{\boldsymbol{v}} \\ \dot{\boldsymbol{\omega}} \end{bmatrix}+\begin{bmatrix} 0 \\ \boldsymbol{\omega}\times\boldsymbol{I}'\boldsymbol{\omega} \end{bmatrix}=\boldsymbol{G}+[e_1\ e_2 ...\ e_m]\boldsymbol{T}_{m\times1} \tag{7}$$

In the above equation, $\boldsymbol{M}$ is the load mass, $\boldsymbol{I}_3$ is the 3×3 identity matrix, and $\boldsymbol{T}$=[$T_1\ \ T_2 \dots T_m$] is the m-dimensional matrix formed by the tension of *m* cables.

By deforming equation (7), the dynamic equation of the load can be

$$\boldsymbol{AT}=\boldsymbol{C} \tag{8}$$

In the above equation, $\boldsymbol{C}$ is an n-dimensional column matrix, and representing the sum of all external spinors acting on the load (including gravity of the load, inertia force, etc.). The matrix $\boldsymbol{A}$ meets the condition:

$$\boldsymbol{A}=[\boldsymbol{e}_1\ \boldsymbol{e}_2 ...\ \boldsymbol{e}_m]\in\boldsymbol{R}^{n\times m} \tag{9}$$

The matrix $\boldsymbol{A}$ consists of m spinors, and each of $\boldsymbol{e}_i$ is

$$\boldsymbol{e}_i = \frac{1}{\|\boldsymbol{l}_i\|_2}\begin{bmatrix} \boldsymbol{l}_i \\ (\boldsymbol{R}\boldsymbol{B}_i)\times\boldsymbol{l}_i \end{bmatrix} \tag{10}$$

$$l_i = \|\boldsymbol{P}_i - \boldsymbol{B}_i\| \tag{11}$$

In the above equation, $\boldsymbol{P}_i$ represents the position vector of helicopter position P_i in the global coordinate system, $\boldsymbol{B}_i$ represents the position vector of the connection point B_i between the cable and the load in the global coordinate system, $\boldsymbol{r} = [x\ \ y\ \ z]^{\mathrm{T}}$ represents the position vector of centroid of the load O' in the global coordinate system, $i = 1,2,\ldots m$.

3. Research on Tension Optimization in Inverse Dynamics

The towing system is working, the minimum pretension and maximum allowable tension of the cable in the tensioning state are set as $T_{\min}$ and $T_{\max}$ respectively, then

$$T_i \in (T_{\min},\ T_{\max}) \quad i = 1,2,\ldots m \tag{12}$$

In inverse dynamics, the position of the helicopter P_i (i=1, 2, …m) is the undetermined quantity, but considering the range of motion of the helicopter and preventing the interference of multiple cables in the process, the range of motion of each helicopter can be defined in advance, so the constraint of the range of motion of each helicopter can be expressed as:

$$\boldsymbol{r}_{\min}^{i} \le \boldsymbol{P}_i \le \boldsymbol{r}_{\max}^{i} \tag{13}$$

In which $\boldsymbol{r}_{\min}^{i} = \left(r_{x\min}^{i}, r_{y\min}^{i}, r_{z\min}^{i}\right)^{\mathrm{T}}$, $\boldsymbol{r}_{\max}^{i} = \left(r_{x\max}^{i}, r_{y\max}^{i}, r_{z\max}^{i}\right)^{\mathrm{T}}$.

Under the above constraints, in obtaining the desired trajectory of the load, the acceptable cable tension $\boldsymbol{T}$ is not unique, and the solution is infinite. Therefore, it is necessary to find acceptable minimum and maximum solutions, which are discussed below.

Suppose $\boldsymbol{F}_{\min} = (T_{1\min},\ldots T_{m\min})^{\mathrm{T}}$, $\boldsymbol{F}_{\max} = (T_{1\max},\ldots T_{m\min})^{\mathrm{T}}$. In the reference[7], it proposes that when the moving platform (the load) is in some specific position and posture, if the lowest solution and the highest solution are not unique, the optimal solution will not be unique. Therefore, p-norm substitution is always consecutive except for singularities. The minimum and maximum acceptable solutions of the cable tension $\boldsymbol{T}$ can be converted into nonlinear programming problems[11].

According to system dynamics equation and cable length equation, the system should also satisfy the constraint equations (8) and (11).

For the lowest solution, it can be expressed as:

$$\min \quad f = \|\boldsymbol{T}\|_p$$
$$\text{sub.to} \quad \|\boldsymbol{P}_i - \boldsymbol{B}_i\| = l_i$$
$$\boldsymbol{AT} = \boldsymbol{C}$$
$$\boldsymbol{F}_{\min} \le \boldsymbol{T} \le \boldsymbol{F}_{\max}, \quad \boldsymbol{r}^i_{\min} \le \boldsymbol{P}_i \le \boldsymbol{r}^i_{\max}, \quad i=1,2, \ldots m$$

In which, the $\boldsymbol{T}$ value that minimizes f is the lowest solution, $\boldsymbol{T}_{low}$.

For the highest solution, it can be expressed as:

$$\max \quad f = \|\boldsymbol{T}\|_p$$
$$\text{sub.to} \quad \|\boldsymbol{P}_i - \boldsymbol{B}_i\| = l_i$$
$$\boldsymbol{AT} = \boldsymbol{C}$$
$$\boldsymbol{F}_{\min} \le \boldsymbol{T} \le \boldsymbol{F}_{\max}, \quad \boldsymbol{r}^i_{\min} \le \boldsymbol{P}_i \le \boldsymbol{r}^i_{\max}, \quad i=1,2, \ldots m$$

In which, the value of $\boldsymbol{T}$ that maximizes f is the highest solution, $\boldsymbol{T}_{high}$.

It should be noted that the constraints in the lowest solution and the highest solution are not linear equation constraints, but nonlinear equation function constraints. If the cable length $\boldsymbol{l}_i$ is not fixed, it is necessary to add function constraints or range constraints of the cable length, such as $\boldsymbol{l}^i_{\min} \le \boldsymbol{l}_i \le \boldsymbol{l}^i_{\max}$. Proper p can be found through adjustment, so as to obtain a desirable continuity of the acceptable minimum and maximum solutions for the cable tension $\boldsymbol{T}$, or a desirable continuity for the trajectory of the helicopter $\boldsymbol{P}_i$.

After obtaining the minimum solution and the maximum solution, the condition of the ultimate tension of each cable at every moment can be known when the desired trajectory of the load is realized, so that the layout can be reasonable. The optimal solution $\boldsymbol{T}(\lambda)$ can be represented by linear interpolation, as shown below. The optimal solution can be selected by adjusting λ.

$$\boldsymbol{T}(\lambda) = \lambda \boldsymbol{T}_{high} + (1-\lambda)\boldsymbol{T}_{low} \quad 0<\lambda<1 \tag{14}$$

4. Analysis of Special Cases

In order to ensure that the cable tension is greater than 0, many scholars have adopted redundant mechanism (m>n). However, for the system discussed in this paper, when m>n, the previous research method is applicable, only the trajectory of the helicopter needs to be considered, which is not detailed in this paper. When m=n, there is a unique solution, and there is no problem of optimization. We can directly determine whether the cable tension $\boldsymbol{T}$ meets equation (12) through the solution obtained. When m<n, there is no guarantee that the cable tension is always greater than 0. But in practical application, for a specific trajectory of the load, in order to ensure normal operation, the cable must always be in a state of tension, that is, the cable tension is always greater than 0. Therefore, theoretical verification is needed before practical application.

In the reference[12], it is pointed out that the singularity type of the parallel mechanism of cable traction only has the singularity of over-motion, under the premise that the position and posture of the load doesn't have force spinor failure, the structural matrix $\boldsymbol{A}$ in equation (8) should be full rank matrix, i.e. rank($\boldsymbol{A}$)=min{m, n}, without rank reduction. Since the case $m<n$ is studied, the structural matrix $\boldsymbol{A}$ is a $n\times m$ matrix, so the structural matrix $\boldsymbol{A}$ should be a full rank matrix, i.e. rank($\boldsymbol{A}$)=m, which is also equal to the number of the unknown quantity of cable tension, but rank($\boldsymbol{A}$) is not necessarily equal to rank($\boldsymbol{A}$， $\boldsymbol{C}$). In special cases, the cable length can be directly figured out. Therefore, equation (8) is a system of linear equations.

According to the knowledge of linear algebra and matrix, linear equations (8) have two cases: compatible or incompatible. If the system (8) is incompatible, namely rank($\boldsymbol{A}$) ≠rank($\boldsymbol{A}$， $\boldsymbol{C}$), then the equations have no solution. In general, the least squares solution of such equations is not unique, but the least squares solution of linear equations with full rank of coefficient matrix is also the only minimal norm solution, that is, the unique approximate solution can be obtained. If the equation (8) is compatible, namely rank($\boldsymbol{A}$) =rank($\boldsymbol{A}$， $\boldsymbol{C}$), then there is unique solution, which is consistent with the least squares solution. Therefore, the unique least squares solution of the equation (8) can be obtained regardless of its compatibility or incompatibility in general. At the same time, it is shown that this kind of under-constrained system can either have unique solution or no solution, so the tension of the cable is uncertain.

Therefore, when solving for this kind of specific trajectory, the least squares method can be used to find the unique $\boldsymbol{T}$ of the cable tension as the optimal solution, and it can be approximately determined whether $\boldsymbol{T}$ meets equation (12). If the normal equation is used, the unique least squares solution $\boldsymbol{T}$ can be expressed as equation (15), and the equation are not required to be solved, with high computational efficiency.

$$\boldsymbol{T} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{C}) \qquad (15)$$

If expressed by nonlinear programming, it can be expressed as equation (16).

$$\begin{aligned} &\min \quad \|\boldsymbol{A}\boldsymbol{T}-\boldsymbol{C}\|_2^2 \\ &\text{sub.to } \|\boldsymbol{P}_i - \boldsymbol{B}_i\| = l_i \end{aligned} \qquad (16)$$

If the obtained solution of the cable tension $\boldsymbol{T}$ does not satisfy equation (11), it means that this specific trajectory cannot be realized, and the parameters need to be re-adjusted and solved again.

5. Analysis of Simulation

5.1. Simulation of Inverse Problem

Take three small unmanned helicopters that lift a weight of three degrees of freedom by cables as example. The position of the unmanned helicopters is represented by $\boldsymbol{P}_i$ (i =1,2,3), and the range of motion of each unmanned helicopter is shown in equation (17) ~ (19).The length of all three cables is L=2 m, the minimum pretension of each cable is $T_{\min}$ =1N, and the maximum allowable tension is $T_{\max}$ =200N. The mass of the weight is M=1 kg, and the position of the weight is expressed by B. The expected trajectory equation of the position of the weight B is shown in equation (20).

$$\begin{cases} -1.5m \le P_{x1} \le 1.5m \\ 0.5m \le P_{y1} \le 2.5m \\ 1m \le P_{z1} \le 2m \end{cases} \tag{17}$$

$$\begin{cases} -2.5m \le P_{x2} \le -0.1m \\ -2.5m \le P_{y2} \le -0.5m \\ 1m \le P_{z2} \le 2m \end{cases} \tag{18}$$

$$\begin{cases} 0.1m \le P_{x3} \le 2.5m \\ -2.5m \le P_{y3} \le -0.5m \\ 1m \le P_{z3} \le 2m \end{cases} \tag{19}$$

$$\begin{cases} B_x = 0.5\cos(0.4\pi t) \\ B_y = 0.5\sin(0.4\pi t) \quad 0 \le t \le 5(s) \\ B_z = 0.5 \end{cases} \tag{20}$$

Suppose p=3, and it can be obtained that the minimum tension of f is the minimum solution $\boldsymbol{T}_{low}$ when the desired trajectory of the weight is realized. $\boldsymbol{T}_{low}$ changing with time is shown in Fig.2 and the value of f changing with time is shown in Fig.3. At this point, the trajectory of each helicopter is in the plane Z=2m, the trajectory of the weight is in the plane Z=0.5m, and the projection on the XOY plane is shown in Fig.4.

Similarly, suppose p=3, it can be obtained that the maximum tension of f is the maximum solution $\boldsymbol{T}_{high}$ when the desired trajectory of the weight is realized. $\boldsymbol{T}_{high}$ changing with time is shown in Fig.5, and the value of f changing with time is shown in Fig.6. At this time, the trajectory of each helicopter is in the plane Z=1m, the trajectory of the weight is in the plane Z=0.5m, and its projection on the XOY plane is shown in Fig.7.

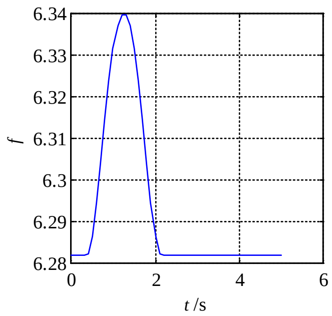


Fig. 2 $\boldsymbol{T}_{low}$ changing with time

Fig. 3 The value of f changing with time

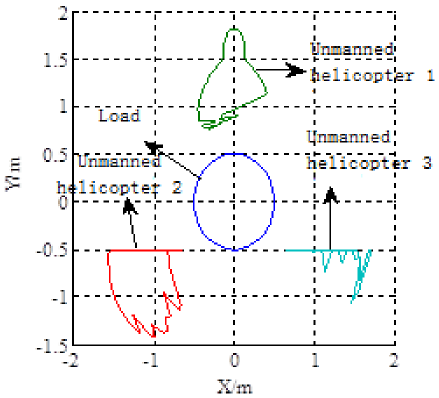
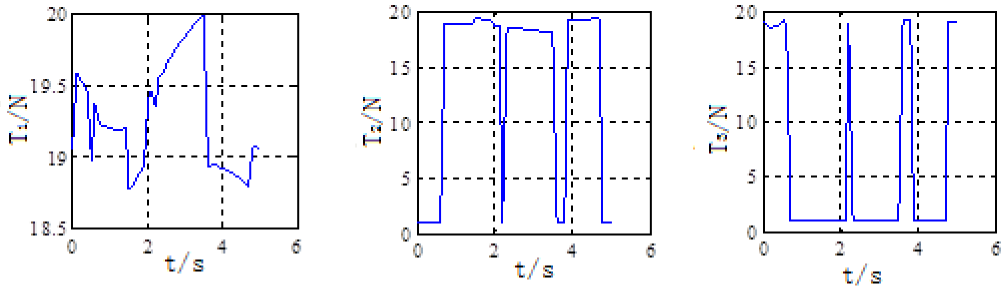


Fig. 4 Unmanned helicopters and trajectory of the load

It can be seen from Fig.2 that the tension of the three cables is continuous and smooth when $\boldsymbol{T}_{low}$ is reached. Therefore, the value of f in the corresponding Fig.3 is also continuous and smooth. In Fig.4, the positions of the unmanned helicopters are all in the plane of boundary Z=2m. This is because the angle between the cables and the Z-axis is small when the unmanned helicopters are at the highest point. The total force required is small when overcoming the same gravity. Therefore, the position of the unmanned helicopters are always at the highest point when $\boldsymbol{T}_{low}$ is reached.

It can be seen from Fig.5 that the tension of the three cables is continuous but not smooth when $\boldsymbol{T}_{high}$ is reached. Therefore, the value of f in the corresponding Fig.6 is also continuous but not smooth. In Fig.7, the positions of the unmanned helicopters are all in the plane of boundary Z=1m. This is because the angle between the cables and the Z-axis is large when the unmanned helicopters are at the lowest point. The total force required is large when overcoming the same gravity. Therefore, the position of the unmanned helicopters are always at the lowest point when $\boldsymbol{T}_{high}$ is reached.



(a) (b) (c)

Fig.5 $\boldsymbol{T}_{high}$ changing with time

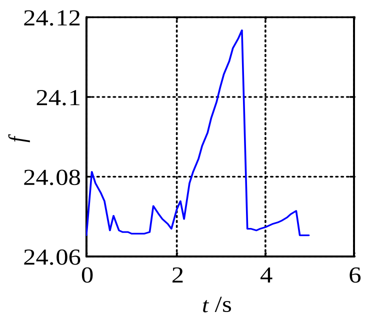


Fig.6 The corresponding f value of $\boldsymbol{T}_{high}$ changing with time

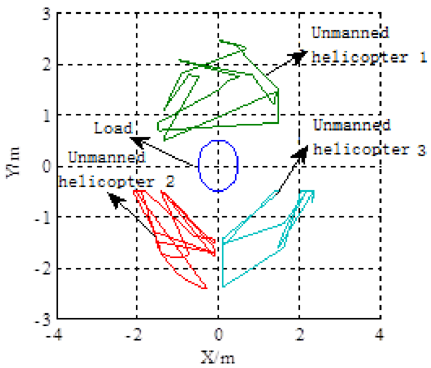


Fig.7 Unmanned helicopters and trajectory of the load

Comparing Fig.2 and Fig.5, it can be seen that in the lowest solution, the tension is between 4 ~ 5N, while in the highest solution, the tension is between 1 ~ 20 N. Sometimes in individual cables, $\boldsymbol{T}_{high}$ is lower than $\boldsymbol{T}_{low}$. It is because when the $\boldsymbol{T}_{high}$ is reached, $f = \|\boldsymbol{T}\|_p$ should be the largest, rather than maximizing the tension of individual cable. Comparing Fig.3 and Fig.6, it can be seen that in Fig.3 the value of *f* maintains between 6.28 ~ 6.34, while in Fig.6 the value of *f* maintains between 24.06 ~ 24.12.

5.2. Simulation of Special Case

Three unmanned helicopters were used to lift a load of 6 degrees of freedom by cables as example. According to previous discussion, there may be no solution at this time. In order to ensure that there is a solution, the trajectory of six degrees of freedom of the load is divided into two steps. First, there are 3 translational degrees of freedom and then 3 degrees of freedom of position and posture change, finally the real value of the tension is obtained. The cable tension *T* is approximated by the least squares method as the optimal solution to realize the desired position and posture of the load as shown in equation (21). Suppose the load be a rigid equilateral triangle object and the connection point between the cable and the load be the vertex of the triangle. The distance from the centroid of the load to the vertex of the triangle is 0.1m, and the weight of the load is 1kg.The expected trajectory of the load is shown in Fig.8.

$$\begin{cases} x = 0.5\sin(6\pi t/10) \\ y = 0.5\cos(6\pi t/10) + 0.3t \\ z = 0.05t \\ \alpha = \pi \times t/100 \\ \beta = \pi \times t/100 \\ \gamma = \pi \times t/100 \\ 0 \le t \le 10(s) \end{cases} \tag{21}$$

The trajectory of the three unmanned helicopters is shown in equation (22).

$$\begin{cases} P_{x1} = 0 \\ P_{y1} = 5/\sqrt{3} + 0.3\times t \\ P_{z1} = 1.5 \end{cases}$$
$$\begin{cases} P_{x2} = -2.5 \\ P_{y2} = -5\sqrt{3}/6 + 0.3\times t \\ P_{z2} = 1.5 \end{cases} \quad (22)$$
$$\begin{cases} P_{x3} = 2.5 \\ P_{y3} = -5\sqrt{3}/6 + 0.3\times t \\ P_{z3} = 1.5 \end{cases}$$
$$0 \le t \le 10(s)$$

Suppose for each cable, the minimum pretightening force T_{min} =1N and the maximum allowable tension T_{max} =200N. Equation (15) or (16) can be used to obtain the change of the cable tension with time, as shown in Fig.9, and the change of the corresponding cable length is shown in Fig.10.

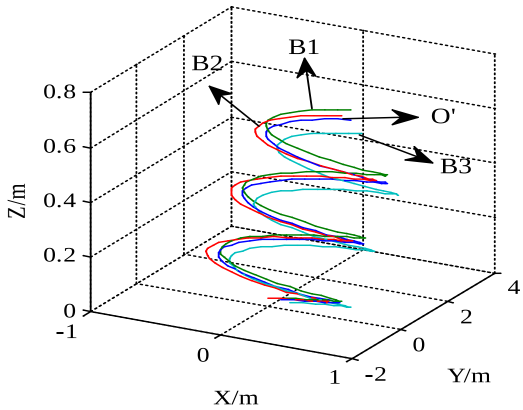


Fig. 8 Trajectory of the load

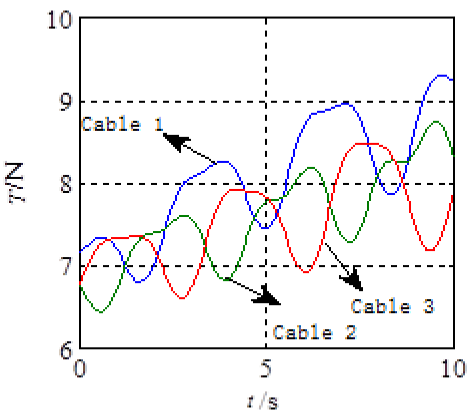


Fig.9 The change of cable tension

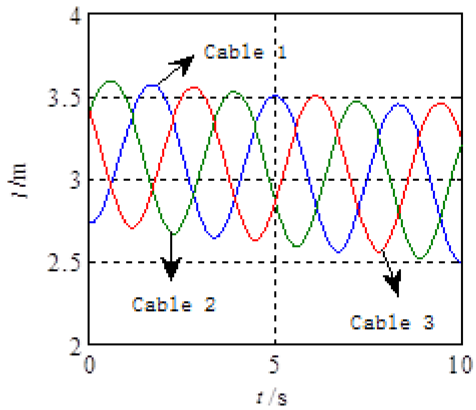


Fig. 10 The change of cable length

It can be seen from equation (21) and (22), as well as in Fig.8, the load is helically moved with the unmanned helicopters. Therefore, it is shown in Fig.9 and Fig.10 that the changes of cable length and cable tension are similar to sine curve or cosine curve. At the same time, it can be seen from Fig.9 that the tension of each cable is within the range of minimum pretightening force and maximum allowable tension, which meets all the requirements. Therefore, the trajectory of the load can be approximately realized.

5. Conclusion

Based on the analysis of tension optimization of cable-traction system with multi-helicopter in inverse dynamics, the following conclusions are drawn:

(1) In the inverse dynamics, if the constraint is not considered, infinite solutions exist in most cases, and the cable tension cannot be directly calculated. In order to ensure that the cable is always in a state of tension, the optimization of cable tension should be considered.

(2) In the process of tension optimization in inverse dynamics, in order to ensure that the cable tension is within a certain range, the lowest and highest acceptable solutions of the cable tension can be transformed into a nonlinear programming problem, and the final optimal solution can be expressed in the form of linear interpolation. The optimal solution can be selected by adjusting λ.

(3) In the process of transforming the lowest solution and the highest solution into a nonlinear programming problem, in order to ensure the continuity of optimal solution, the optimized objective function is approximately expressed by p-norm.

In this paper, the optimization is only carried out from the perspective of rope tension. In fact, through the analysis of the dynamics of the system, it can be seen that the flexible cable tension is not the only factor that affects the movement of the lifted object. The flexible cable tension is closely related to the movement of the helicopter and the length of the rope, that is to say, there are multiple inverse dynamics solutions of the system. Therefore, on the basis of the tension optimization, the optimization objective function can be changed Helicopter motion and rope length are optimized, which is convenient for system control.

Acknowledgements

This project is supported by National Natural Science Foundation of China (Grant No.51265021,11462011).

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